Lepton number violation in future lepton colliders and neutrinoless double beta decay

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Based on collaboration with Takehiko Asaka, and Hiroyuki Ishida
(work in progress)

Contents

Neutrino masses

- Neutrino masses have been confirmed by oscillation experiments, which are not accommodated within the standard model
- Mass-squared differences and mixing angles have been measured precisely

Normal Hierarchy(NH)	Inverted Hierarchy(IH)
$m_1 < m_2 < m_3$	$m_3 < m_1 < m_2$
$\begin{cases} \Delta m^2_{21} = 7.49 \times 10^{-5}\mathrm{eV}^2 \\ \Delta m^2_{31} = 2.513 \times 10^{-3}\mathrm{eV}^2 \end{cases}$	$\begin{cases} \Delta m^2_{21} = 7.49 \times 10^{-5}\mathrm{eV}^2 \\ \Delta m^2_{32} = -2.484 \times 10^{-3}\mathrm{eV}^2 \end{cases}$

JHEP 12 (2024) 216[arXiv:2410.05380]
(NuFIT 6.0 (2024), www.nu-fit.org.

We still don't understand

- Mass hierarchy (ordering)
- CP violation in the lepton sector
- Origin of neutrino masses
- Neutrino is Dirac or Majorana

In this talk, we consider

- The origin of neutrino masses
- Its experimental tests

Seesaw mechanism

[Minkowski(1977), T.Yanagida(1979),
Gell-Mann,Ramond, Slansky(1979), Glashow(1979)]

- We consider the seesaw mechanism for neutrino masses by introducing right-handed neutrinos (RHNs)
  - It can naturally explain the smallness of neutrino masses
  - RHNs can generate the baryon asymmetry via lepton asymmetry creation [M. Fukugita, T. Yanagida(1986), Akhmedov et al.(1998]
  - RHN can be a DM candidate [S. Dodelson, L. M. Widrow(1994)]
  - RHN naturally appears as part of the 16-dim. spinor representation in SO(10) GUT [H. Fritzsch, P. Minkowski(1975)]

Short summary

- We consider the seesaw mechanism with two RHNs, which is the minimal option to explain the two mass square differences observed by oscillation experiments.

  At least two generations of active neutrinos must have non-zero masses

- Especially we consider the TeV scale RHNs, which can be probed by future experiments

- Since the seesaw mechanism induces lepton number violation (LNV), we investigate the LNV processes like $e^\pm e^\pm \to W^\pm W^\pm$, $\mu^\pm \mu^\pm \to W^\pm W^\pm$ $(\Delta L = \pm 2)$

Seesaw mechanism with two right-handed neutrinos

Model

- SM + two RHNs($I = 1,2$)

$$\mathcal{L} = \mathcal{L}_{SM} + \overline{N_{RI}} i\partial_\mu \gamma^\mu N_{RI} - \left(Y_{\alpha I} \overline{L_{L\alpha}} H N_{RI} + \frac{M_I}{2} \overline{N^c_{RI}} N_{RI} + h.c. \right)$$

$Y_{\alpha I}$:Yukawa coupling H:Higgs N_R:Right-handed neutrino

- Two types of neutrino masses

$$\begin{cases} 1.\ \text{Dirac type}: [M_D]_{\alpha I} = Y_{\alpha I} v \\ 2.\ \text{Majorana type}: [M_I]_{IJ} = M_I \delta_{IJ} \end{cases} \qquad v = \langle H \rangle$$

- Seesaw mechanism($M_D \ll M_I$)

Three active neutrinos(ν_i)

$$M_\nu = -M_D M_I^{-1} M_D^T$$

(The lightest one is massless)

Two heavy neutral leptons(N_I)

$$M_I$$

Neutrino mixing

- Neutrino mixing in the charged current interaction

$$\nu_{L\alpha} = \sum_i^3 U_{\alpha i}\nu_i + \sum_I^2 \Theta_{\alpha I} N_I^c$$

$U_{\alpha i}$:PMNS matrix ν_i, N_I,:Mass-eigenstate neutrinos $\Theta_{\alpha I}$:Mixing elements

- Mixing elements of heavy neutral leptons

$$\Theta_{\alpha I} = \frac{[M_D]_{\alpha I}}{M_I} = \frac{v Y_{\alpha I}}{M_I}$$

They are proportional to the Yukawa couplings

Parametrization of Yukawa couplings

[Casas,Ibarra(2001)]

- Yukawa couplings

$$Y = \frac{i}{v} U D_{\sqrt{m}} \Omega D_{\sqrt{M}}$$

$$D_{\sqrt{m}} = \mathrm{diag}(\sqrt{m_1}, \sqrt{m_2}, \sqrt{m_3}), \quad D_{\sqrt{M}} = \mathrm{diag}(\sqrt{M_1}, \sqrt{M_2})$$

$$\Omega = \begin{pmatrix} 0 & 0 \\ \cos\omega & -\sin\omega \\ \sin\omega & \cos\omega \end{pmatrix} \text{ for NH,} \quad \Omega = \begin{pmatrix} \cos\omega & -\sin\omega \\ \sin\omega & \cos\omega \\ 0 & 0 \end{pmatrix} \text{ for IH}$$

ω:complex free parameter

- PMNS matrix

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\eta} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$s_{ij} = \sin\theta_{ij}, \quad c_{ij} = \cos\theta_{ij}$

η:Majorana phase

TeV scale seesaw mechanism

- Active neutrinos and heavy neutral leptons in the seesaw mechanism are Majorana fermions

- Lepton number is violated at Lagrangian level

- Non standard model LNV processes

  - Meson decays ($B^- \to N\mu^- \to \pi^+\mu^-\mu^-$)

  - Neutrinoless double beta ($0\nu2\beta$) decay

  - $e^\pm e^\pm \to W^\pm W^\pm$, $\mu^\pm\mu^\pm \to W^\pm W^\pm$

TeV scale RHNs can be probed by future lepton colliders

LNV processes in the seesaw mechanism

$0\nu 2\beta$ decay by massive Majorana active neutrinos

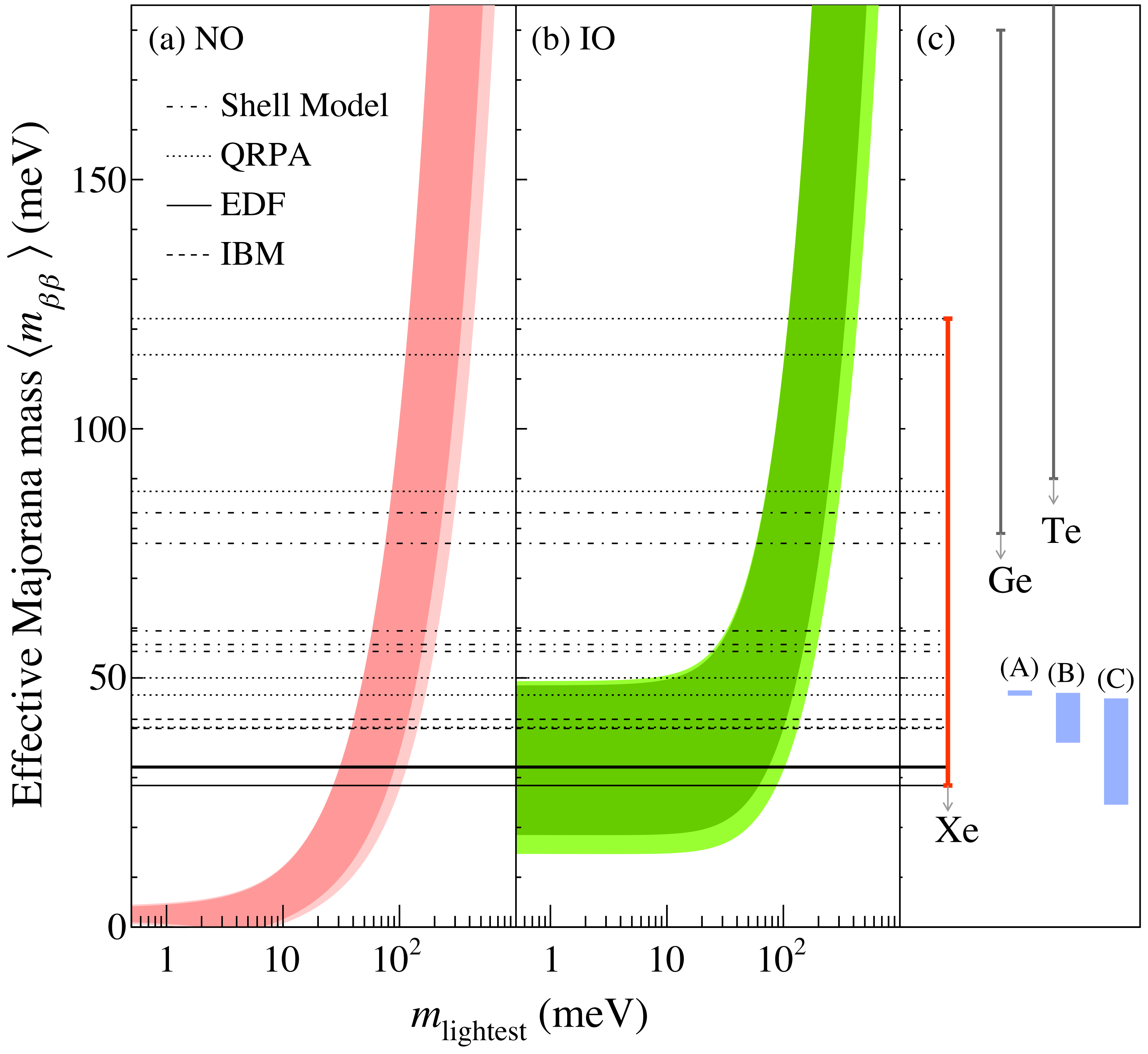
- Process

$$(\mathrm{A, Z}) \to (\mathrm{A, Z+2}) + 2e^-$$

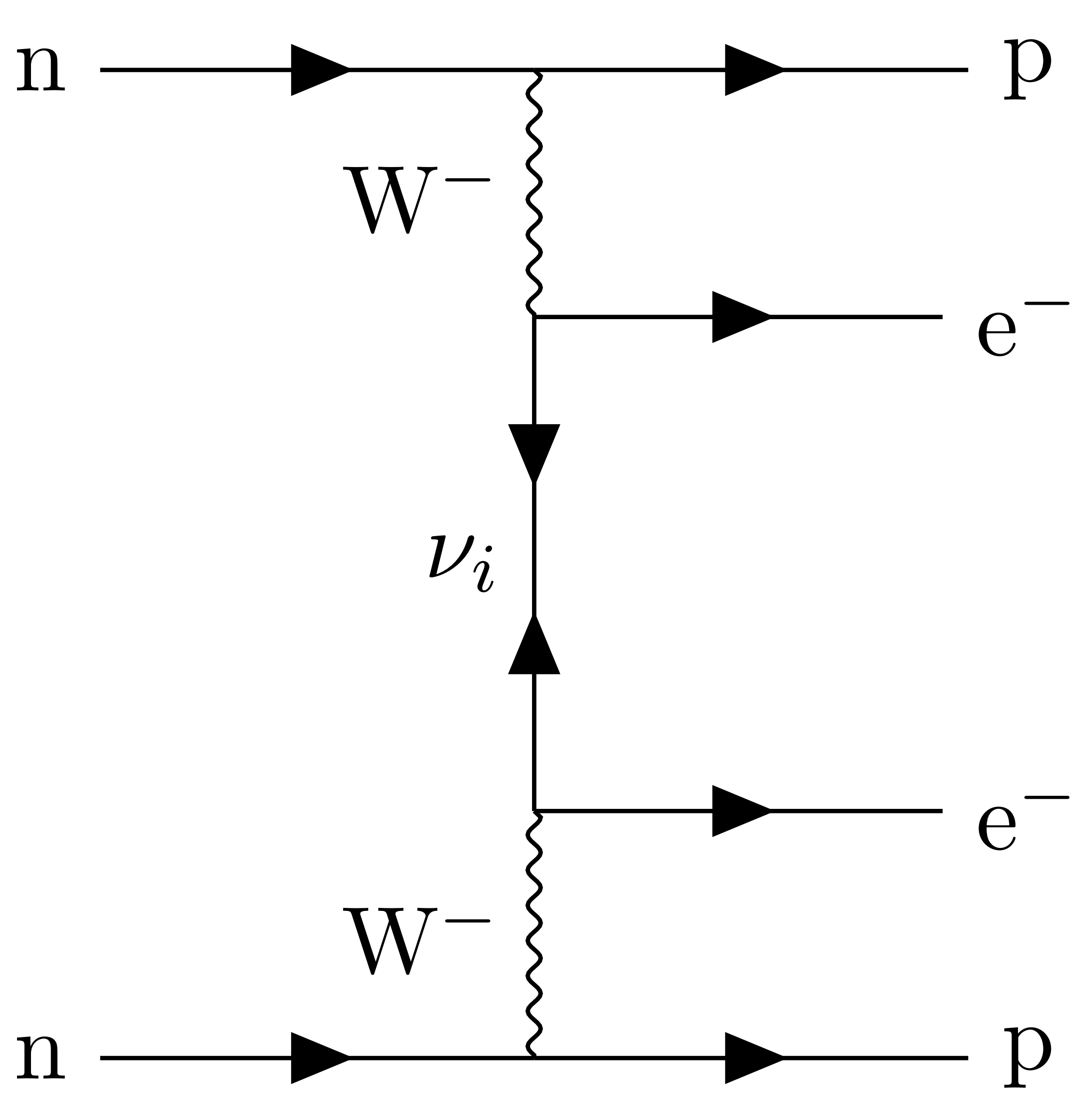
A:Mass number, Z:Atomic number

- Effective mass

$$m_{\mathrm{eff}} = \sum_i U_{ei}^2 m_i$$



- Current experimental bound



$$m_{\mathrm{eff}} < (28 - 122)\ \mathrm{meV}(^{136}\mathrm{Xe})$$

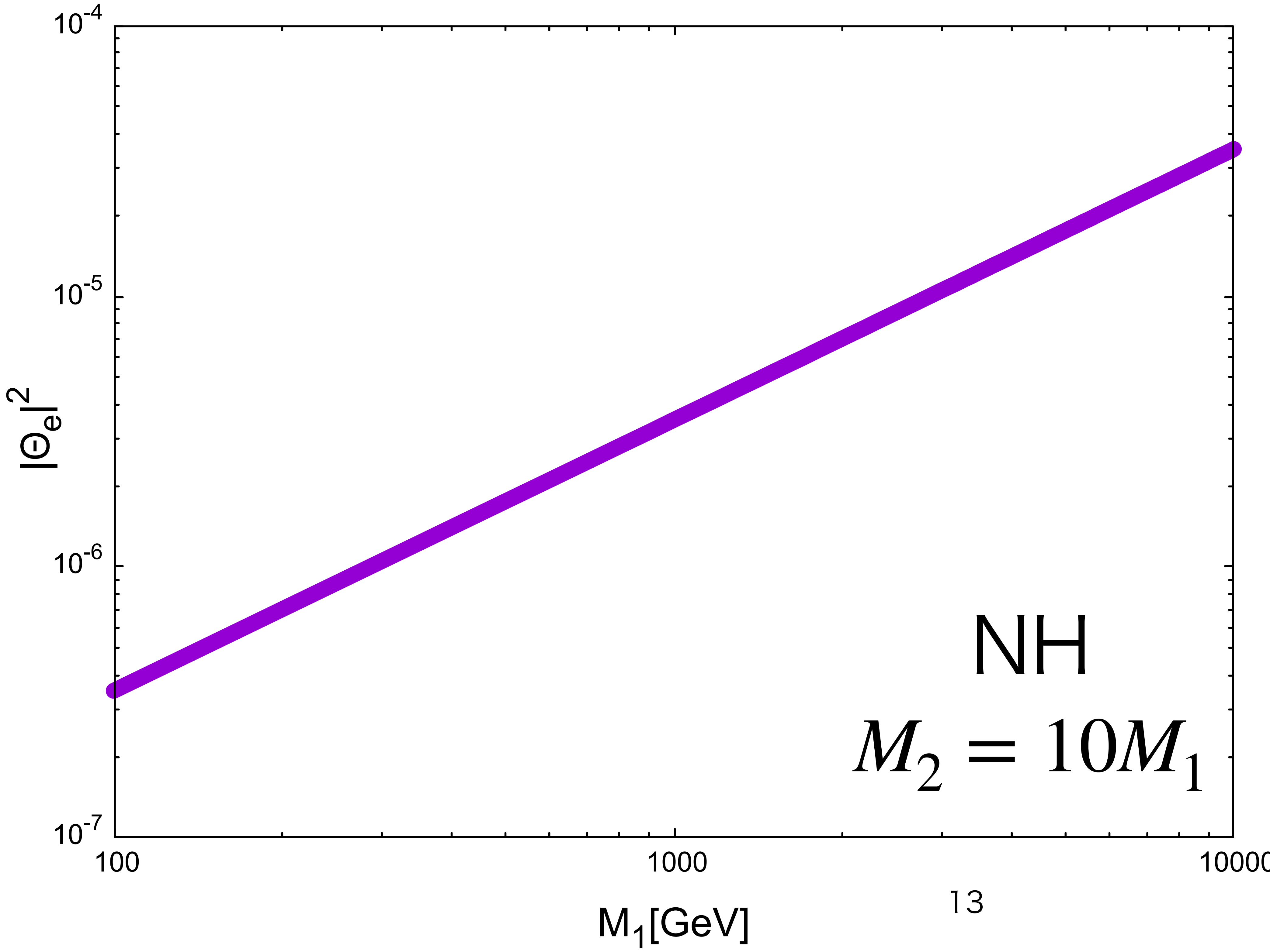
[KamLAND-Zen(2024)]

$0\nu2\beta$ decay in the seesaw mechanism

- $m_{\text{eff}} = \sum_i U_{ei}^2 m_i + \sum_I \Theta_{eI}^2 f(M_I) M_I$

  $f(M_I) \sim \frac{\langle p^2 \rangle}{\langle p^2 \rangle + M_I^2}$: suppression factor, $\sqrt{\langle p^2 \rangle} \sim O(100\text{MeV})$

- $0\nu2\beta$ decay gives upper bound on Θ_{eI}



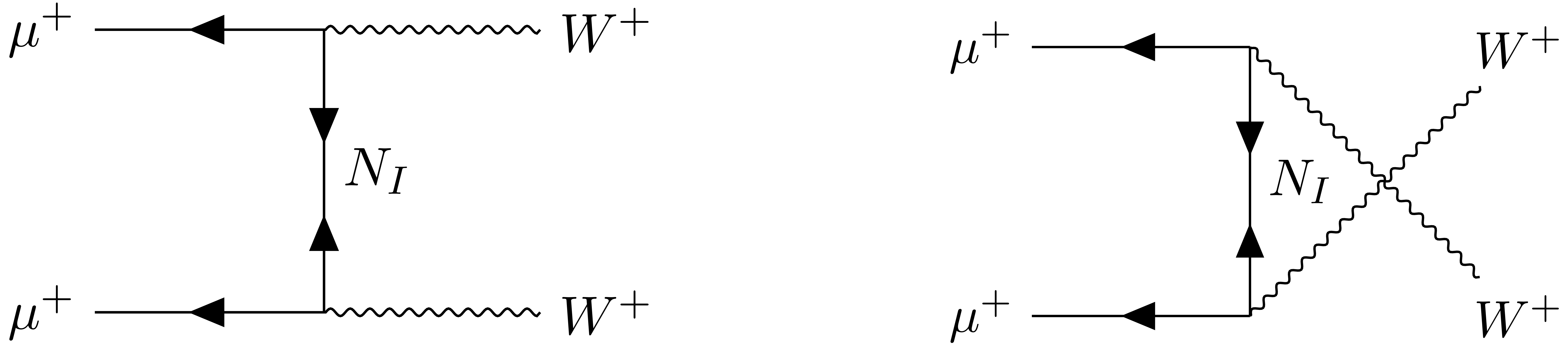
Constraint

- $m_{\text{eff}} < 122\text{meV}$

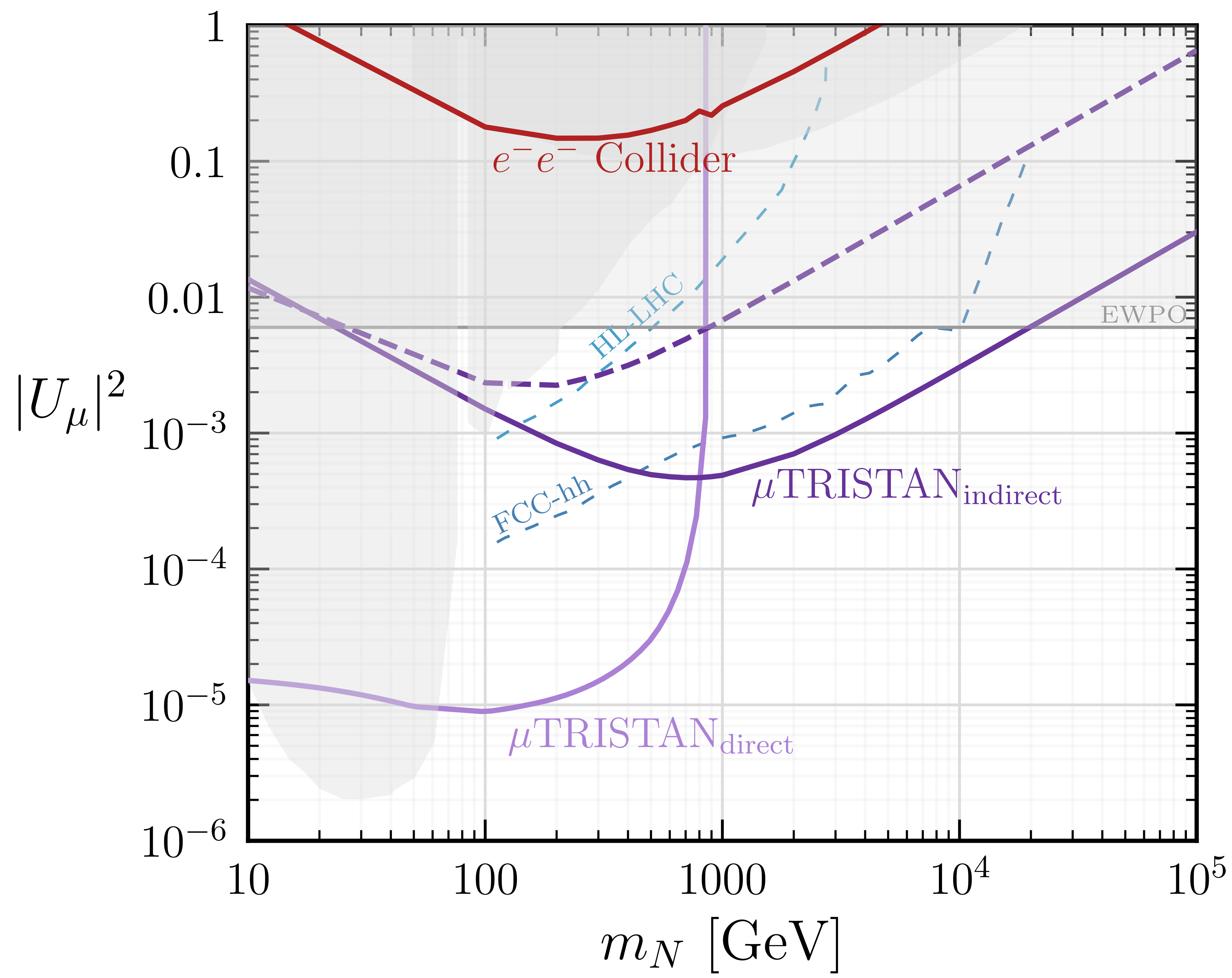
$|\Theta_e|^2 = |\Theta_{e1}|^2 + |\Theta_{e2}|^2$

$e^\pm e^\pm \to W^\pm W^\pm$, $\mu^\pm\mu^\pm \to W^\pm W^\pm$ at future lepton collider

- $\Delta L = \pm 2$ processes
- Advantages over $0\nu 2\beta$ decay:
    - Clean experimental signature
    - Free from uncertainties in nuclear matrix elements
- A good target for searching heavy neutral leptons at TeV scale

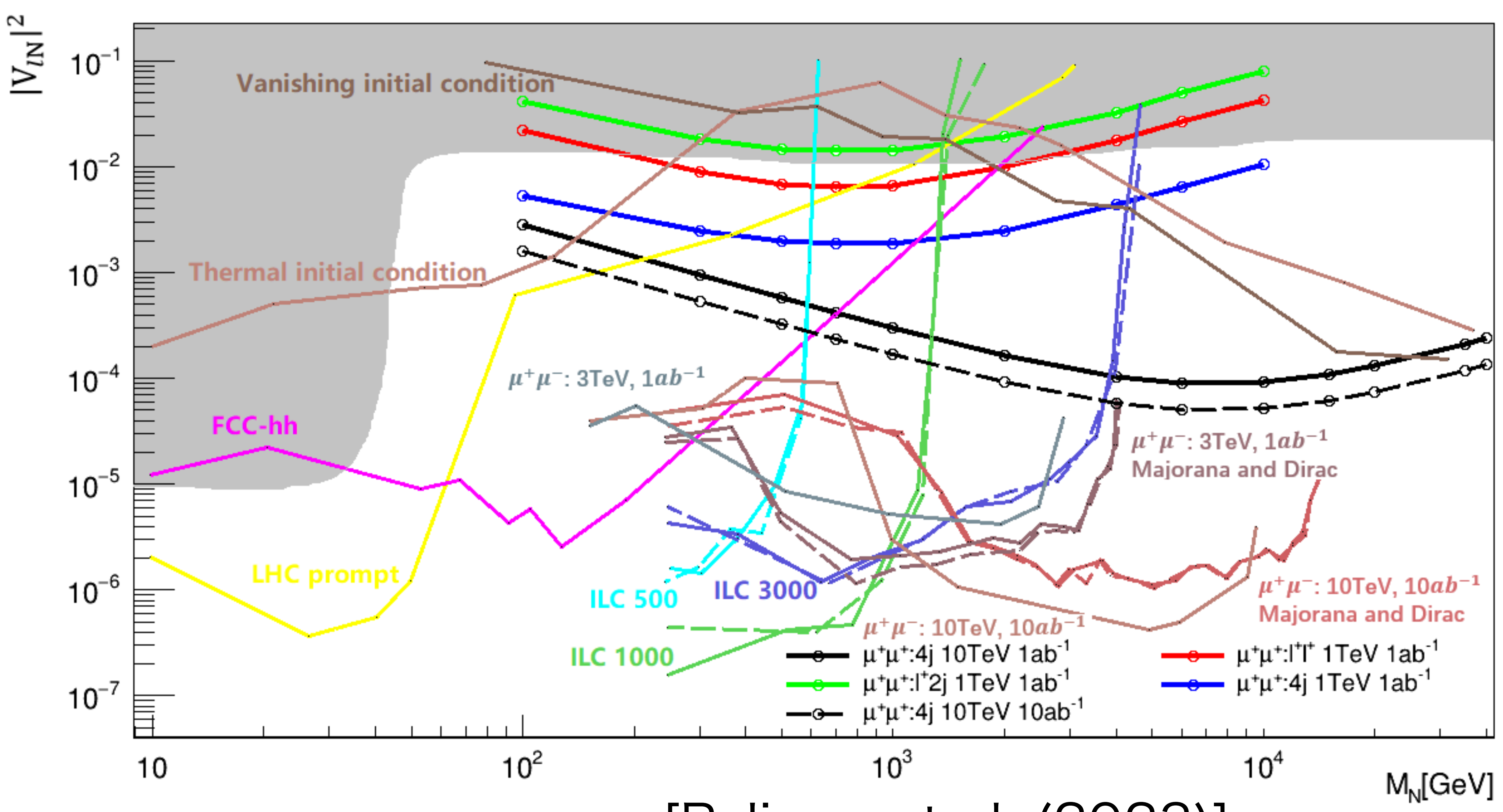


Sensitivity at future lepton colliders



[C.Lima et al. (2024)]

$$\mu^\pm\mu^\pm \to W^\pm W^\pm$$



[R.Jiang et al. (2023)]

- Target: RHNs with TeV scale mass and large mixing elements

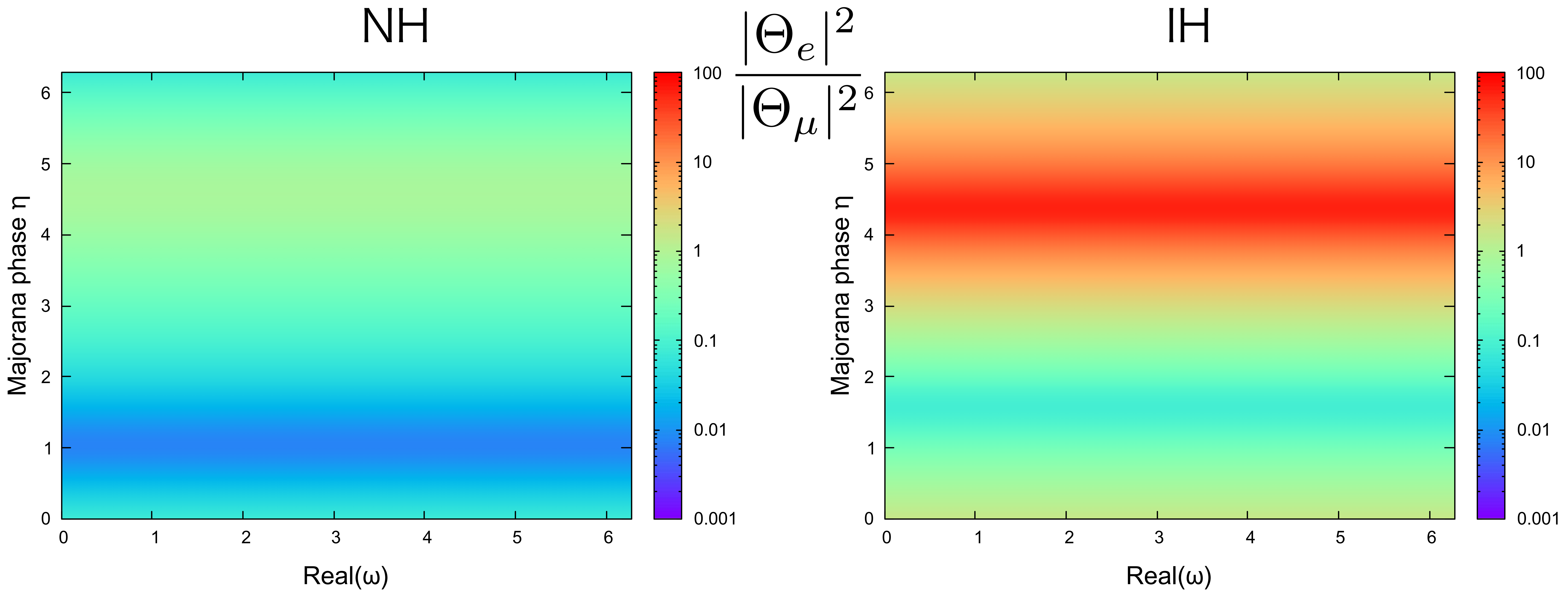
Upper bounds on

$$\sigma\left(e^{\pm}e^{\pm} \to W^{\pm}W^{\pm}, \mu^{\pm}\mu^{\pm} \to W^{\pm}W^{\pm}\right)$$

Case for the seesaw mechanism

- $l^\pm l^\pm \to W^\pm W^\pm$ is induced at $|\Theta_l|^4$ effect, and large Θ_l is required
- $e^\pm e^\pm \to W^\pm W^\pm$ is severely restricted by stringent constraint on Θ_e from $0\nu 2\beta$ decay [Asaka, Tsuyuki(2015)]
- As for $\mu^\pm \mu^\pm \to W^\pm W^\pm$, such a constraint might be avoided and cross section might be large at detectable level
- However, Θ_e and Θ_μ are not independent free parameters and are related to each other in the seesaw mechanism
- In this analysis, we estimate the upper bounds on cross sections of $e^\pm e^\pm \to W^\pm W^\pm$ and $\mu^\pm \mu^\pm \to W^\pm W^\pm$ by imposing the constraints from the $0\nu 2\beta$ decay as well as the electroweak precision measurements

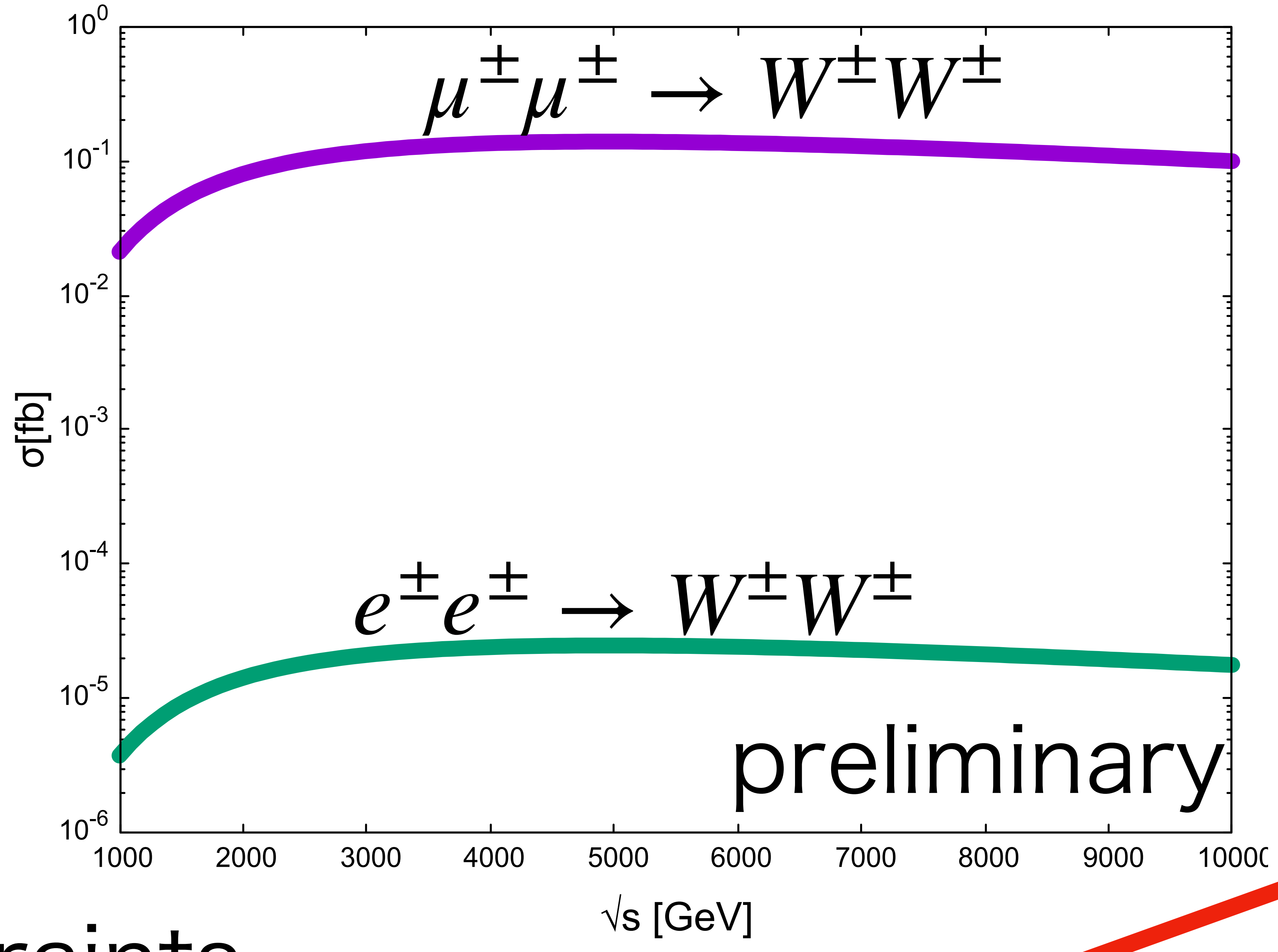
Ratio between mixing elements



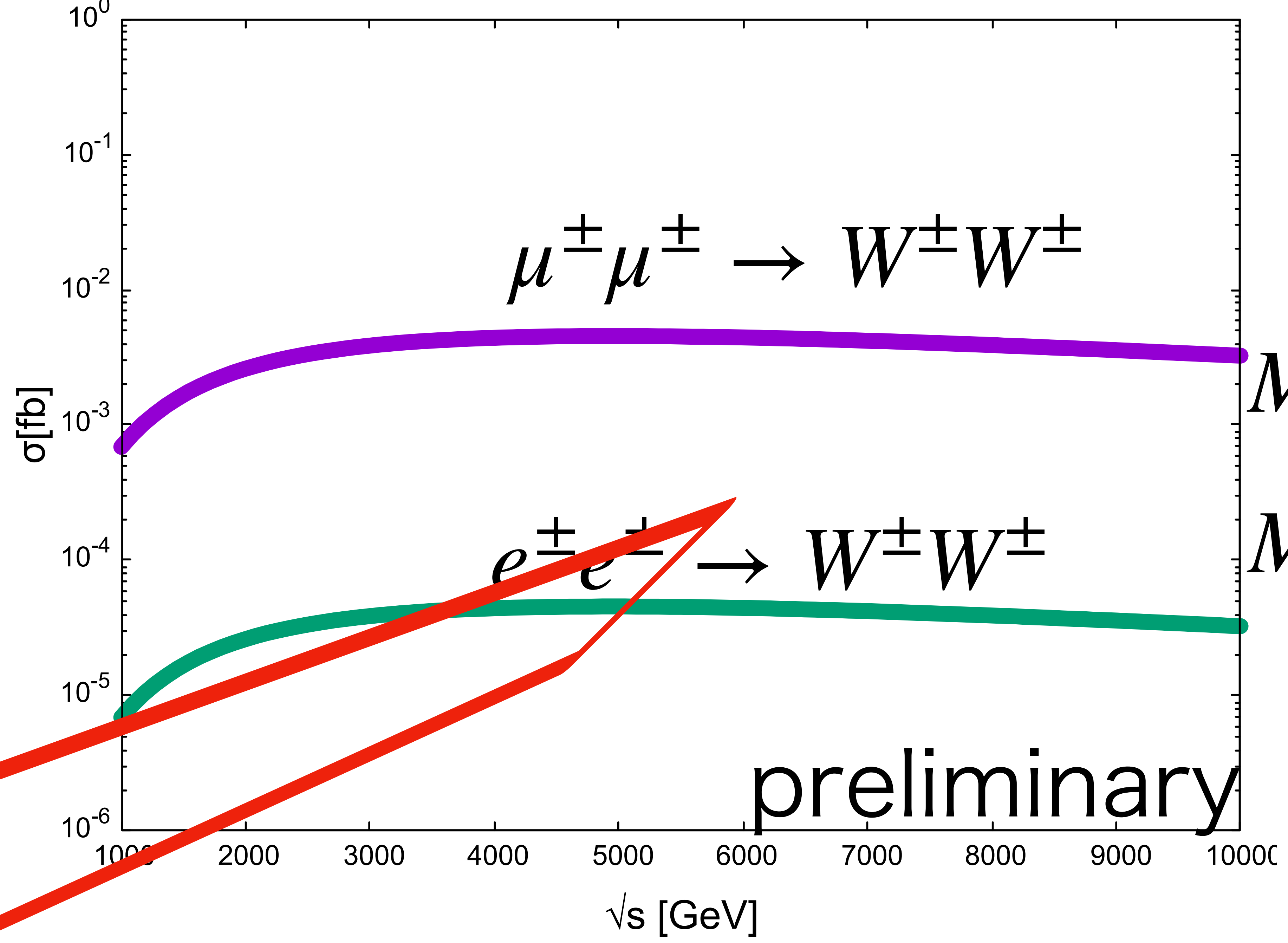
- In the seesaw mechanism, the mixing elements $|\Theta_e|^2$ and $|\Theta_\mu|^2$ are related to each other
- In the NH case, $|\Theta_e|^2$ can be much smaller than $|\Theta_\mu|^2$, and thus a stringent bound on $|\Theta_e|^2$ from the $0\nu2\beta$ decay does not affect $|\Theta_\mu|^2$

Upper bounds on cross sections

NH



IH



$M_1 = 1$TeV,

$M_2 = 10$TeV

Constraints

1.$0\nu2\beta$ decay:m_{eff} <122meV [KamLAND-Zen(2024)]

2.EW precision measurements: upper bounds on $|\Theta_{\alpha I}\Theta^*_{\beta I}|$ [M.Blennow(2023)]

In the seesaw mechanism with two RHNs,

$\mu^\pm\mu^\pm \to W^\pm W^\pm$ is a good target of future muon colliders for the NH

Summary

- We considered the seesaw mechanism with two RHNs and investigated the lepton number violating processes when RHNs are TeV scale
- Especially, we discussed $e^{\pm}e^{\pm} \to W^{\pm}W^{\pm}$, $\mu^{\pm}\mu^{\pm} \to W^{\pm}W^{\pm}$ at future lepton colliders
- The cross sections of $e^{\pm}e^{\pm} \to W^{\pm}W^{\pm}$, $\mu^{\pm}\mu^{\pm} \to W^{\pm}W^{\pm}$ are severely restricted by the $0\nu2\beta$ decay
- We found upper bound on $\mu^{\pm}\mu^{\pm} \to W^{\pm}W^{\pm}$ is relaxed for the NH, because $\frac{|\Theta_e|^2}{|\Theta_\mu|^2} = O(10^{-2} - 1)$ for NH while $O(10^{-1} - 10)$ for IH
- $\mu^{\pm}\mu^{\pm} \to W^{\pm}W^{\pm}$ is a good target of future muon colliders for the NH

Back up

Constraints from electroweak precision measurements

[M.Blennow et al. (2023)]

2N-SS	Normal Ordering		Inverted Ordering	
	68%CL	95%CL	68%CL	95%CL
$\eta_{ee} = \frac{\lvert\theta_e\rvert^2}{2}$	$6.4 \cdot 10^{-6}$	$9.4 \cdot 10^{-6}$	$[0.98, 4.4] \cdot 10^{-4}$	$5.5 \cdot 10^{-4}$
$\eta_{\mu\mu} = \frac{\lvert\theta_\mu\rvert^2}{2}$	$6.9 \cdot 10^{-5}$	$1.3 \cdot 10^{-4}$	$[0.20, 1.0] \cdot 10^{-6}$	$3.2 \cdot 10^{-5}$
$\eta_{\tau\tau} = \frac{\lvert\theta_\tau\rvert^2}{2}$	$8.6 \cdot 10^{-5}$	$2.1 \cdot 10^{-4}$	$[0.94, 2.8] \cdot 10^{-5}$	$4.5 \cdot 10^{-5}$
$\mathrm{Tr}\,[\eta] = \frac{\lvert\theta\rvert^2}{2}$	$1.6 \cdot 10^{-4}$	$2.9 \cdot 10^{-4}$	$[1.1, 4.8] \cdot 10^{-4}$	$6.0 \cdot 10^{-4}$
$\lvert\eta_{e\mu}\rvert = \frac{\lvert\theta_e\theta_\mu^*\rvert}{2}$	$8.3 \cdot 10^{-6}$	$1.2 \cdot 10^{-5}$	$[0.37, 1.0] \cdot 10^{-5}$	$1.3 \cdot 10^{-5}$
$\lvert\eta_{e\tau}\rvert = \frac{\lvert\theta_e\theta_\tau^*\rvert}{2}$	$1.5 \cdot 10^{-5}$	$2.2 \cdot 10^{-5}$	$[0.25, 1.2] \cdot 10^{-4}$	$1.4 \cdot 10^{-4}$
$\lvert\eta_{\mu\tau}\rvert = \frac{\lvert\theta_\mu\theta_\tau^*\rvert}{2}$	$7.2 \cdot 10^{-5}$	$1.3 \cdot 10^{-4}$	$[0.38, 3.0] \cdot 10^{-6}$	$3.5 \cdot 10^{-5}$

$$\theta_\alpha \theta_\beta^* = \sum_I \Theta_{\alpha I} \Theta_{\beta I}^*$$

To get large mixing and cross section

$$\Theta_{\mu I} = \frac{Y_{\mu I} v}{M_I}$$

⇒ Want to get large Yukawa coupling

Use Casas-Ibarra parametrization

$$Y \propto \Omega = \begin{pmatrix} 0 & 0 \\ \cos\omega & -\sin\omega \\ \sin\omega & \cos\omega \end{pmatrix}, \begin{pmatrix} \cos\omega & -\sin\omega \\ \sin\omega & \cos\omega \\ 0 & 0 \end{pmatrix}$$

$$\cos\omega, \sin\omega \sim e^{-i(Re\omega)} e^{Im\omega}$$

$$\Downarrow$$

We can enhance $X_\omega \equiv e^{Im\omega}$

⇒Both mixing and cross section increase

Experimental upper bound on mixing

- Mixing is constrained by various experiments.

- Data come from direct search (beam dump, collider) and indirect search ($0\nu2\beta$, EW precision, meson decays)

Flavor	Main experiments	Bounds
electron	$0\nu2\beta$, TRISTAN, DELPHI	$10^{-8} - 10^{-3}$
muon	CHARM-II , NuTeV, Bell II	$10^{-8} - 10^{-3}$
tauon	Bell, DELPHI, ATLAS	$10^{-5} - 10^{-2}$

Prospects of lepton colliders

- ILC(Japan)

  e^-e^+ collider(250-500 GeV), 1-2ab^{-1}

- CILC(CERN)

  e^-e^+ collider(Max 3 TeV), $5ab^{-1}$

- μTRISTAN(Japan)

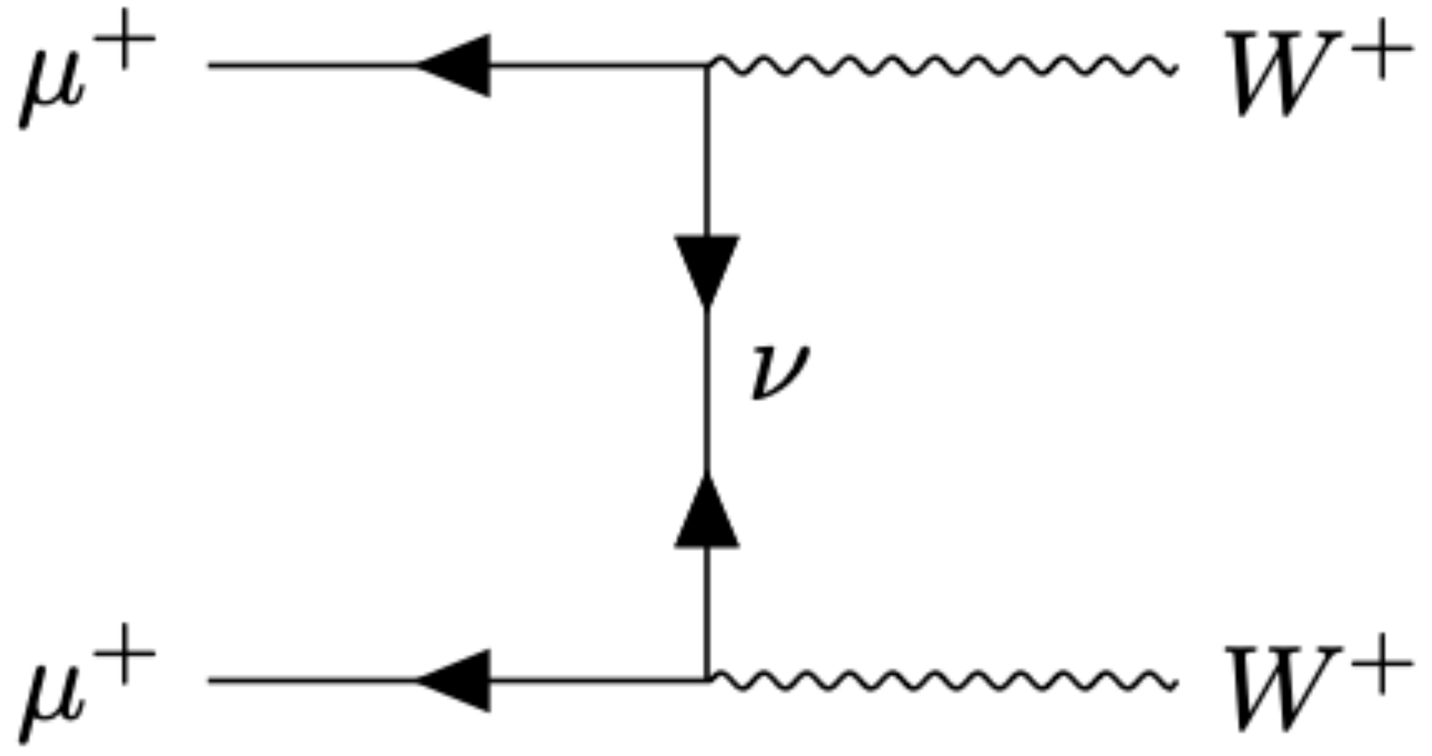
  $\mu^+\mu^+, \mu^+e^-$ collider(Max 3 TeV), $6 \times 10^{-7} fb^{-1} s^{-1}$

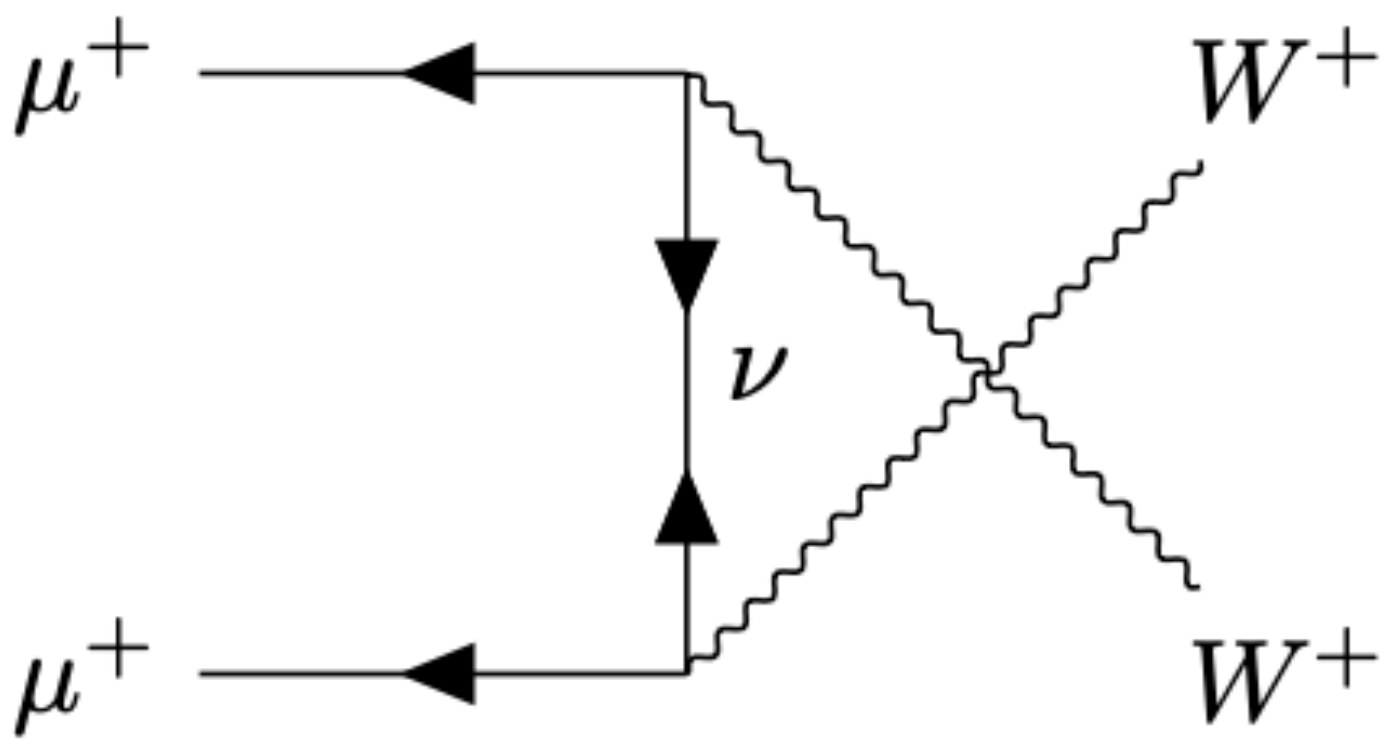
- Muon collider(USA)

  $\mu^+\mu^-$collider(Max 10 TeV), $10ab^{-1}$

Processes

$$\mathcal{L} = +\frac{g}{\sqrt{2}} W_\mu^+ \bar{\nu}_\mu^c \gamma^\mu P_R \mu^c + h.c.$$





These processes violate lepton number and are forbidden in the SM

- Key parameter

$$\Theta_{\mu I} = \frac{Y_{\mu I} v}{M_I}$$