Analysis of Inflationary models in Higher-dimensional uniform inflation

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Based on JHEP 04 (2025) 77

(arXiv:2501.13581[hep-ph])

August 21st, 2025 Summer Institute 2025

Contents

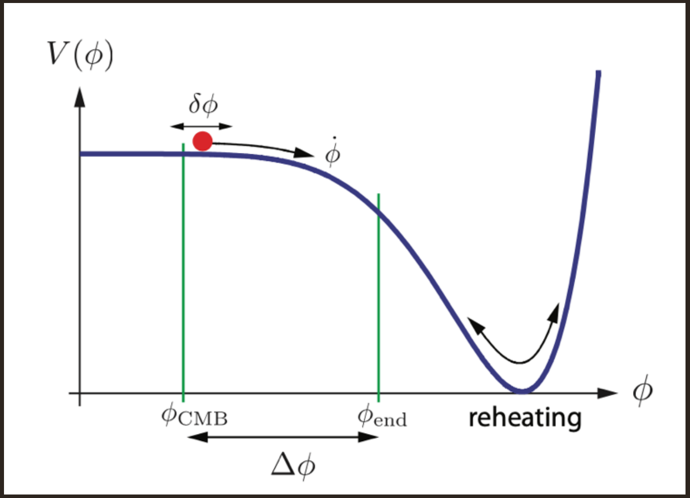
1. Basic inflation

Inflation solves the puzzle of Big Bang theory!

- Horizon problem:
  Exponential expansion helps the causality.
- Flatness problem:
  Exponential expansion flattens the curvature.

Slow-roll inflation

Inflation needs an inflaton ϕ, which has a potential.



Baumann, TASI lecture (2009)

Slow-roll parameters

$$\epsilon_V = \frac{M_{pl}^2}{2}\left(\frac{V_\phi}{V}\right)^2$$

$$\eta_V = M_{pl}^2 \frac{V_{\phi\phi}}{V}$$

1. Basic inflation

The information of the inflationary dynamics can be converted into the observables n_s, r.

- spectral index: n_s

  The power spectrum of scalar perturbation

$$\mathcal{P}_\mathcal{R} \propto k^{n_s - 1}, \quad n_s = 1 - 6\epsilon_V + 2\eta_V$$

- tensor-scalar ratio: r

  The ratio of the power spectra of tensor and scalar

↙the power spectrum of the tensor perturbation

$$r = \frac{\mathcal{P}_h}{\mathcal{P}_\mathcal{R}} = 16\epsilon_V$$

↖the power spectrum of the scalar perturbation

1. Basic inflation

credit:ESA

n_s and r are constrained by the CMB observations.

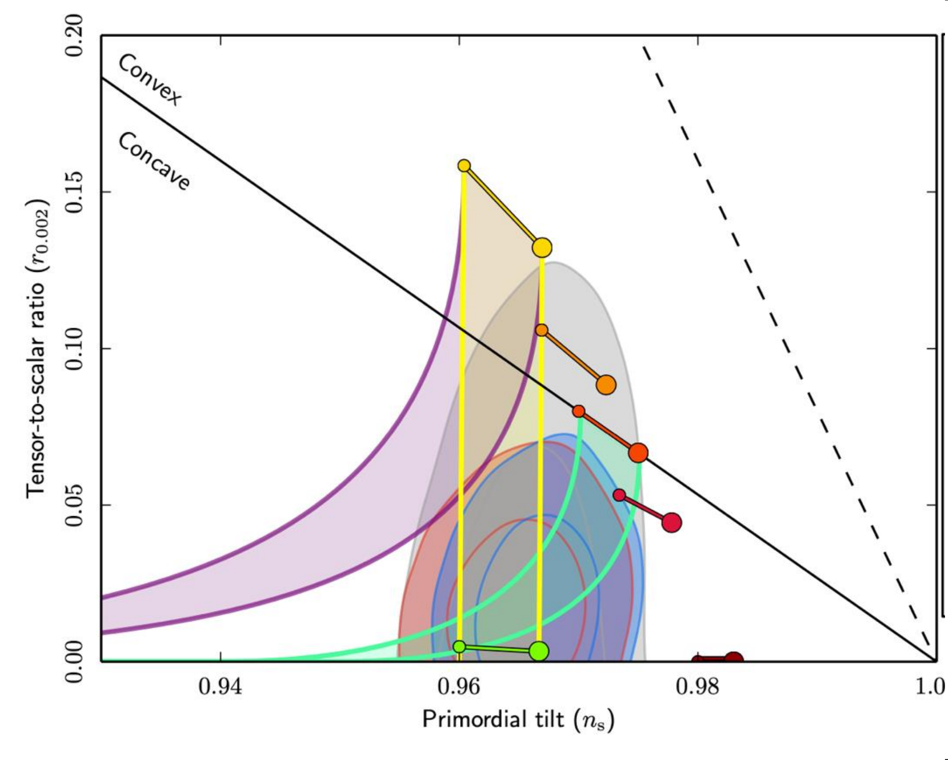
Planck 2018

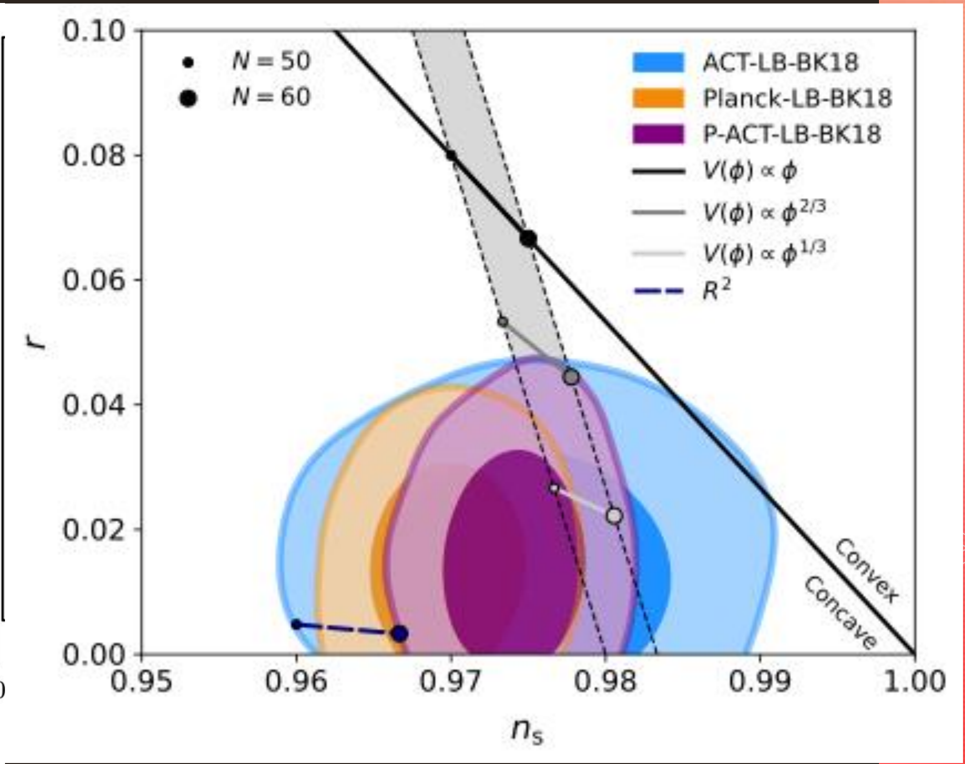
$$n_s = 0.9649 \pm 0.0042$$

$$r < 0.10$$

ACT DR6 (Mar. 2025)

$$n_s = 0.9743 \pm 0.0034$$





2. Higher-dim uniform inflation

If a theory has extra dimensions, do they expand during inflation?

Pioneering studies

- L. A. Anchordoqui, I. Antoniadis (2023)
  Large extra dimensions from higher-dimensional inflation
- I. Antoniadis, J. Cunat, A. Guillen (2023)
  Cosmological perturbations from five-dimensional inflation

2. Higher-dim uniform inflation

- L. A. Anchordoqui, I. Antoniadis (2023)
  FRW metric with extra dim

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}_3^2 + b(t)^2 dy^2$$

$a(t)$: 3D scale factor

$b(t)$: extra-dim scale factor

The expansion rates of $a(t)$ and $b(t)$ are same

$$H = \frac{\dot{a}}{a} = \frac{\dot{b}}{b}$$

➡

Higher-dimensional uniform inflation

※ The different expansion rates are studied [N. Arkani-Hamed, S. Dimopoulos et.al (2000)]

2. Higher-dim uniform inflation

- I. Antoniadis, J. Cunat, A. Guillen (2023)
  Cosmological perturbations in 5D uniform inflation

$$ds^2 = a(t)^2\Big\{-(1+2\Phi)d\tau^2 + \Big((1+\underline{2\mathcal{R}})\delta_{ij} + \underline{2h_{ij}}\Big)dx^i dx^j + (b_0^2 - \underline{\Xi})dy^2\Big\}$$

scalar perturbation

tensor perturbation

extra-dim scalar perturbation

b_0: initial radius of extra space

The spectral index and tensor-scalar ratio are…

$$n_s = 1 - 7\epsilon_V + 2\eta_V, \quad r = 24\epsilon_V$$

n_s and r are slightly changed !!

2. Higher-dim uniform inflation

My work

1. Extend 5D uniform inflation to $D+4$ dimensions
   Extra-dim space: $S^1 \times S^1 \times \cdots S^1$ for simplicity
2. Derive the general results of spectral index and tensor-scalar ratio
3. Investigate three inflationary models
   Compare their n_s, r with Planck 2018 constraints

- Chaotic inflation
- Natural inflation
- Quartic hilltop inflation
- Spontaneously broken SUSY model
- R^2 inflation (Starobinsky inflation)

3. My work (Extension)

Set up

- Extra dimension: $S^1 \times S^1 \times \cdots S^1$ (all initial radii: b_0)
- The expansion rates of $a(t)$ and $b(t)$ are same

$$a(t) = e^{Ht}, \quad b(t) = b_0 e^{Ht}, \quad H = \frac{\dot{a}}{a} = \frac{\dot{b}}{b}$$

- Metric: $ds^2 = a^2(\tau)\left(-d\tau^2 + d\vec{x}_3^2 + b_0^2 d\vec{y}_D^2\right)$
- Friedmann eq. [N. Arkani-Hamed, S. Dimopoulos, et.al (2000)]

$$\frac{(D+2)(D+3)}{2}\mathcal{H}^2 = \frac{a^2(\tau)\rho}{M_{pl}^2}, \quad (D+2)\left(\mathcal{H}' + \frac{D+1}{2}\mathcal{H}^2\right) = -\frac{a^2(\tau)p}{M_{pl}^2}$$

τ : conformal time
$\mathcal{H}$: conformal Hubble parameter

3. My work (Extension)

Inflationary parameters (inflaton = ϕ)

- Slow-roll parameters

$$\epsilon_V = \frac{D+2}{4} M_{pl}^2 \left(\frac{V_\phi}{V} \right)^2 , \quad \eta_V = \frac{D+2}{2} M_{pl}^2 \frac{V_{\phi\phi}}{V}$$

- The number of e-folds

$$N_* = \ln \frac{a_{\rm end}}{a} = \frac{2}{(D+2) M_{pl}^2} \int_{\phi_{\rm end}}^{\phi_*} \frac{V}{V_\phi} d\phi$$

ϕ_*: field value of CMB observation

Typical range of N_*: $40 \leq N_* \leq 60$

3. My work (Extension)

Metric perturbations

$$ds^2 = a(t)^2\Big\{-(1+2\Phi)d\tau^2 + \Big((1+2\mathcal{R})\delta_{ij} + 2h_{ij}\Big)dx^i dx^j + (b_0^2 - 2\Xi)\eta_{mn}dy^m dy^n\Big\}$$

We perform the Fourier transformation of perturbations

$$A(\tau, x^i, y_D^m) = \int d^3k \sum_{\vec{n}} A_{k,n}(k,\tau) e^{i\vec{k}\cdot\vec{x}_3} e^{i\vec{n}\cdot\vec{y}_D}$$

We also define Kaluza-Klein (KK) mass $m_{k,n}^2$ as

$$m_{k,n}^2 = k^2 + \frac{|\vec{n}|^2}{b_0^2}$$

3. My work (Extension)

The same eq. of the tensor perturbation

E.O.M. of the scalar perturbations

$$\Theta'' + (D+2)\mathcal{H}\Theta' + m_{k,n}^2\Theta = 0$$

$$\Omega'' + \left[(D+2)\mathcal{H} + \frac{2(\mathcal{H}')^2 - \mathcal{H}\mathcal{H}''}{\mathcal{H}(\mathcal{H}^2 - \mathcal{H}')}\right]\Omega' + m_{k,n}^2\Omega = 0$$

Here,

$$\Theta \equiv \mathcal{R} + \frac{\Xi}{b_0^2}$$

$$\Omega \equiv \frac{1}{(D+2)m_{k,n}^2}\left[\left(2m_{k,n}^2 + \frac{|\vec{n}|^2}{b_0^2}\right)\mathcal{R} - \left(Dm_{k,n}^2 - \frac{|\vec{n}|^2}{b_0^2}\right)\frac{\Xi}{b_0^2}\right]$$

If we could get the solutions of Θ and Ω,
we derive the solution of the scalar perturbation $\mathcal{R}$!!

3. My work (Derivation)

$$\Theta''_{k,n} + (D+2)\mathcal{H}\Theta'_{k,n} + m^2_{k,n}\Theta_{k,n} = 0$$

appropriate variable transformations

$$y = a^{(D+2)/2}, \quad \theta_{k,n} = y\Theta_{k,n}$$

$$\theta''_{k,n} + \left(m^2_{k,n} - \frac{y''}{y}\right)\theta_{k,n} = 0$$

Mukhanov-Sasaki eq.

- The solution of this eq. has been known.
- The same procedure is true for $\Omega_{k,n}$.

We can derive the scalar perturbation !

3. My work (Derivation)

From the perturbations, we derive n_s and r.

- The power spectrum

scalar: $$\mathcal{P}_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2} \sum_{\vec{n}} |\mathcal{R}_{k,n}|^2$$

tensor: $$\mathcal{P}_h(k) = \frac{k^3}{2\pi^2} \frac{2 \cdot 4}{M_{pl}^2} \sum_{\vec{n}} |h_{ij}|^2,$$

After hard calculation…

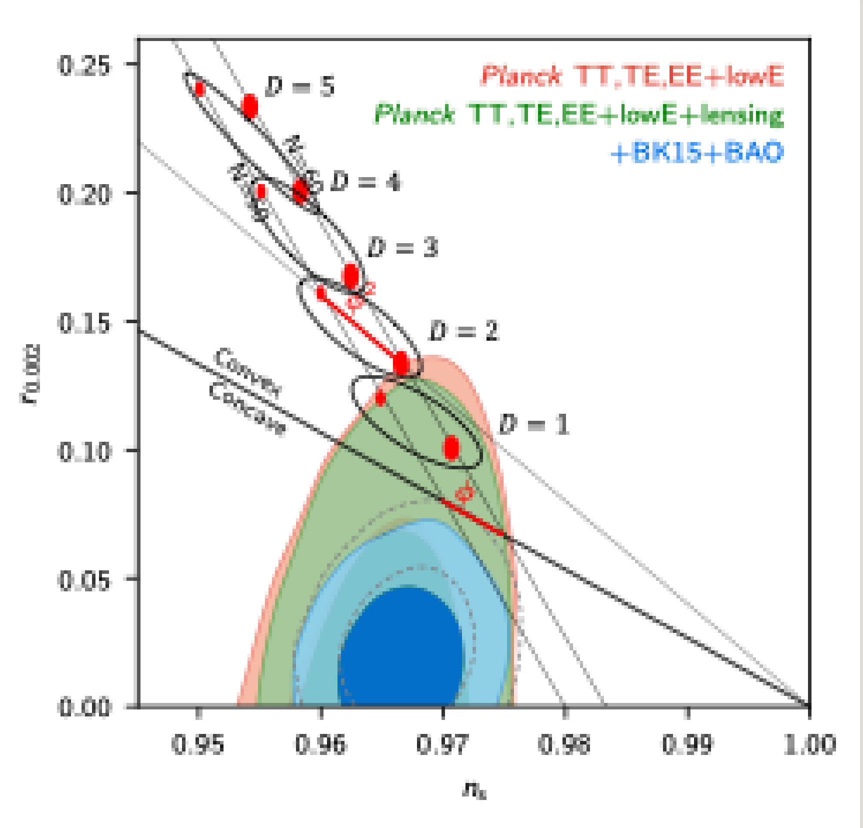
$$n_s = 1 - (D+6)\epsilon_V + 2\eta_V, \quad r = 8(D+2)\epsilon_V \quad (b_0 k \gg 1)$$

3. My work (Comparison)

- Chaotic inflation [A. D. Linde (1983)]

$$V(\phi) = \frac{\lambda}{n}\phi^n \qquad (\lambda\text{: a coupling})$$

$n = 1$ case



- When $n = 2$ and $D = 1$,
  $n_s = 0.9505,\ r = 0.24\ (N_* = 50)$
  $n_s = 0.9587,\ r = 0.20\ (N_* = 60)$

Rule out the case of
$n \geq 2$ and $D \geq 1$

- Possibility: $n \leq 1,\ D \geq 1$

Rule out the case of
$D \geq 2$ if $n = 1$

- When $n = 1$ and $D = 1$,
  $n_s = 0.9652,\ r = 0.12\ (N_* = 50)$
  $n_s = 0.9710,\ r = 0.10\ (N_* = 60)$

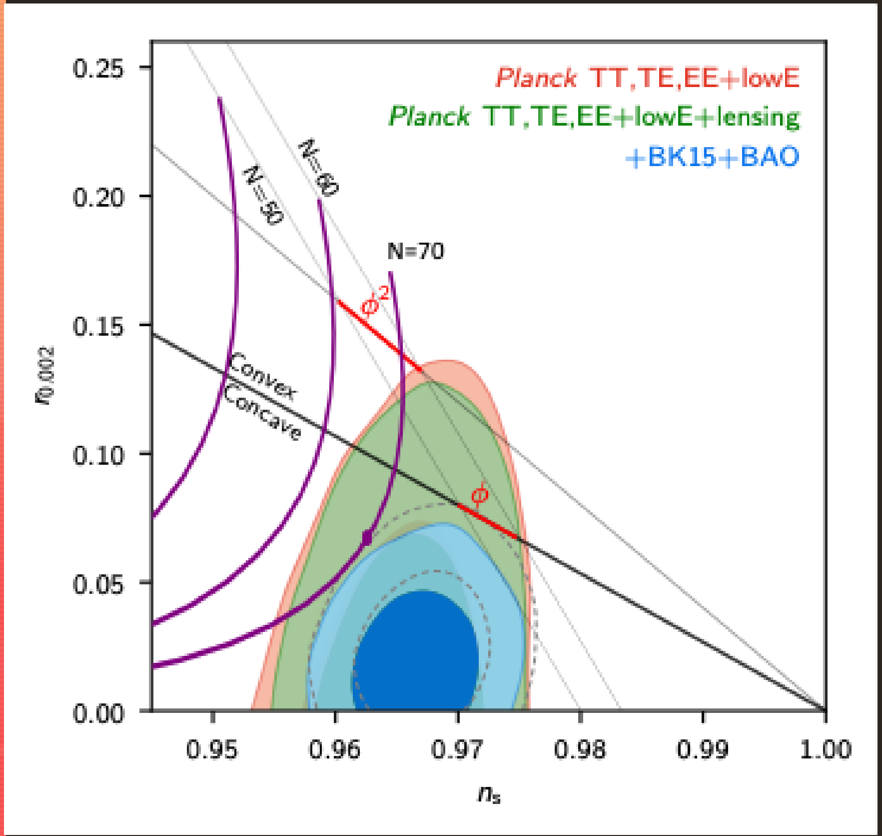
3. My work (Comparison)

- Natural inflation [K. Freese, J. A. Frieman, A. V. Olinto (1990)]

$$V(\phi) = V_0\left(1+\cos\left(\frac{\phi}{f}\right)\right)$$

(f: a scale)

$D = 1$ case



- When $D \geq 2$, no lines within the allowed region

Rule out the case of $D \geq 1$ and $N_* \leq 60$

- Only the case $D = 1$ and $N_* = 70$ is within the allowed region
- When $D = 1$ and $f = 8M_{pl}$, $n_s = 0.9627,\ r = 0.067\ (N_* = 70)$

3. My work (Comparison)

- R^2 inflation (Starobinsky inflation) [A. A. Starobinsky (1980)]
- The R^2 inflationary potential in $D+4$ dimension

[S. P. Otero, F. G. Pedro, C. Wieck (2017)]

$$V(\phi) = V_0 \exp\left[\frac{D}{\sqrt{(D+2)(D+3)}}\frac{\phi}{M_{pl}}\right]\left(1-\exp\left[-\sqrt{\frac{D+2}{D+3}}\frac{\phi}{M_{pl}}\right]\right)^2$$

- When $D > 0$ and taking the limit as $\phi/M_{pl} \rightarrow \infty$,

$$n_s \rightarrow 1 - \frac{D^2(D+2)}{4(D+3)}, \quad r \rightarrow \frac{2D^2(D+2)}{D+3}$$

R^2 inflation with expanding extra dimension is excluded !!

4. Summary

- We consider higher-dimensional uniform inflation.

  Extra space: $S^1 \times S^1 \times \cdots \times S^1$

- We derive the general results of n_s and r.

$$n_s = 1 - (D+6)\epsilon_V + 2\eta_V, \quad r = 8(D+2)\epsilon_V \quad (b_0 k \gg 1)$$

- We investigate three inflationary models, and compare their n_s, r with Planck 2018 constraints
  - Chaotic inflation
  - Natural inflation
  - Quartic hilltop inflation

Only $D=1$ case are allowed

  - SSB SUSY model
  - R^2 inflation

Exclude in any dimension!!

4. Discussion

- The expansion of extra-dim space is NOT favored by Planck results.
    - It is favored that 3D non-compact space is expanded, while extra-dim compact space is not expanded.
    - Revisit different expansion rate? [N. Arkani-Hamed, S. Dimopoulos et.al. (2000)]
- Only the expansion of 1D extra-dim space may be allowed.
    - Related to Dark Dimension proposal or Swampland ?
    - Reheating?
- Another inflationary model
    - Extra-natural inflation [N. Arkani-hamed, H. C. Cheng, P. Creminelli, L. Randall (2003)]
    - Modular inflation [T. Kobayashi, D. Nitta, Y. Urakawa (2016)]
    - Revisit SSB SUSY model in $D + 4$ dimensions, and R^n inflation with $n \geq 3$
    - Analysis with radion stabilization [L. A. Anchordoqui, I. Antoniadis (2023)]

Appendix

- Sasaki-Mukhanov equation($f = \theta_{k,n}, \omega_{k,n}$)

$$f(\tau) \to e^{i\pi(\nu_f - \frac{1}{2})} 2^{\nu_f - 1} \sqrt{\frac{\tau}{\pi}} \Gamma(\nu_f) \frac{1}{(m_{k,n}\tau)^{\nu_f}}$$

※1 Bunch-Davis vacuum
※2 long wavelength limit
$m_{k,n}\tau \to 0$

- Power spectrum: scalar

$$\mathcal{P}_{\mathcal{R}}(k) \simeq \frac{2^D \Gamma^2(\frac{D+3}{2})}{\pi^3 (D+2)\epsilon M_{pl}^2} b_0^D (b_0 k)^3 H^{D+2} \tau^{D+3} \times \Big[\tau^{-2\nu_\omega} S_{\nu_\omega}((b_0 k)^2) + \frac{\epsilon M_{pl}^2}{D+2} \tau^{-2\nu_\theta} \Big((D-1)^2 S_{\nu_\theta}((b_0 k)^2) + 2(D-1)(b_0 k)^2 S_{\nu_\theta + 1}((b_0 k)^2) + (b_0 k)^4 S_{\nu_\theta + 2}((b_0 k)^2) \Big) \Big]$$

Here,

$$S_\nu(x) \equiv \sum_{\vec{n}} \frac{1}{(|\vec{n}|^2 + x)^\nu}$$

$$\nu_\theta = \frac{D+3}{2} + \frac{D+2}{2}\epsilon, \quad \nu_\omega = \frac{D+3}{2} + \frac{D+6}{2}\epsilon - \eta$$

Appendix

- Power spectrum: scalar
  The behavior of $S_\nu(x^2)$ can be understood by Schwinger representation, Poisson resummation and Bessel function.

$$S_\nu(x^2) = \sum_{\vec{n}} \frac{1}{\Gamma(\nu)} \int_0^\infty dt\, t^{\nu-1} \exp\left[-(|\vec{n}|^2 - x^2)t\right] \qquad \leftarrow \text{Schwinger representation}$$

$$= \frac{1}{\Gamma(\nu)} \int_0^\infty dt\, t^{\nu-1} e^{-x^2 t} \left(\sum_{n=-\infty}^{\infty} e^{-n^2 t} \right)^D$$

Poisson resummation
↓

$$= \frac{\pi^{D/2}}{\Gamma(\nu)} x^{D-2\nu} \sum_{\vec{n}} \int_0^\infty du\, u^{\frac{2\nu-D}{2}-1} \exp\left[-u - \frac{1}{4u}(2\pi x|\vec{n}|)^2\right]$$

$$= \frac{2\pi^{D/2}}{\Gamma(\nu)} x^{D-2\nu} \sum_{\vec{n}} \left(\frac{z}{2}\right)^{\frac{2\nu-D}{2}} K_{\frac{2\nu-D}{2}}(z) \qquad \leftarrow \text{Modified Bessel function}$$

$$\simeq \frac{\pi^{D/2}\Gamma\left(\nu - \frac{D}{2}\right)}{\Gamma(\nu)} x^{D-2\nu} \qquad (x \gg 1)$$

Appendix

Exact results

- Power spectrum: scalar

$$\mathcal{P}_{\mathcal{R}}(k) \simeq \begin{cases} \frac{2^D\Gamma^2\left(\frac{D+3}{2}\right)}{\pi^3(D+2)\epsilon M_{pl}^2}\frac{H^{D+2}}{k^D}\left[\left(\frac{k}{aH}\right)^{-(D+6)\epsilon+2\eta}+\frac{D}{D+2}\epsilon M_{pl}^2\left(\frac{k}{aH}\right)^{-(D+2)\epsilon}\right] & (b_0k \ll 1) \\ \frac{2^{D-1}\pi^{\frac{D-5}{2}}\Gamma\left(\frac{D+3}{2}\right)}{(D+2)\epsilon M_{pl}^2}b_0^D H^{D+2}\left[\left(\frac{k}{aH}\right)^{-(D+6)\epsilon+2\eta}+\frac{f(D)}{D+2}\epsilon M_{pl}^2\left(\frac{k}{aH}\right)^{-(D+2)\epsilon}\right] & (b_0k \gg 1) \end{cases}$$

Here,

$$f(D) = (D-1)^2 + \frac{6(D-1)}{D+3} + \frac{15}{(D+3)(D+5)}.$$

- Power spectrum: tensor

$$\mathcal{P}_h(k) \simeq \begin{cases} \frac{2^{D+3}\Gamma^2\left(\frac{D+3}{2}\right)}{\pi^3 M_{pl}^2}\frac{H^{D+2}}{k^D}\left(\frac{k}{aH}\right)^{-(D+2)\epsilon} & (b_0k \ll 1) \\ \frac{2^{D+2}\pi^{\frac{D-5}{2}}\Gamma\left(\frac{D+3}{2}\right)}{M_{pl}^2}b_0^D H^{D+2}\left(\frac{k}{aH}\right)^{-(D+2)\epsilon} & (b_0k \gg 1) \end{cases}$$

Appendix

Exact results

- Spectral index n_s

$$n_s = 1 - (D+6)\epsilon_V + 2\eta_V - D \quad (b_0 k \ll 1)$$
$$n_s = 1 - (D+6)\epsilon_V + 2\eta_V \quad (b_0 k \gg 1)$$

- Tensor-scalar ratio r

$$r = 8(D+2)\epsilon_V$$

- Scalar amplitude A_s

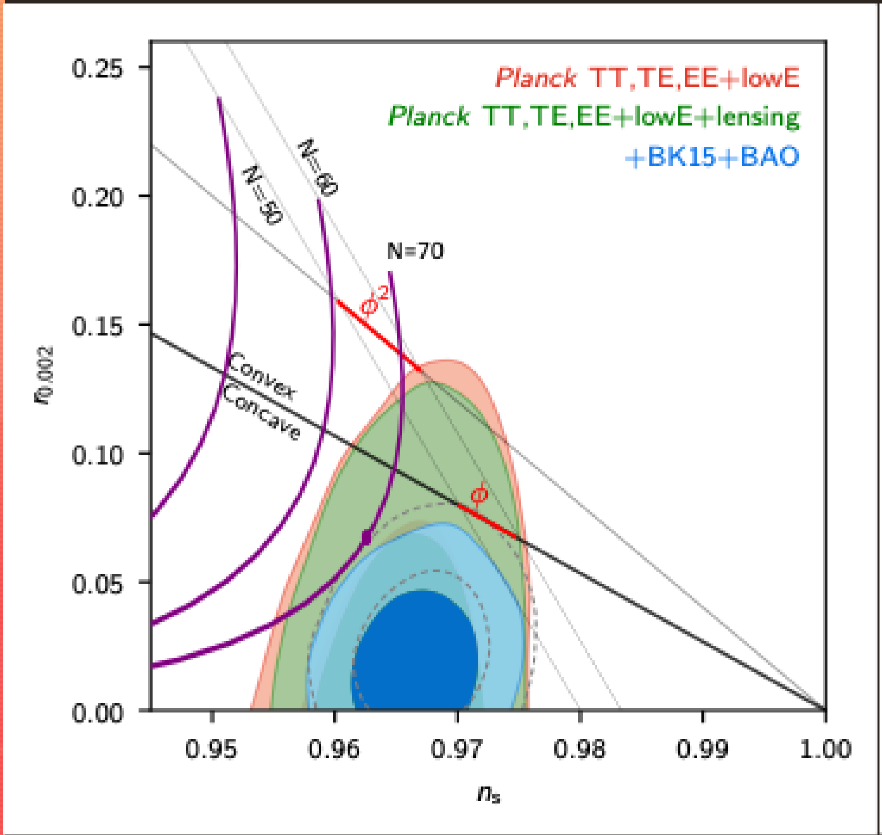
$$A_s = \frac{2^{D-1}\pi^{\frac{D-5}{2}}\Gamma\left(\frac{D+3}{2}\right)}{(D+2)\epsilon M_{pl}^2} b_0^D H^{D+2}$$

Appendix

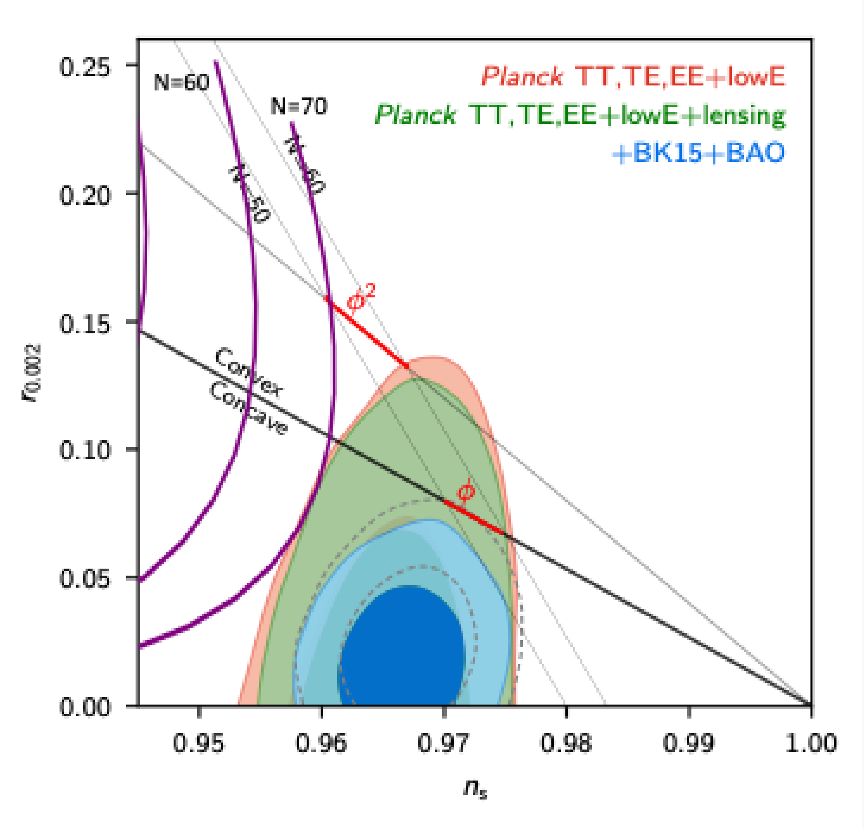
- Natural inflation

$$V(\phi) = V_0\left(1+\cos\left(\frac{\phi}{f}\right)\right)$$ (f: a scale)

$D = 1$ case



$D = 2$ case



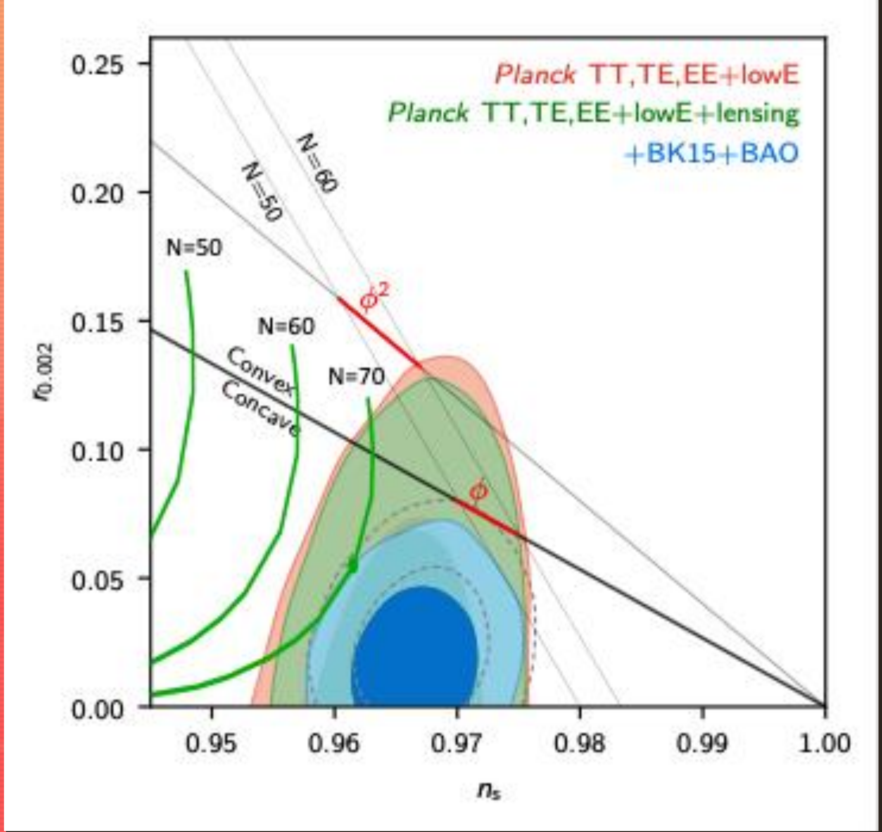
Appendix

- Quartic hilltop inflation [A. D. Linde (1982), K.Dimopoulos (2020)]

$$V(\phi) = V_0\left(1 - \lambda\frac{\phi^4}{M_{pl}^4}\right)$$

(λ: a coupling)

$D = 1$ case



- When $D \geq 2$, no lines within the allowed region

Rule out the case of $D \geq 1$ and $N_* \leq 60$

- Only the case $D = 1$ and $N_* = 70$ is within the allowed region

- When $D = 1$ and $\lambda = 10^{-6}$, $n_s = 0.9616$, $r = 0.054$ ($N_* = 70$)

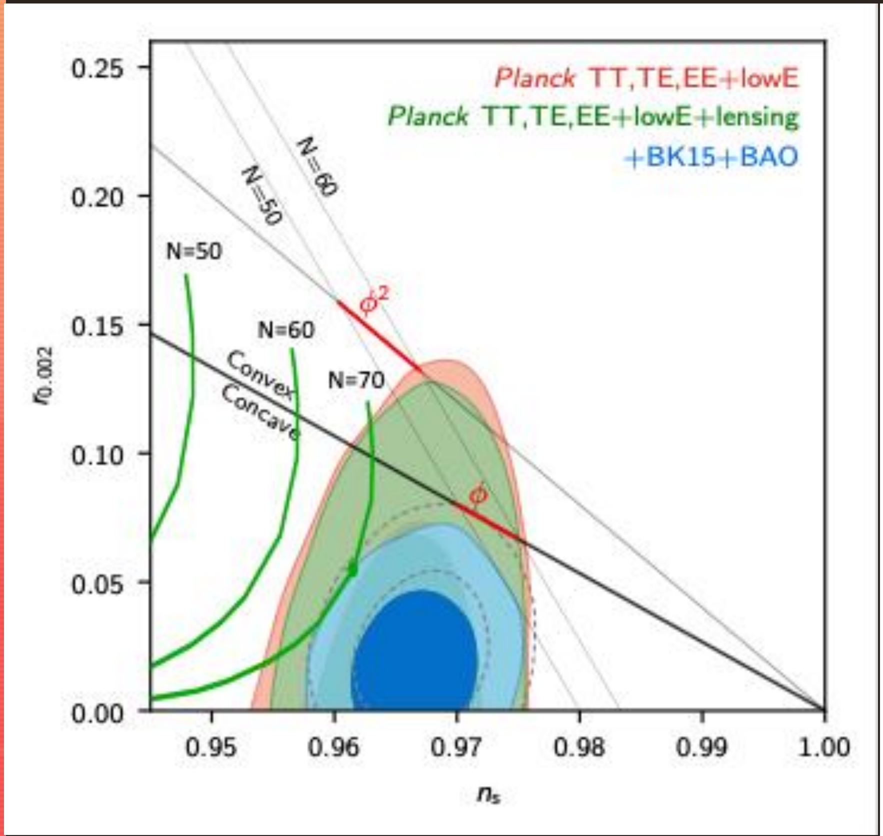
Appendix

- Quartic hilltop inflation

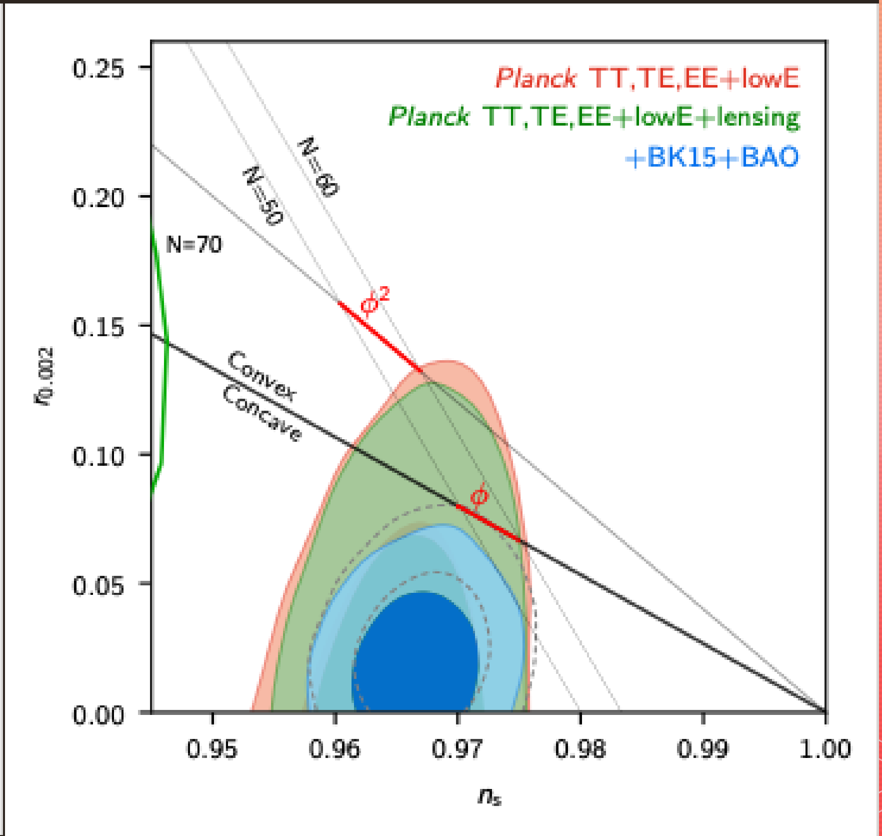
$$V(\phi) = V_0\left(1 - \lambda\frac{\phi^4}{M_{pl}^4}\right)$$

(λ: a coupling)

$D = 1$ case



$D = 2$ case



Appendix

- Spontaneously broken SUSY model

[G. R. Dvali, Q. Shafi, R. K. Schaefer (1994)]

$$V(\phi) = V_0 \left(1 + \alpha_h \ln \frac{\phi}{M_{pl}}\right)$$

(α_h: a parameter)

- This model predicts r takes a very small value.
- However, this model is dismissed since n_s is large.
- Regardless of the influence of dimensions,

$$n_s \approx 0.980 \; (N_* = 50)$$
$$n_s \approx 0.983 \; (N_* = 60)$$

$$(\alpha_h \leq 10^{-1})$$

The spontaneously broken SUSY model
is not revived.

Another motivation

- Dark Dimension scenario [M. Montero, C. Vafa, I. Valenzuela (2023)]

  Motivated by AdS distance conjecture or Swampland

AdS distance conjecture [D. Lust, E. Palti, C. Vafa (2019)]

- Quantum gravity on D-dimensional AdS space with cosmological constant Λ
- An infinte KK tower of states with mass scale m
- as $\Lambda \to 0$, behave as

$$m \sim |\Lambda|^{\alpha},$$

(α: a positive order-one number)

Attempt to solve the hierarchy of the particle physics and cosmology (cosmological hierarchy problem)

※ Original large extra dimensions is related to the gauge hierarchy problem.

Another motivation

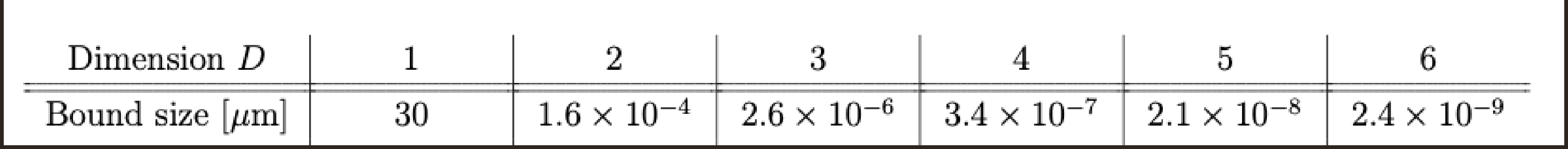
- Dark Dimension scenario [M. Montero, C. Vafa, I. Valenzuela (2023)]
  This conjecture predicts the radius of extra dim as

$$R \sim 0.1\ \mu\text{m} - 10\ \mu\text{m} \Rightarrow$$

Dark Dimension
(one mesoscopic extra dimension)

Can we observe a micro-sized extra dimension?

Current bound of the size of extra dim



Dimension D	1	2	3	4	5	6
Bound size [μm]	30	1.6×10^{-4}	2.6×10^{-6}	3.4×10^{-7}	2.1×10^{-8}	2.4×10^{-9}

↑ torsion balance (1)
astrophysical bound (2–4)
LHC (5–6)