DM relic abundance via multi-Higgs production in the early universe

Seishi Enomoto (Kyushu Sangyo Univ.)

Based on 2504.17127 [hep-ph]

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Kohta Murase (Pennsylvania State Univ.)
Masato Yamanaka (Hosei Univ.)

[Illustrated by *ChatGPT*]

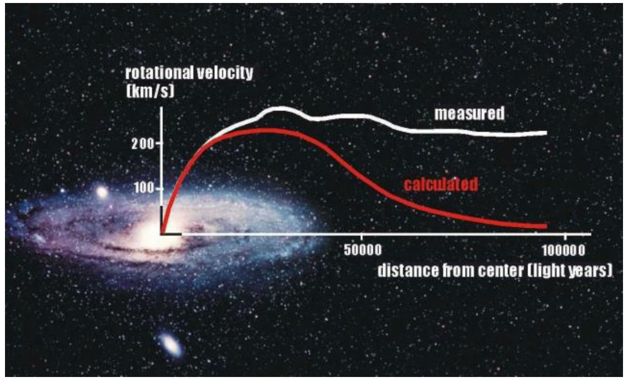
Outlook

1. Introduction
2. Application to DM relic abundance
    1. Model setup and formulation
    2. Numerical result via Boltzmann equation
3. Summary

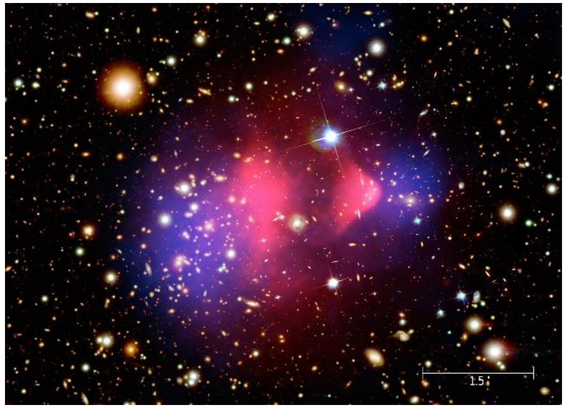
1. Introduction

Evidence of dark matter

- Galaxy rotation curve
- Bullet cluster
- Cosmic microwave background

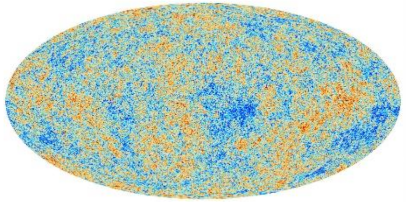


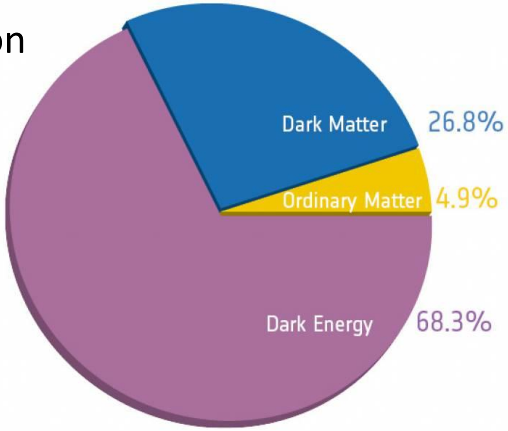
Disagreement in galaxy rotation curves (measured vs calculated)



Mismatch of the mass distribution from X-ray and gravitational lensing

CMB observation



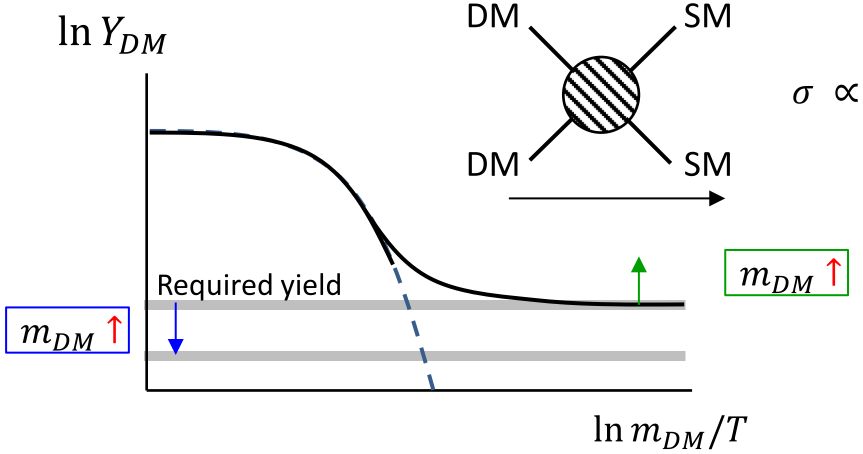


$$\Omega_{CDM} = 0.268$$

$$\Rightarrow \quad Y_{DM} \equiv \frac{n_{DM}}{s} = 4.36 \times 10^{-12} \cdot \frac{100\ \mathrm{GeV}}{m_{DM}}$$

$$\boxed{m_{DM} \uparrow \;\Rightarrow\; Y_{DM} \downarrow}$$

Thermal relic by freeze-out



$$\sigma \propto \frac{y^4}{m_{DM}^2}$$

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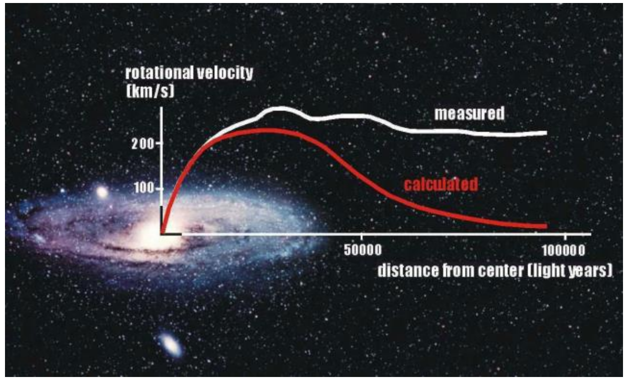

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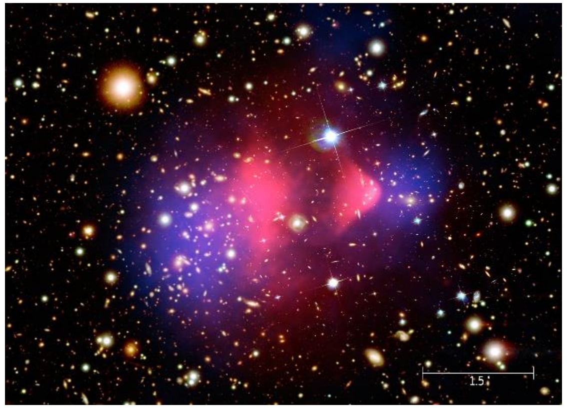

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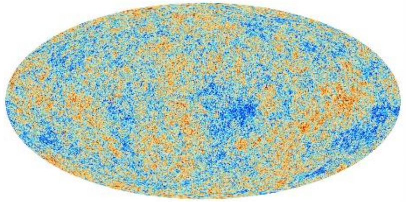

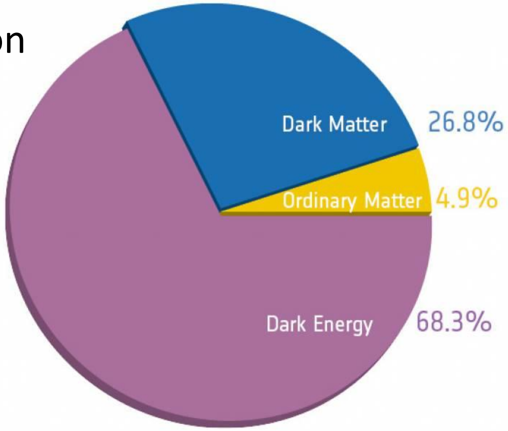

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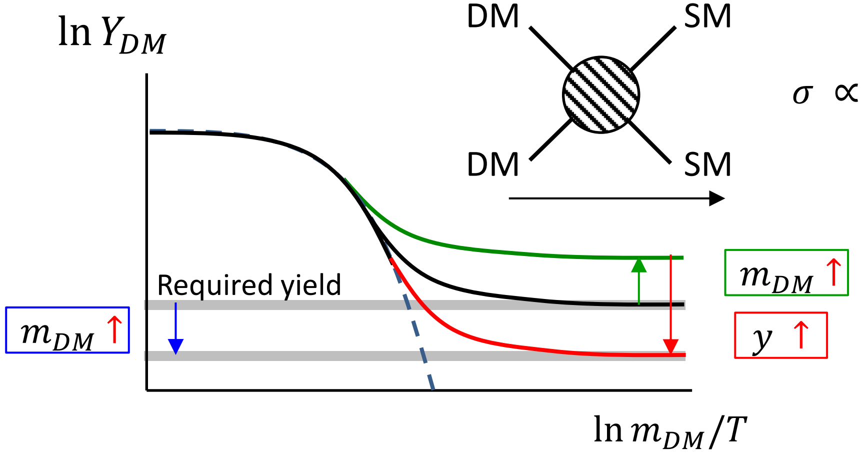

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Worry about the unitarity bound…

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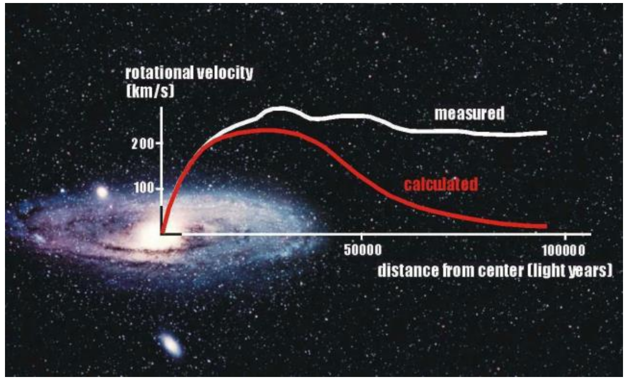

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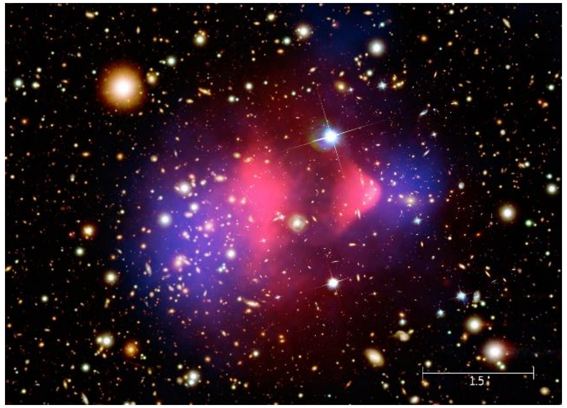

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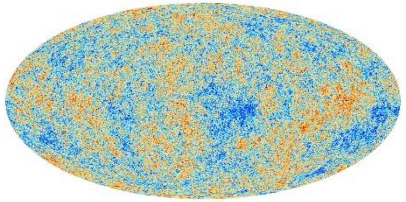

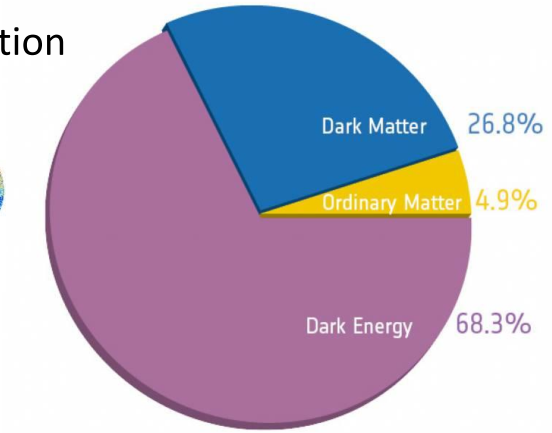

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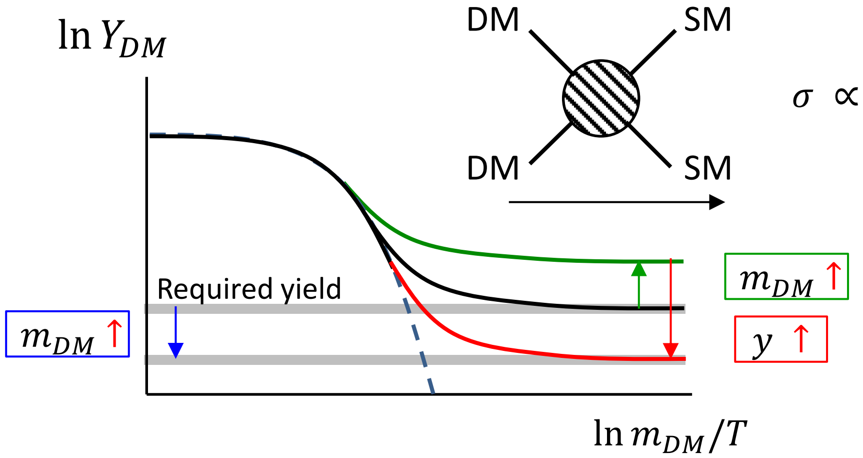


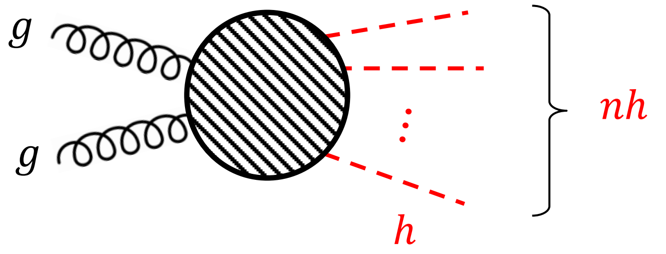
$$\sigma \propto \frac{y^4}{m_{DM}^2}$$

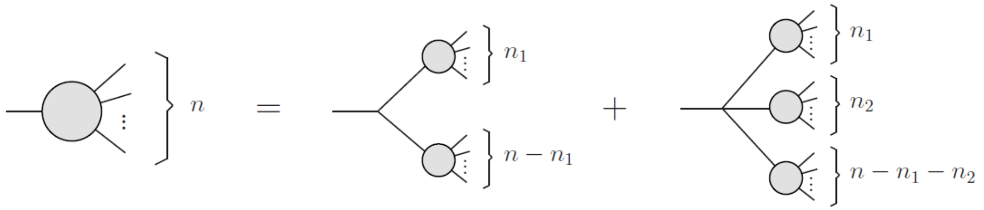
Worry about the unitarity bound...

$\boxed{\text{Is there a mechanism to enhance the cross-section?}}$ ← Higgsplosion!

Higgsplosion [C. Degrande, V. V. Khoze, O. Mattelaer; PRD 94 (2016)]

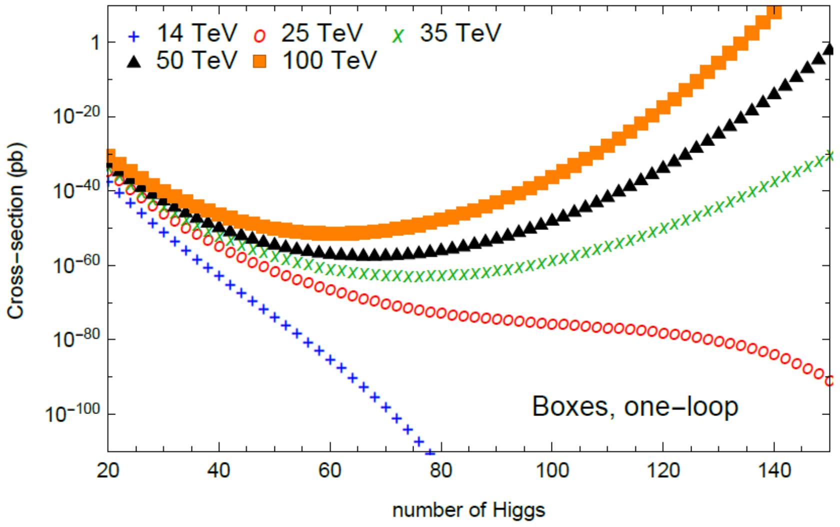
- A hypothesis that the explosive Higgs production is caused above a certain energy
- Essences:
  - bosonic statistical effect
  - (3- or 4-pt) self-interaction





$$
\begin{aligned}
i\mathcal{M}(1\to n) \;=\; & \sum_{n_1=1}^{n-1}\left(-i\sqrt{\frac{9}{2}\lambda m_\varphi^2}\right)\cdot\frac{n!}{n_1!(n-n_1)!}\times\frac{i}{s_{n_1}-m_\varphi^2}\cdot i\mathcal{M}(1\to n_1)\\
& \times\frac{i}{s_{n-n_1}-m_\varphi^2}\cdot i\mathcal{M}(1\to n-n_1)\\
& +\sum_{n_1=1}^{n-2}\sum_{n_2=1}^{n-n_1-1}(-i\lambda)\cdot\frac{n!}{n_1!n_2!(n-n_1-n_2)!}\;\frac{i}{s_{n_1}-m_\varphi^2}\cdot i\mathcal{M}(1\to n_1)\\
& \times\frac{i}{s_{n_2}-m_\varphi^2}\cdot i\mathcal{M}(1\to n_2)\;\;\frac{i}{s_{n-n_1-n_2}-m_\varphi^2}\cdot i\mathcal{M}(1\to n-n_1-n_2)
\end{aligned}
$$

$$\boxed{\mathcal{M}(h^*\to nh)\;\sim\;\lambda^{n/2}\,n!\cdot f_n(s)}$$



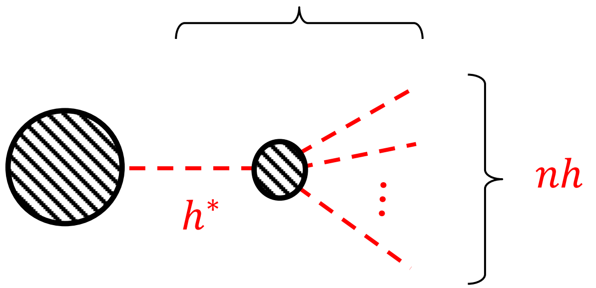
- Blow up around $\sigma(gg\to 60h)$
- $\sigma(gg\to h)\sim\sigma(gg\to 135h)$

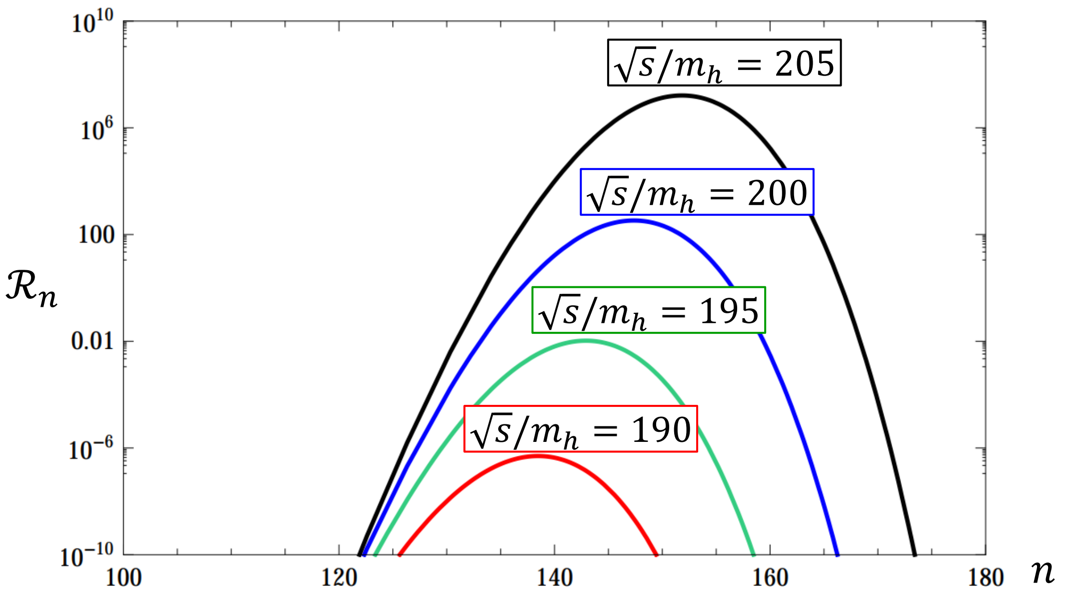
"Decay rate" for $h^* \to nh$ near the n-particle mass threshold ($\sqrt{s} \sim nm_h$)

[V. V. Khoze and M. Spannowsky; NPB 926 (2018)]

$$\mathcal{R}_n \equiv \frac{\Gamma_{h^*\to nh}(s)}{m_n} = \exp\left[n\left(\ln\frac{\lambda n}{4e} + + \frac{3}{2}\ln\frac{e\epsilon}{3\pi} - \frac{25}{12}\epsilon + 3.02\sqrt{\lambda n}\right)\right]$$

$$\left(\epsilon \equiv \frac{\sqrt{s} - nm_h}{nm_h} \ll 1\right)$$

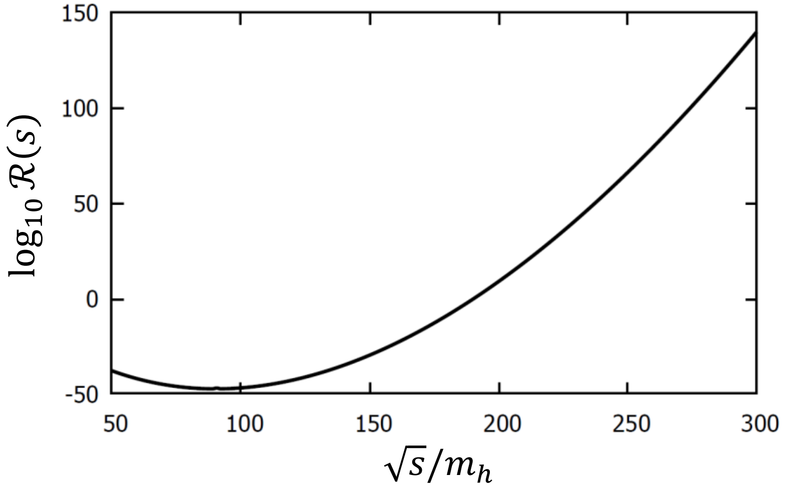




Total rate

$$\mathcal{R}(s) \equiv \sum_n \mathcal{R}_n \cdot \theta(\sqrt{s} > nm_h)$$

Exp growing for high s



We consider the higgsplosion effect in the DM relic abundance

2. Application to DM relic abundance

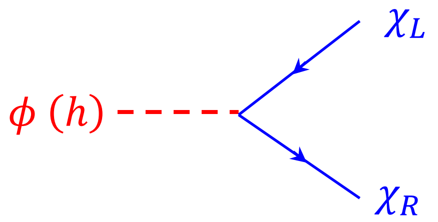
Model setup (higgs portal DM model)

Can we naturally derive this setup…?
→ Later if I have time

Lagrangian (ϕ: Higgs, χ_L, χ_R: DMs)

$$\mathcal{L} \supset \underbrace{\frac{1}{2}(\partial\phi)^2 - \frac{1}{4}\lambda(\phi^2 - v^2)^2}_{\text{Higgs part}} + \underbrace{\bar{\chi}(\not{\partial} - m_\chi)\chi}_{\text{DM part}} - \underbrace{\left(y_\chi \phi \cdot \overline{\chi_R}\chi_L + (h.c.)\right)}_{\text{Higgs-DM interaction}}$$

(y_χ : complex Yukawa coupling)



$\Downarrow$ (SSB: $\phi = v + h$)

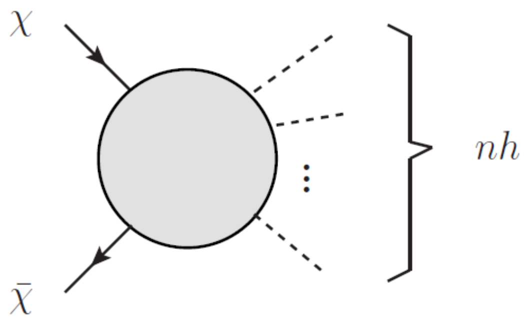
$$= \frac{1}{2}(\partial h)^2 - \frac{1}{2}m_h^2 h^2 - \lambda v h^3 - \frac{1}{4}\lambda h^4 + \bar{\chi}(\not{\partial} - M_\chi)\chi - h\left(y_\chi \cdot \overline{\chi_R}\chi_L + (h.c.)\right)$$

($M_\chi \equiv m_\chi + y_\chi v$: complex)

$\Downarrow$ (Re-phasing: $\chi_L e^{i \arg M_\chi} \rightarrow \chi_L$)

$$= \frac{1}{2}(\partial h)^2 - \frac{1}{2}m_h^2 h^2 - \lambda v h^3 - \frac{1}{4}\lambda h^4 + \bar{\chi}(\not{\partial} - |M_\chi|)\chi - h\left(\tilde{y}_\chi \cdot \overline{\chi_R}\chi_L + (h.c.)\right)$$

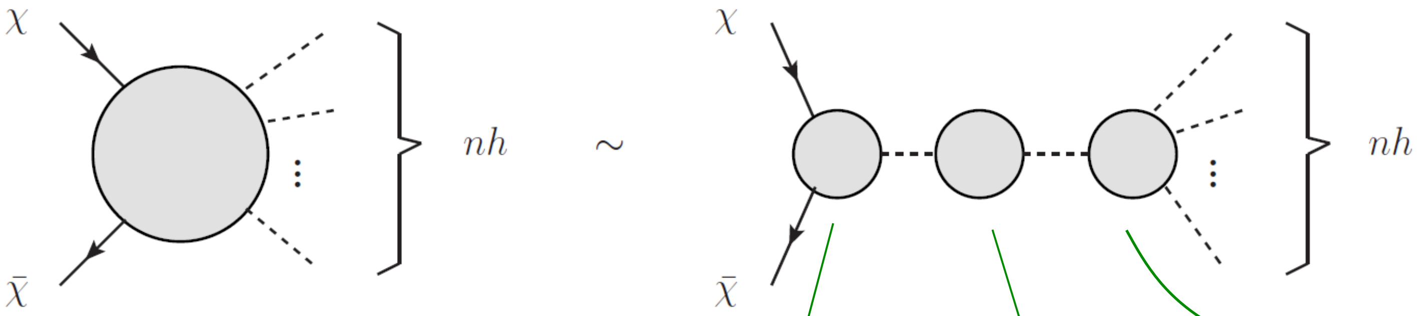
($\tilde{y}_\chi \equiv y_\chi e^{-i \arg M_\chi}$: complex)



Scattering amplitude

$$\mathcal{L} \supset -\tilde{y}_\chi h \overline{\chi_R}\chi_L$$

$$\theta_{\tilde{y}} \equiv \arg \tilde{y}_\chi$$



$$\sum_{\text{spins}} |\mathcal{M}(\chi\chi \to nh)|^2 \sim \sum_{\text{spins}} \left| \mathcal{M}(\chi\chi \to h) \frac{1}{s - M_h(s)^2 - iM_h(s)\Gamma_h(s)} \mathcal{M}(h \to nh) \right|^2$$

$M_h(s) \sim m_h$

$s \gg m_h$

$$\sim 2|\tilde{y}_\chi|^2 (s - 4|M_\chi|^2 \cos^2\theta_{\tilde{y}}) \frac{1}{s^2 + m_h^2 \Gamma_h(s)^2} |\mathcal{M}(h \to nh)|^2 ,$$

$$\sum_n \int d\Pi_n \times \ldots$$

$\sim \Gamma_h(s)$

$\theta_{\tilde{y}} = 0$: p-wave

$\boxed{\theta_{\tilde{y}} = \pm\pi/2 : \ s\text{-wave}}$

for the maximized cross section

Scattering amplitude

$$\mathcal{L} \supset -\tilde{y}_\chi h \overline{\chi_R} \chi_L$$

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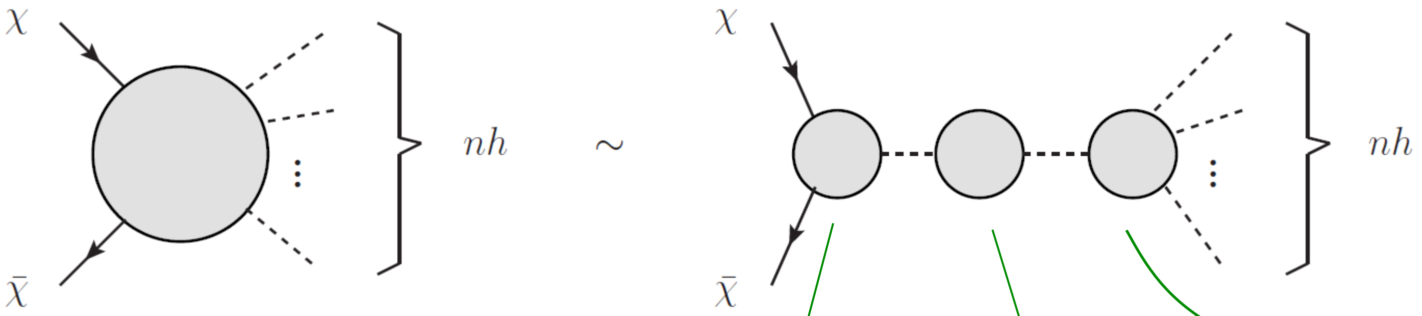

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$M_h(s) \sim m_h$

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$\sum_n \int d\Pi_n \times \ldots$

$\sim \Gamma_h(s)$

$\theta_{\tilde{y}} = 0 :$ p-wave

$\boxed{\theta_{\tilde{y}} = \pm\pi/2 : \; s\text{-wave}}$

for the maximized cross section

Forms a "window"

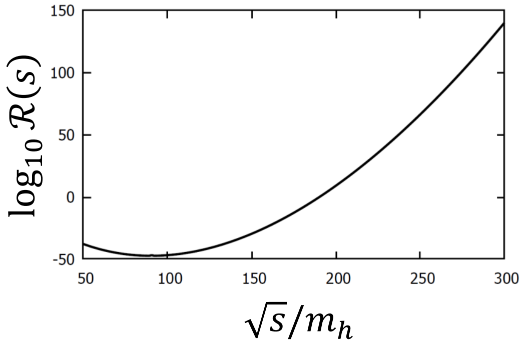
Cross section

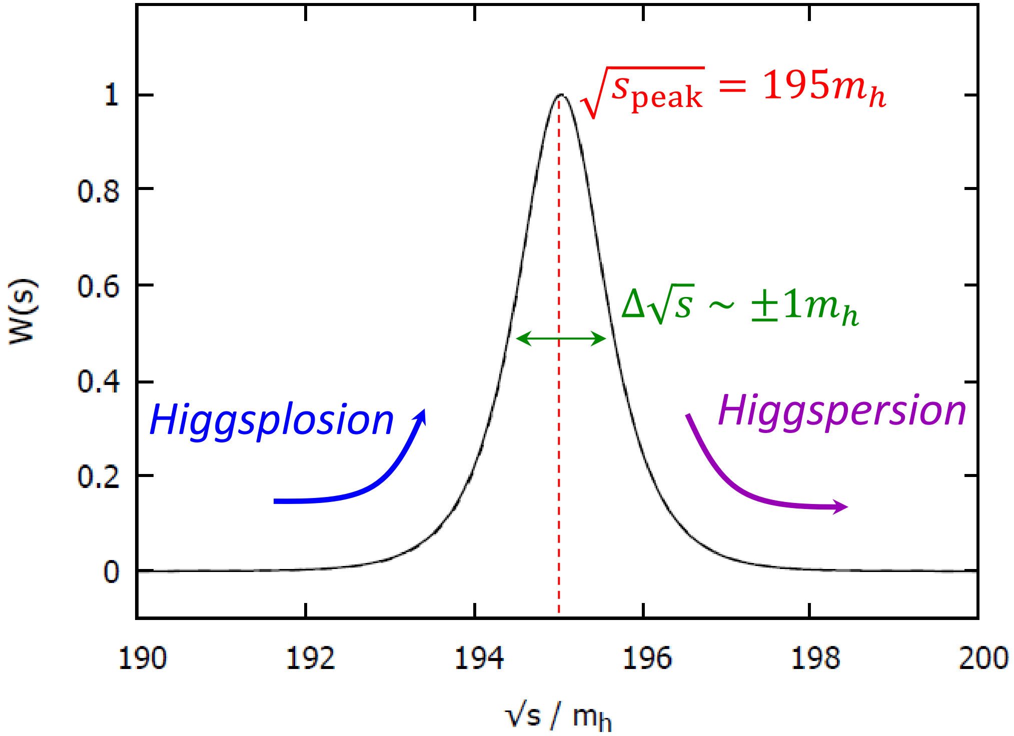
$$\sigma = \sum_n \sigma_{\chi\chi\to nh} = \frac{1}{\sqrt{s(s-4|M_\chi|^2)}} \cdot \frac{|\tilde{y}_\chi|^2}{4}\left(1 - \frac{4|M_\chi|^2\cos^2\theta_{\tilde{y}}}{s}\right) \underline{\underline{W(s)}}$$

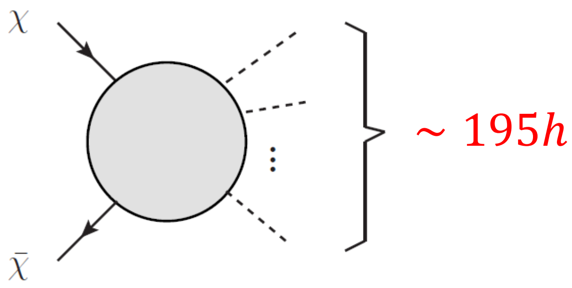
"Window" function

$$W(s) = \frac{2sm_h^2\mathcal{R}(s)}{s^2 + m_h^4\mathcal{R}(s)^2}, \qquad 0 \le W(s) \le 1$$

$\left(\mathcal{R}(s) \equiv \Gamma_h(s)/m_h \right)$







$\sqrt{s} \sim 2M_\chi \sim 195m_h$

$\therefore M_\chi \sim 100\, m_h$

for the maximized cross section

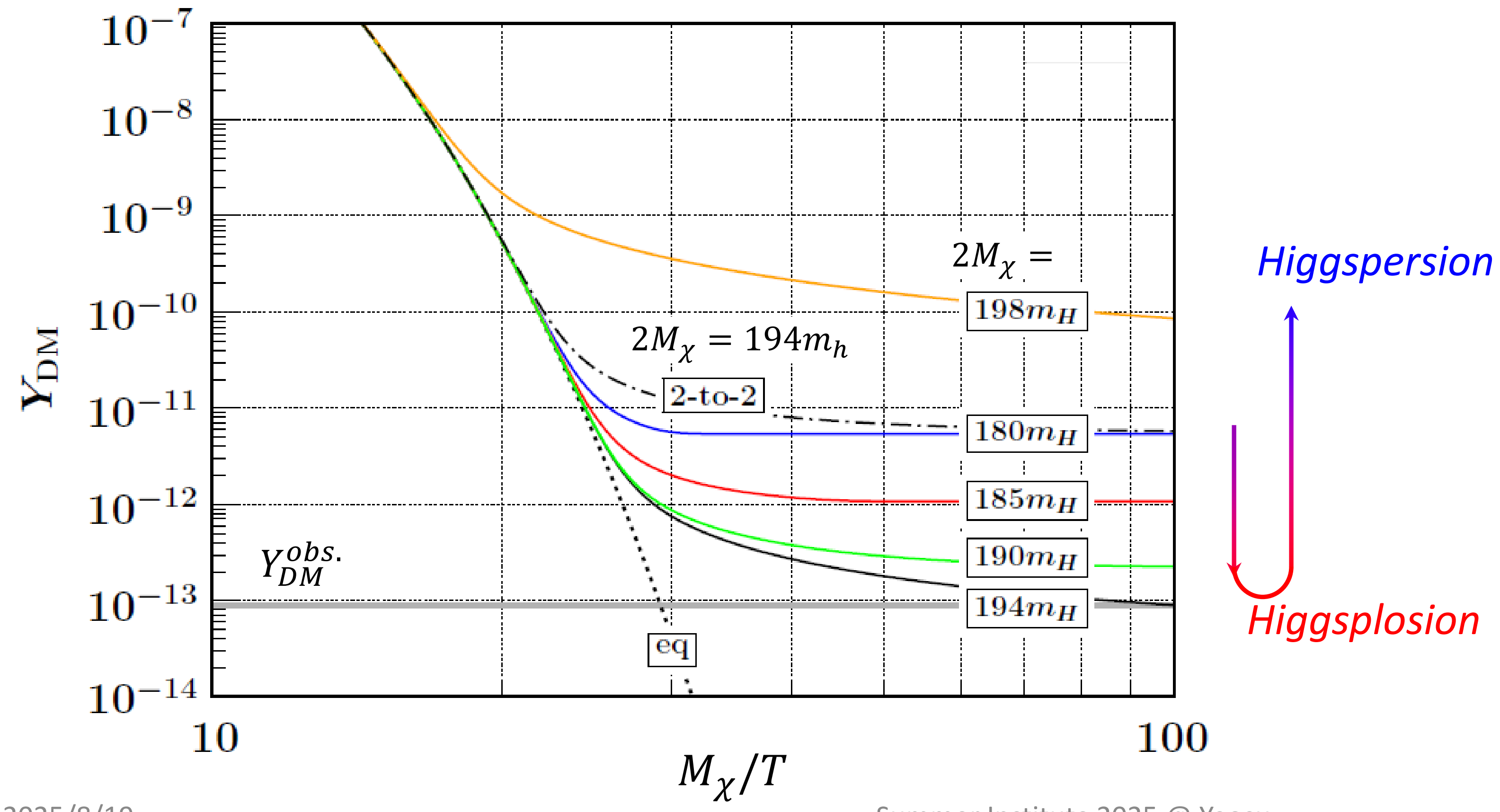
DM (thermal) relic abundance

$$\left(\mathcal{L} \supset -\tilde{y}_\chi h \overline{\chi_R}\chi_L\right)$$

- Boltzmann equation: $\dot{n}_\chi + 3Hn_\chi = -\langle\sigma v\rangle\left(n_\chi^2 - \left(n_\chi^{eq}\right)^2\right)$

- Numerical result

Evolution of DM abundance $(\tilde{y}_\chi = 1.3i)$



- Scale of freeze-out

$T \sim M_\chi/25$

$\sim 4m_h$ $\quad M_\chi \sim 100m_h$

~ 500 GeV $\quad (m_h = 125$ GeV$)$

Failed (∵ before SSB)

$T \sim 200$ GeV $\quad (m_h = 50$ GeV$)$

OK (during SSB)

$\Rightarrow \quad M_\chi \sim 100m_h \sim$ 5 TeV

A model to provide our setup

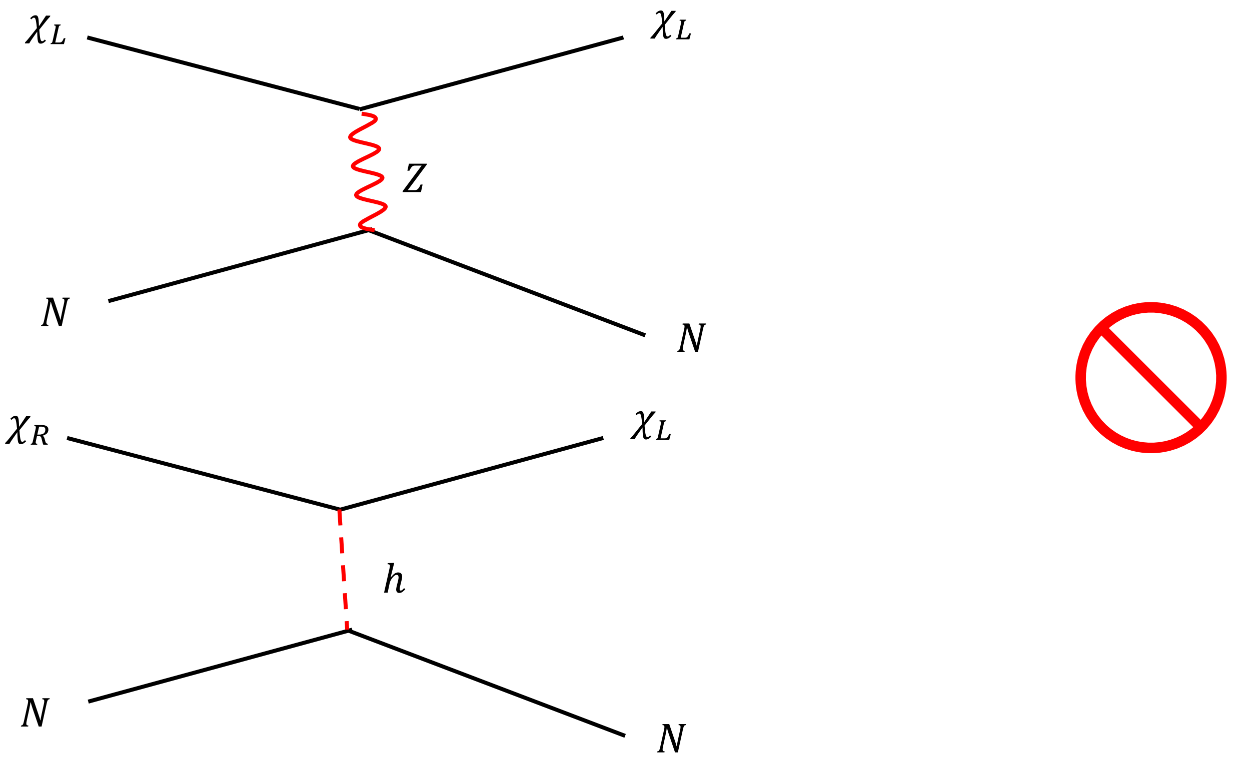
Simple model fails:

$$\mathcal{L} \supset \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}\lambda(\phi^2 - v^2)^2 + \bar{\chi}(\partial\!\!\!/ - m_\chi)\chi - \Big(y_\chi \underset{SU(2)}{\phi} \cdot \overline{\chi_R}\underset{SU(2)}{\chi_L} + (h.c.)\Big)$$

(ϕ: Higgs, χ_L, χ_R: DMs)

$$\Rightarrow \quad \mathcal{L} \supset g\bar{\chi}_L \not{Z} \chi_L$$

Ruled out by the direct DM search



Singlet-doublet DM model (+ complex parameters)

[T. Cohen, J. Kearney, A. Pierce, and D. Tucker-Smith; PRD85 (2012)]

Contents	$SU(2)_L$	$U(1)_Y$	Z_2
Ψ	**1**	0	−
$f_L = \begin{pmatrix} f^0 \\ f^- \end{pmatrix}$	**2**	$-1/2$	−
$f_R^c = \begin{pmatrix} f'^+ \\ f'^0 \end{pmatrix}$	**2**	$+1/2$	−

$$\mathcal{L} \supset -\frac{1}{2} m_\psi \psi\psi - m_D f_R^c f_L - y_L \psi f_L \cdot H - y_R \psi f_R^c \cdot H^\dagger + (h.c.)$$

(m_ψ: complex, others: real)

SSB ($H = (0, v+h)^T/\sqrt{2}$)

$$\supset -\frac{1}{2} (\psi\ f^0\ f'^0)\, \mathbf{M} \begin{pmatrix} \psi \\ f^0 \\ f'^0 \end{pmatrix}$$

$$\left(\mathbf{M} = \begin{pmatrix} m_\psi & m_L & m_R \\ m_L & 0 & m_D \\ m_R & m_D & 0 \end{pmatrix}, \quad m_{L(R)} = \frac{1}{\sqrt{2}} y_{L(R)} v \right)$$

$$= -\frac{1}{2} (\chi_1\ \chi_2\ \chi_3) \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix} \qquad (m_1 \le m_2 \le m_3)$$

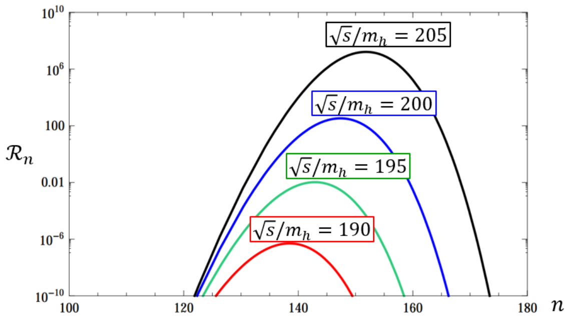
Case of $\mathrm{Re}\, m_\psi = 0,\ m_L = m_R,$ and $m_D \gg |m_\psi|, m_L, m_R$

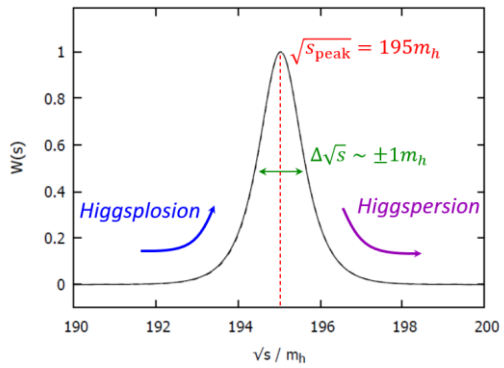
- $m_1 = |m_\psi| \ll m_2 \sim m_3 \sim m_D$: χ_1 is LSP = DM
- Coefficients relating to the direct detection: $\mathcal{L} \supset c_h \cdot h \bar{\chi}_1 \chi_1 + c_Z \cdot \bar{\chi}_1 \not{Z} \gamma^5 \chi_1$

$$c_h \sim \frac{2 m_L m_R}{m_D v} \frac{\mathrm{Re}\, m_\psi}{|m_\psi|} = 0, \quad c_Z \sim \frac{g_2}{4 \cos\theta_W} \frac{|m_L^2 - m_R^2|}{m_D^2} = 0$$

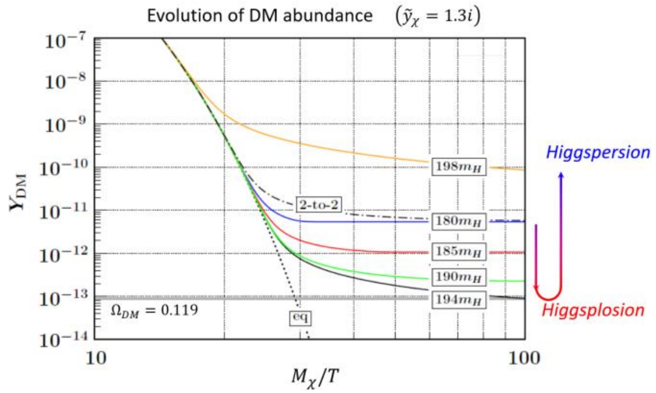
4. Summary

- Higgsplosion:
  - Explosive Higgs production occurs above a certain energy
    - $\sigma(gg \to h) \sim \sigma(gg \to 135h)$ @ 100 TeV p-p collider
    - must be confirmed at the future experiments





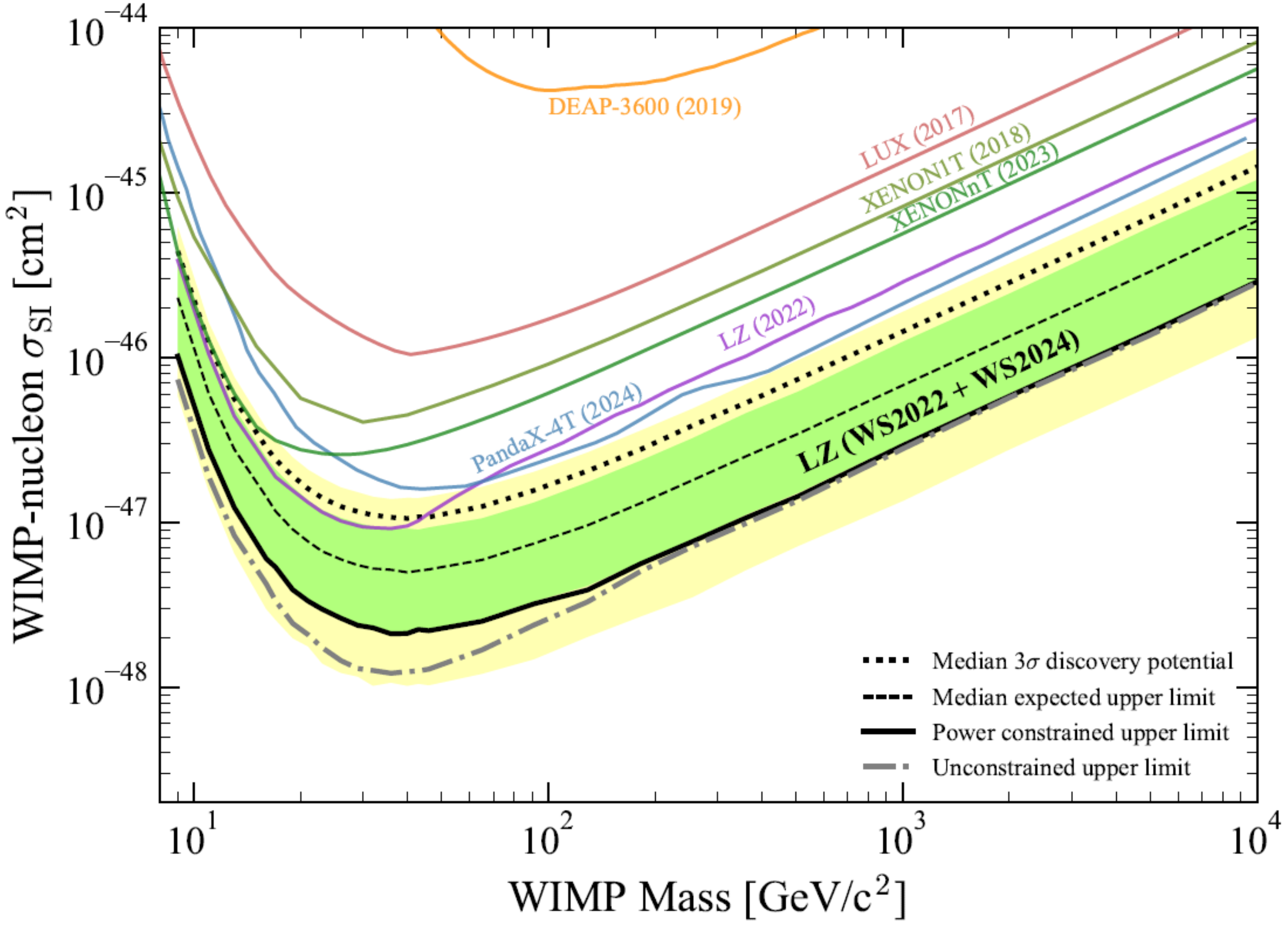
- Application to DM relic abundance
  - The "decay rate" including the higgsplosion forms a *window* in the cross section
  - Higgsplosion effect is maximized at $\sqrt{s} = 195 m_h$
  - $M_{DM} \sim 5\ \mathrm{TeV}$ higgs portal model is possible if $m_h \sim 50$ GeV at the freeze-out scale (during SSB, $T \sim 200$ GeV)
  - Singlet-doublet model is better to evade the direct detection



Backup

Present bound for DM : Naïve WIMP scenario is almost ruled out

Direct search [LZ collabolation, 2410.17036 [hep-ex]]

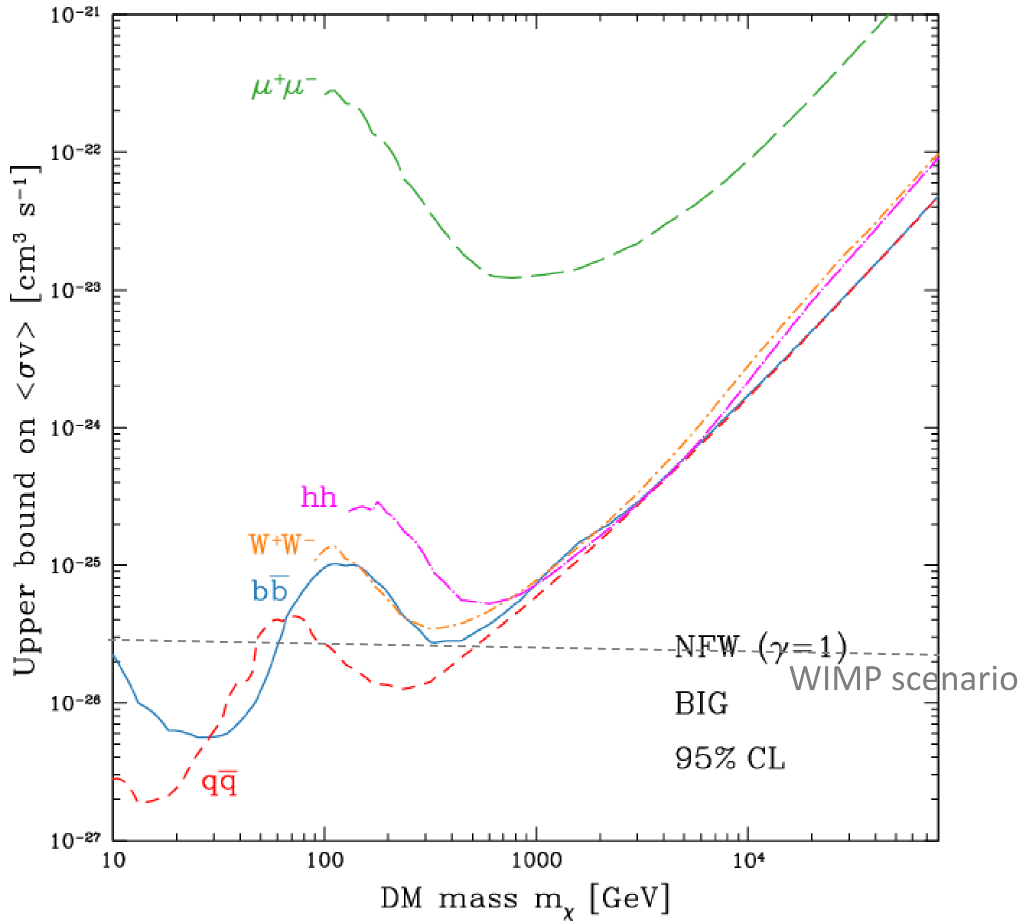


Typical scale by WIMP: $\sigma_{SI} \sim 10^{-44}\ \mathrm{cm}^2$

→ DM-SM coupling must be quite small

Indirect search

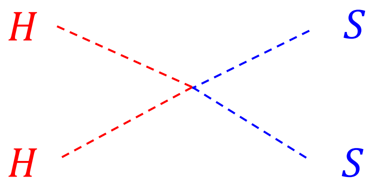
[F. Calore *et al.*, SciPost Phys. 12 (2022) 5, 163]



Possible model: Higgs portal dark matter scenario + resonance

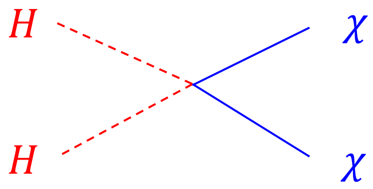
[G. Arcadi, A. Djouadi, M. Kado, EPJC 81 (2021) 7, 653]]

- Scalar DM model

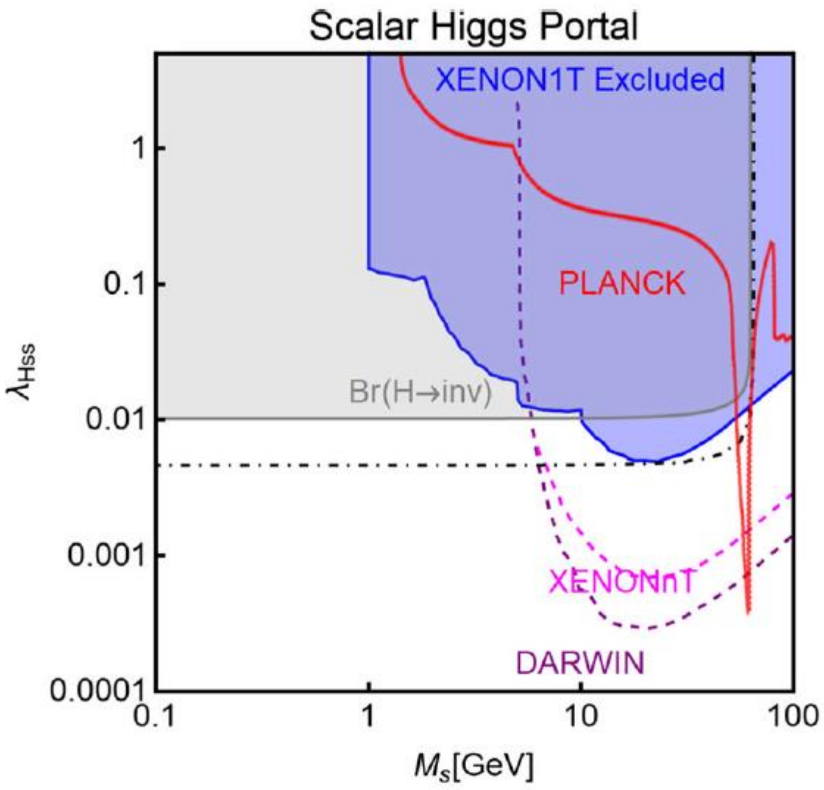


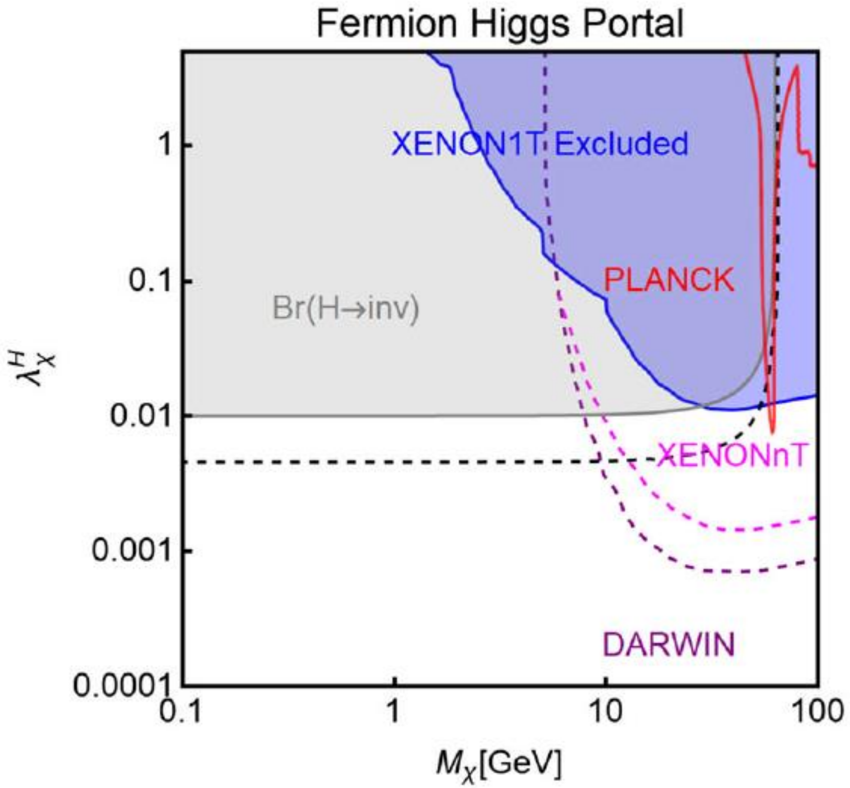
$= -i\lambda_{HSS}$

- Fermion DM model

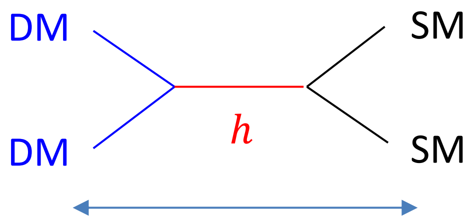


$= -i\,\frac{\lambda_\chi^H}{\Lambda}$





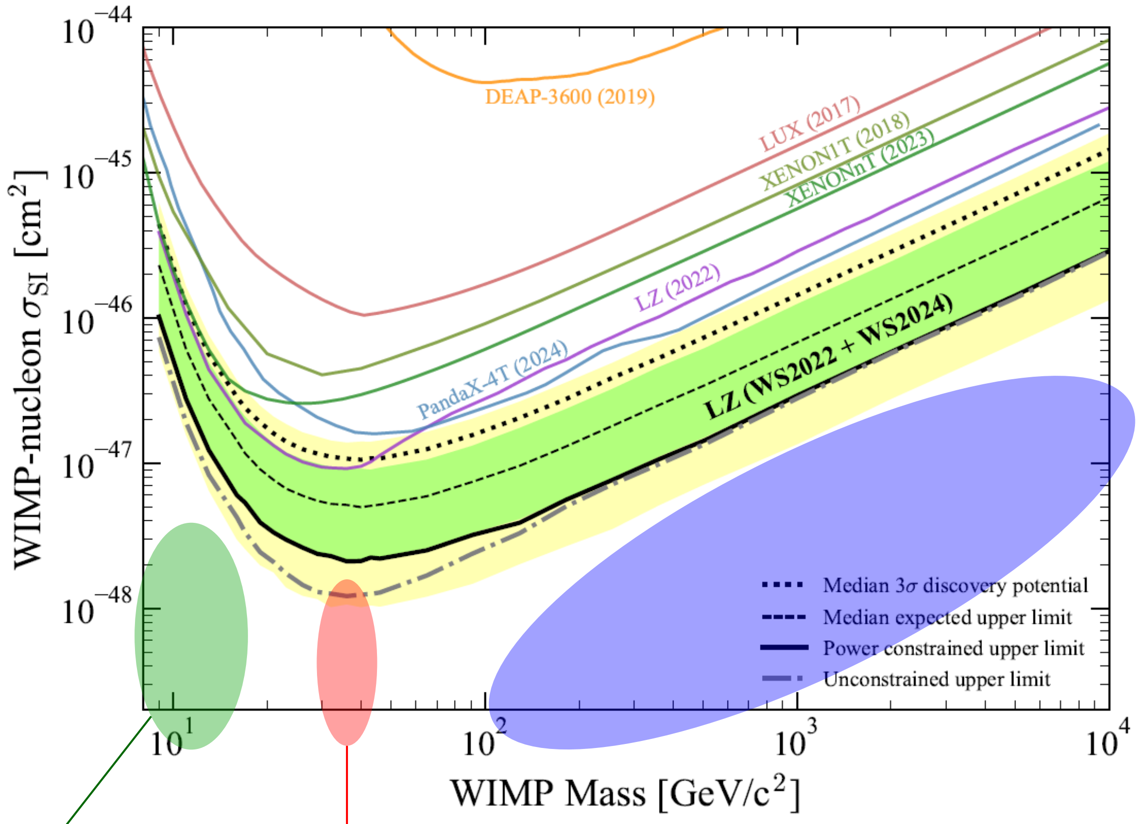
$\boxed{m_{DM} \sim m_H/2 \text{ is allowed}}$



DM annihilation cross section is enhanced

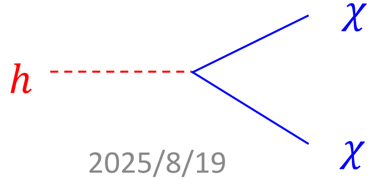
→ Small coupling is allowed

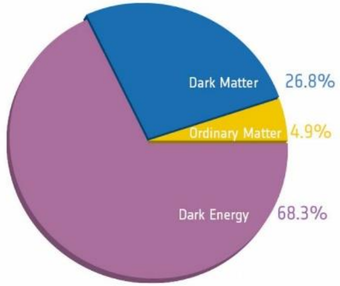
How is the other mass range?



Possible by the resonant enhancement

Limited by collider exp.
because $h \to$ 2DM opens





$$\Omega_{CDM} = 0.268$$

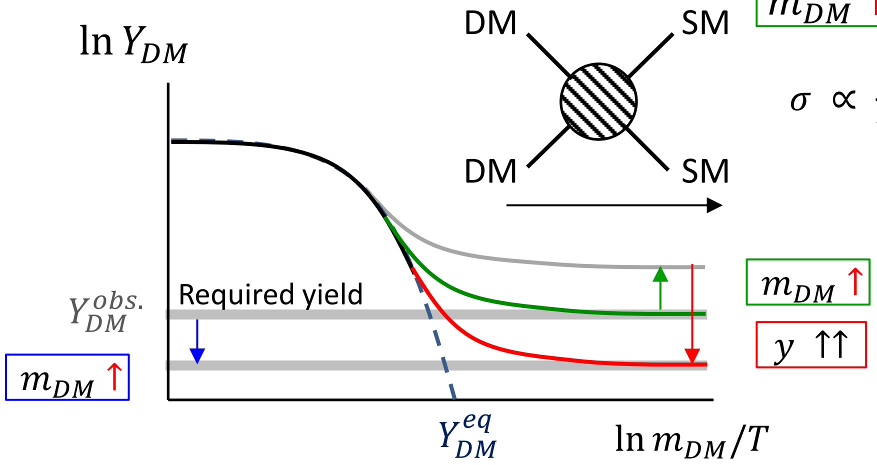
$$\Rightarrow \quad Y_{DM} \equiv \frac{n_{DM}}{s} = 4.36 \times 10^{-12} \cdot \frac{100\ \text{GeV}}{m_{DM}}$$

$$\boxed{m_{DM} \uparrow \ \Rightarrow \ Y_{DM} \downarrow}$$

Thermal relic by freeze-out

$$\boxed{m_{DM} \uparrow \ \Rightarrow \ \sigma \downarrow}$$

$$\sigma \propto \frac{y^4}{m_{DM}^2}$$



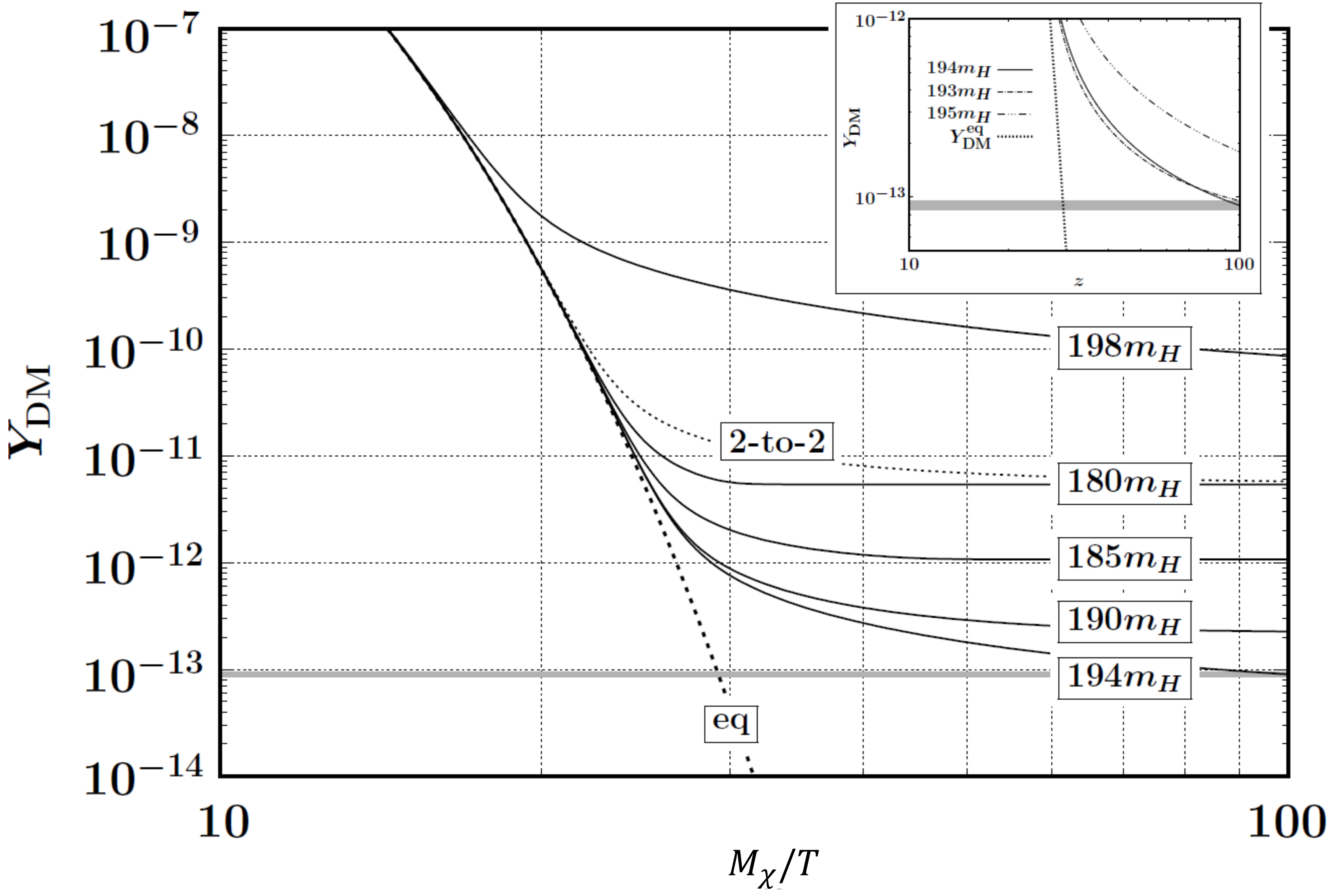
- Too large a coupling breaks the unitarity…
- Heavier DM is impossible …?

→ Possible by Higgsplosion!

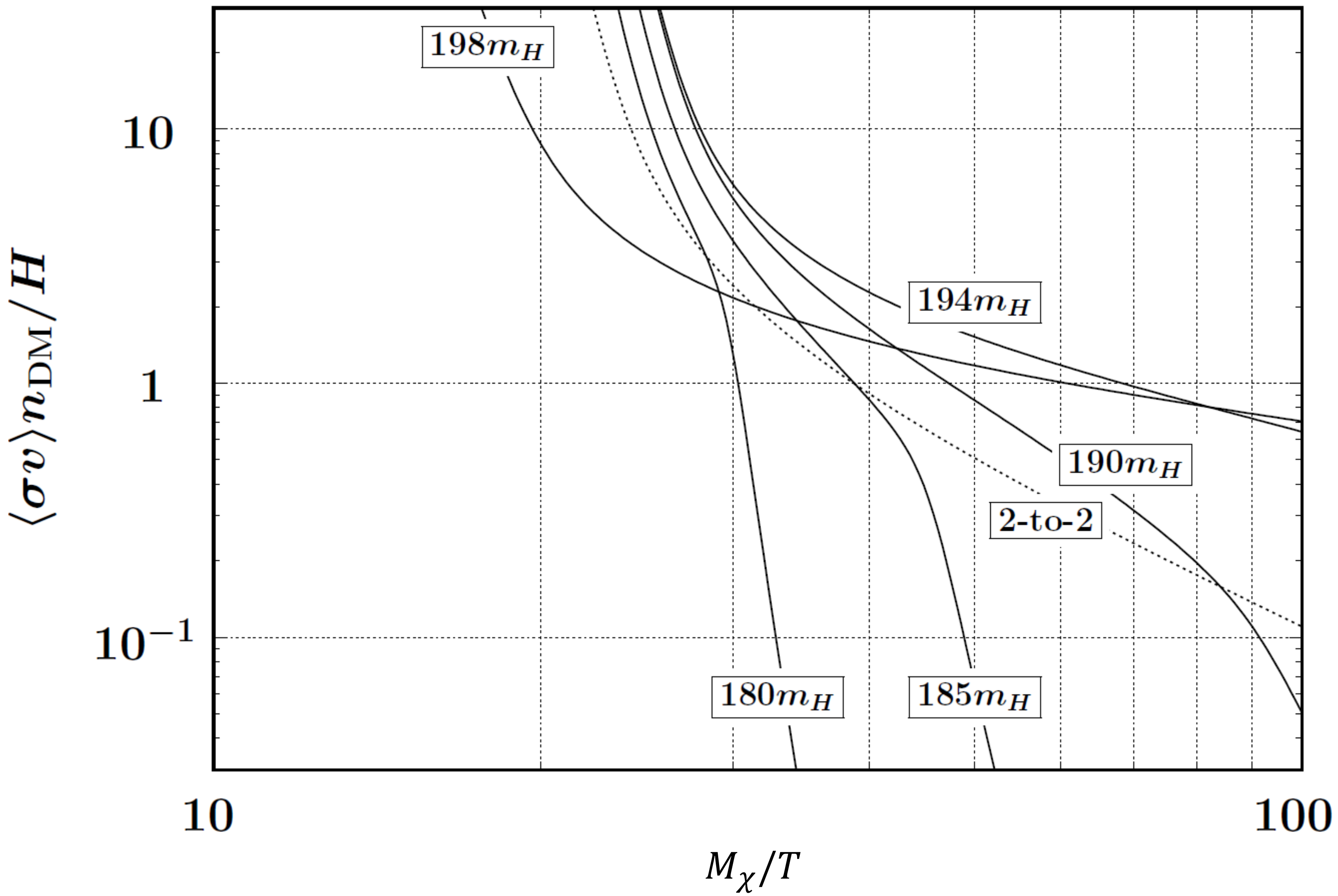
Cross sections via triangles, boxes, pentagons, …

	$\sigma_{gg\to hh}$	$\sigma_{gg\to hhh}$	$\sigma_{gg\to hhhh}$
Triangles	$y_t^2 \frac{m_t^2 M_h^2}{s^3} \log^4\left(\frac{m_t}{\sqrt{s}}\right) \frac{M_h^2}{v^2}$	$y_t^2 \frac{m_t^2}{s^2} \log^4\left(\frac{m_t}{\sqrt{s}}\right) \frac{M_h^4}{v^4}$	$y_t^2 \frac{m_t^2}{s^2} \log^4\left(\frac{m_t}{\sqrt{s}}\right) \frac{M_h^6}{v^6}$
Boxes	$y_t^4 \frac{1}{s}$	$y_t^4 \frac{1}{s} \frac{M_h^2}{v^2}$	$y_t^4 \frac{1}{s} \frac{M_h^4}{v^4}$
Pentagons	–	$y_t^6 \frac{m_t^2}{s^2} \log^4\left(\frac{m_t}{\sqrt{s}}\right)$	$y_t^6 \frac{m_t^2}{s^2} \log^4\left(\frac{m_t}{\sqrt{s}}\right) \frac{M_h^2}{v^2}$
Hexagons	–	–	$y_t^8 \frac{1}{s}$

Evolution of relic abundance

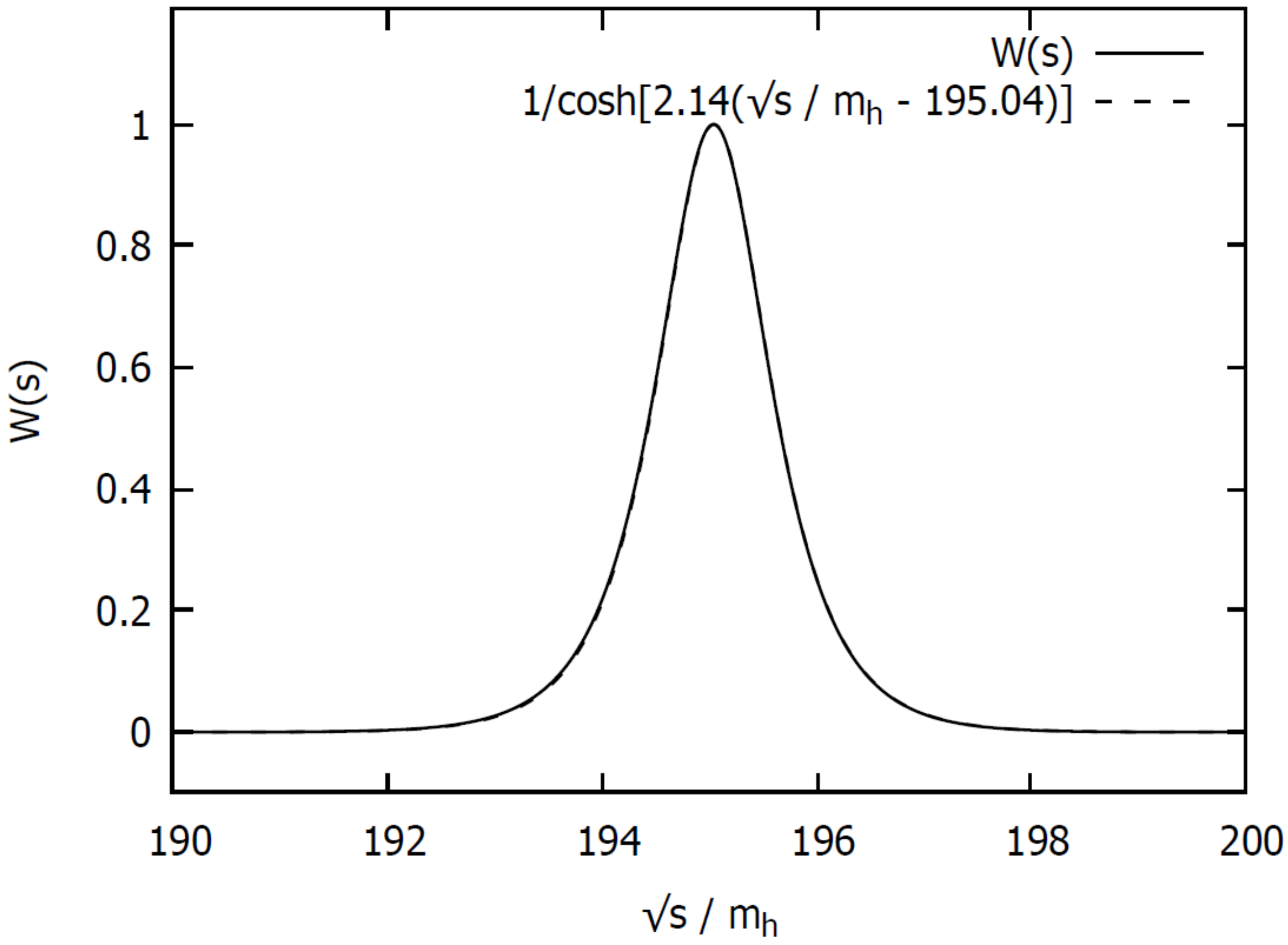


Reaction rate v.s. Hubble



How do we evaluate analytically?

EOM for wave function



Singlet-doublet DM model (+ complex parameters)

[T. Cohen, J. Kearney, A. Pierce, and D. Tucker-Smith; PRD85 (2012)]

$$\mathcal{L} \supset -\frac{1}{2}\overline{\Psi}(m_\psi P_L + m_\psi^* P_R)\Psi - m_D(\overline{f_R}f_L + \overline{f_L}f_R) - (\overline{\Psi}(y_L P_L + y_R P_R)H\cdot f + (h.c.))$$

(m_ψ: complex, others: real)

($\Psi = \Psi^c$: singlet fermion, $f_L = \begin{pmatrix} f_L^- \\ f_L^0 \end{pmatrix}$, $f_R = \begin{pmatrix} f_R'^- \\ f_R'^0 \end{pmatrix}$: $SU(2)_L$ doublet fermions)

$$\Rightarrow \mathcal{L} \supset -\frac{1}{2}(\overline{\Psi}\ \overline{f^0}\ \overline{f'^0})(\mathcal{M}P_L + \mathcal{M}^\dagger P_R)\begin{pmatrix} \Psi \\ f^0 \\ f'^0 \end{pmatrix}$$

($\mathcal{M} = \begin{pmatrix} m_\psi & m_L & m_R \\ m_L & 0 & m_D \\ m_R & m_D & 0 \end{pmatrix}$, $m_{L(R)} = \frac{1}{\sqrt{2}}y_{L(R)}v$)

$$= -\frac{1}{2}(\overline{\chi_1}\ \overline{\chi_2}\ \overline{\chi_3})\begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix}\begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix}$$

($m_1 \le m_2 \le m_3$)

Case of $\mathrm{Re}\, m_\psi = 0$, $m_L = m_R$, and $m_D \gg |m_\psi|, m_L, m_R$

- $m_1 \ll m_2 \sim m_3$: χ_1 is LSP = DM
- Coefficients relating to the direct detection: $\mathcal{L} \supset c_h \cdot h\bar{\chi}_1\chi_1 + c_Z \cdot \bar{\chi}_1 \not{Z}\gamma^5\chi_1$

$$c_h \sim \frac{y_L y_R v}{m_D}\frac{\mathrm{Re}\, m_\psi}{|m_\psi|} = 0, \qquad c_Z \sim \frac{g_2}{4\cos\theta_W}\frac{|y_L^2 - y_R^2|^2}{2m_D^2} = 0$$