Summer Institute 2025

A Z4 Symmetric Model for Self-Interacting Dark Matter and its Detection Probes

(TO APPEAR)

Lucca Radicce Justino
Chung Ang University Phd Student - THEP Group
Collaborators: Hyun Min Lee, Seong-Sik Kim, Jun-Ho Song

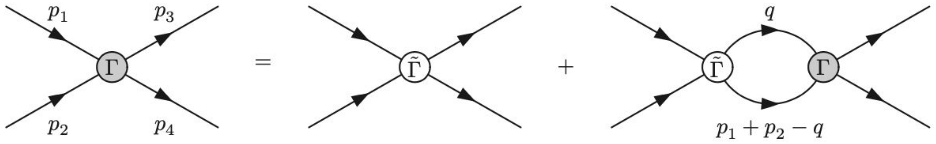
Utop Marina Hotel & Resort, Yeosu, South Korea
August 21st, 2025

Contents

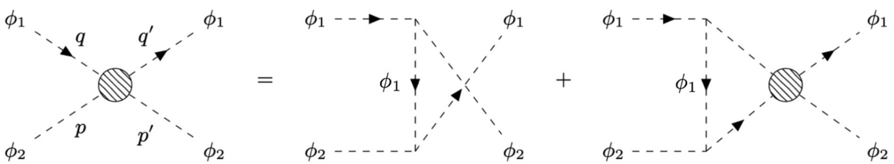
Introduction

- Small velocities of DM in galactic scales lead to **non-perturbative** effects from QFT ->

DM annihilation and self-scattering can be enhanced by the Sommerfeld Effect.



- We propose a model of two-component dark matter which provide a mechanism for the Sommerfeld Enhancement through a resonance in the u-channel.

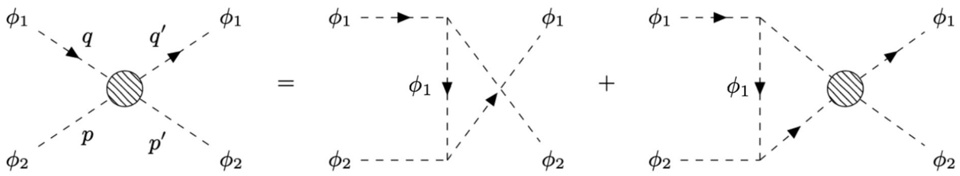


$$\widetilde{\Gamma}_{\phi_1\phi_2\to\phi_1\phi_2}(p,q;p',q') = -\frac{4g_1^2 m_1^2}{(p-q')^2 - m_1^2}$$

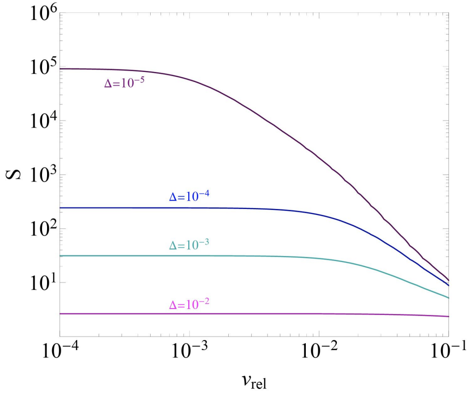
$$\widetilde{\Gamma}_{\phi_1\phi_2^\dagger\to\phi_1\phi_2}(p,q;p',q') = -\frac{4g_1 g_2 m_1^2}{(p-q')^2 - m_1^2}$$

Introduction

- It is possible to enhance DM cross sections considering **two-component DM with a 2:1 mass ratio (self-resonant dark matter)**.



$$\left[-\frac{1}{2\mu}\left(\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}\right)+\frac{l(l+1)}{2\mu r^2}\right]R_l(r)-\frac{\alpha}{r}e^{-Mr}(-1)^l R_l(br)=E\,R_l(r)$$



(work by HM Lee, SS Kim and B Zhu arXiv:2108.06278v2)

- Observables: self-scattering (galactic scale data), nuclear recoil (direct detection), cosmic particles (indirect detection).
- Our proposal: **study a SRDM model stabilized by a Z4 symmetry and its detection limits.**

Z4 symmetric SRDM

- Setup: $$\mathcal{L} = |D_\mu \chi|^2 + |D_\mu \phi_1|^2 + |D_\mu \phi_2|^2 - \frac{1}{4} X_{\mu\nu} X^{\mu\nu} - \frac{1}{2} \sin\xi \, X_{\mu\nu} B^{\mu\nu} - V(\chi, \phi_1, \phi_2) + \mathcal{L}_{H,\text{portal}}$$

$$\begin{aligned} V(\chi, \phi_1, \phi_2) &= m_\chi^2 |\chi|^2 + \lambda_\chi |\chi|^4 + m_1^2 |\phi_1|^2 + \lambda_1 |\phi_1|^4 + m_2^2 |\phi_2|^2 + \lambda_2 |\phi_2|^4 \\ &+ \sum_{i=1,2} \lambda_{\chi i} |\chi|^2 |\phi_i|^2 + \lambda_{12} |\phi_1|^2 |\phi_2|^2 + (g_1 m_1 \phi_2^\dagger \phi_1^2 + \text{h.c.}) \\ &+ \left(\kappa_1 \chi^\dagger \phi_2^2 + \kappa_2 \chi^\dagger \phi_2 \phi_1^2 + \text{h.c.} \right) \end{aligned}$$

	ϕ_1	ϕ_2	χ
$U(1)'$	$+1$	$+2$	$+4$
$U(1)' \supset Z_4$	i	-1	$+1$

(U(1)' symmetry is broken into Z4 by the VEV of the dark Higgs)

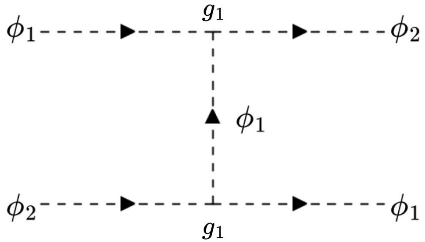
- The real and imaginary component of $\boldsymbol{\phi}_2$ are decoupled because of the induced mass splitting ($\boldsymbol{\phi}_1$ and the lightest $\boldsymbol{\phi}_2$ component will be the DM particle):

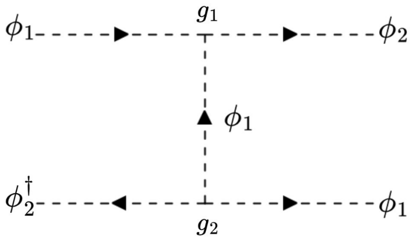
$$\begin{aligned} m_{1,\text{eff}}^2 &\equiv m_1^2 + \frac{1}{2} \lambda_{\chi 1} v_\chi^2, \\ m_{2,\text{eff}}^2 &\equiv m_2^2 + \frac{1}{2} \lambda_{\chi 2} v_\chi^2, \\ m_3^2 &\equiv \frac{1}{\sqrt{2}} \kappa_1 v_\chi, \\ g_2 &\equiv \frac{\kappa_2 v_\chi}{\sqrt{2} m_1}. \end{aligned}$$

$$\begin{aligned} m_s^2 &= m_{2,\text{eff}}^2 + 2 m_3^2, \\ m_a^2 &= m_{2,\text{eff}}^2 - 2 m_3^2. \end{aligned}$$

Z4 symmetric SRDM

- For the Z4 model, two ladder diagrams contribute for the SE:





$$\widetilde{\Gamma}_{\phi_1\phi_2\to\phi_1\phi_2}(p,q;p',q') = -\frac{4g_1^2 m_1^2}{(p-q')^2 - m_1^2}$$

$$\widetilde{\Gamma}_{\phi_1\phi_2^\dagger\to\phi_1\phi_2}(p,q;p',q') = -\frac{4g_1 g_2 m_1^2}{(p-q')^2 - m_1^2}$$

- This defines two Bethe-Salpeter functions (**conversion between states**).

$$\begin{aligned} i\chi_A(p,q) &\simeq -G_2(p)G_1(q)\int\frac{d^4k}{(2\pi)^4}\Big[\widetilde{\Gamma}_{\phi_2\phi_1\to\phi_2\phi_1}(p,q;p+q-k,k)\chi_A(p+q-k,k) \\ &\quad +\widetilde{\Gamma}_{\phi_2\phi_1\to\phi_2^*\phi_1}(p,q;p+q-k,k)\chi_B(p+q-k,k)\Big], \\ i\chi_B(p,q) &\simeq -G_2(p)G_1(q)\int\frac{d^4k}{(2\pi)^4}\Big[\widetilde{\Gamma}_{\phi_2^*\phi_1\to\phi_2\phi_1}(p,q;p+q-k,k)\chi_A(p+q-k,k) \\ &\quad +\widetilde{\Gamma}_{\phi_2^*\phi_1\to\phi_2^*\phi_1}(p,q;p+q-k,k)\chi_B(p+q-k,k)\Big] \end{aligned}$$

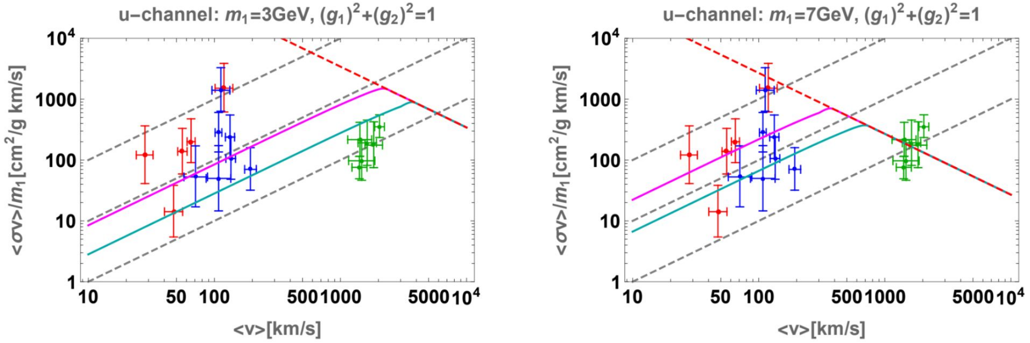
Z4 symmetric SRDM

Defining the position-space wave-function from the Fourier transf., one obtains coupled Schrodinger-like equations:

$$-\frac{1}{2\mu}\nabla^2\begin{pmatrix}\psi_A(\vec{x})\\ \psi_B(\vec{x})\end{pmatrix}+V(\vec{x})\begin{pmatrix}1 & \frac{g_2}{g_1}\\ \frac{g_2}{g_1} & \left(\frac{g_2}{g_1}\right)^2\end{pmatrix}\begin{pmatrix}\psi_A\left(-\frac{m_2}{m_1}\vec{x}\right)\\ \psi_B\left(-\frac{m_2}{m_1}\vec{x}\right)\end{pmatrix}=\begin{pmatrix}E_A & 0\\ 0 & E_B\end{pmatrix}\begin{pmatrix}\psi_A(\vec{x})\\ \psi_B(\vec{x})\end{pmatrix}$$

$$V_{\rm eff}(\vec{x})=-\left(1+\frac{g_2^2}{g_1^2}\right)\frac{\alpha}{r}\,e^{-Mr}\qquad\qquad M\equiv m_2\sqrt{2-\frac{m_2}{m_1}}.$$

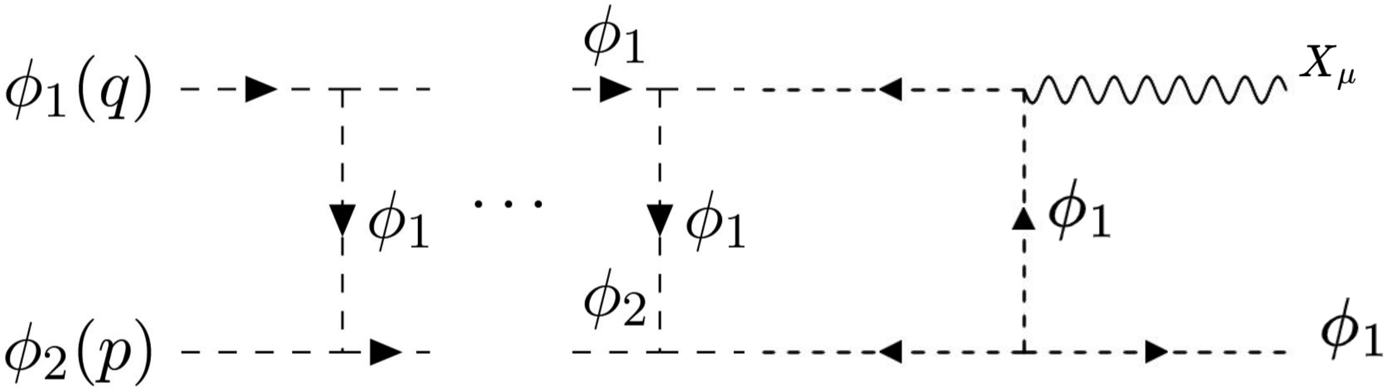
- Fitting of small-scale data:



(Done by Hyun Min Lee and Seong-Sik Kim)

Z4 symmetric SRDM: u-channel resonance

The u-channel resonance enhances processes with initial states ϕ1 and ϕ2. This can be seen through the following ladder diagram:



Abundance benchmarks

The abundance of species are set by

- 1) **DM annihilation to SM/DS**,
- 2) **DM inelastic scattering** and
- 3) **DM semi annihilation.**

$$\begin{aligned}\dot{n}_1 + 3Hn_1 &= -\frac{1}{2}\langle\sigma v\rangle_{\phi_1\phi_1^\dagger \to f\bar{f}, VV, h_1h_1, h_2h_2, h_1h_2, XX}\left(n_1^2 - (n_1^{\rm eq})^2\right) \\ &\quad + \frac{1}{2}\langle\sigma v\rangle_{\phi_2\phi_2^\dagger, \phi_2\phi_2, \phi_2^\dagger\phi_2^\dagger \to \phi_1\phi_1^\dagger}\left(n_2^2 - n_1^2\right),\end{aligned}$$

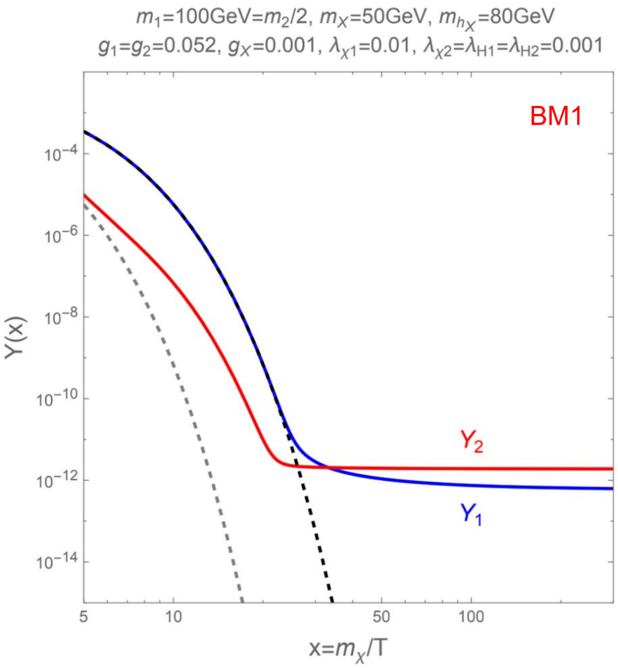
$$\begin{aligned}\dot{n}_2 + 3Hn_2 &= -\frac{1}{2}\langle\sigma v\rangle_{\phi_2\phi_2^\dagger \to f\bar{f}, VV, h_1h_1, h_2h_2, h_1h_2, XX}\left(n_2^2 - (n_2^{\rm eq})^2\right) \\ &\quad - \frac{1}{2}\left(\langle\sigma v\rangle_{\phi_2\phi_2^\dagger \to \phi_1\phi_1^\dagger} + 2\langle\sigma v\rangle_{\phi_2\phi_2 \to \phi_1\phi_1^\dagger}\right)\left(n_2^2 - n_1^2\right) \\ &\quad - \frac{1}{2}\sum_{i = X, h_1, h_2}\left(\langle\sigma v\rangle_{\phi_2\phi_1^\dagger \to \phi_1 i} + \langle\sigma v\rangle_{\phi_2\phi_1 \to \phi_1^\dagger i}\right) n_1\left(n_2 - n_2^{\rm eq}\right).\end{aligned}$$

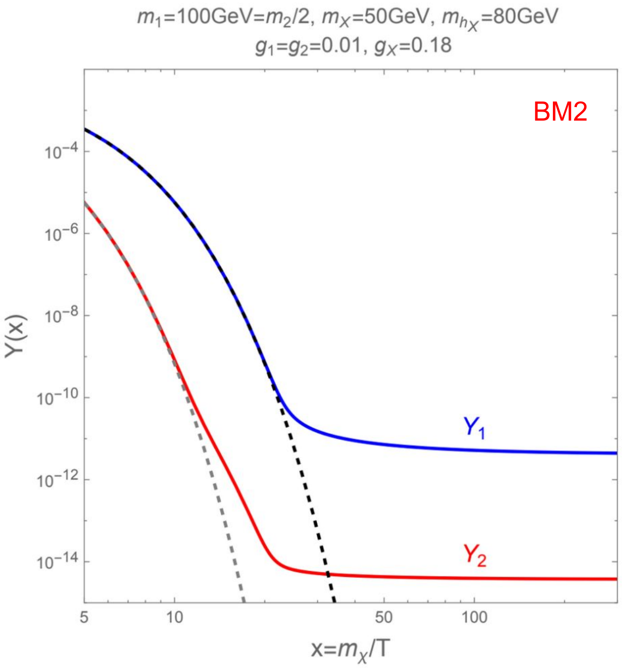
Recasting:

$$\frac{dY_1}{dx} = -\sigma_{11}(Y_1^2 - (Y_1^{eq})^2) + \sigma_{22}\left(Y_2^2 - Y_1^2\right)$$

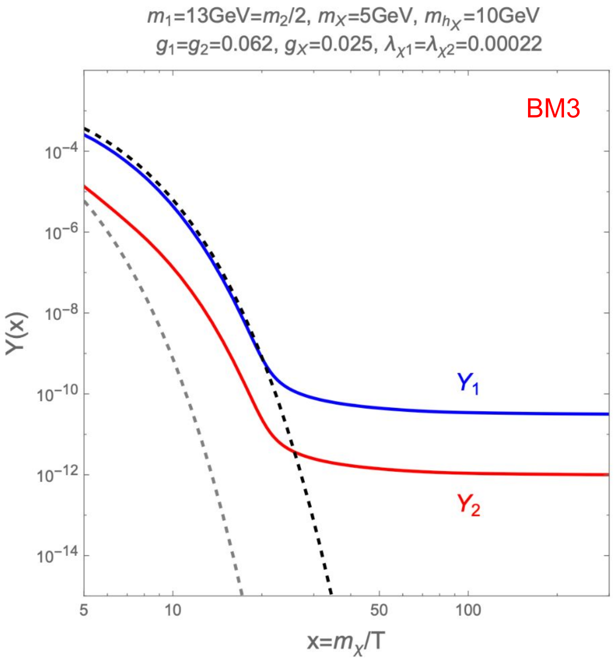
$$\frac{dY_2}{dx} = -\sigma_{21}(Y_1^2 - (Y_1^{eq})^2) - \sigma_{22}\left(Y_2^2 - Y_1^2\right) - \sigma_{23}Y_1\left(Y_2 - Y_2^{eq}\right)$$

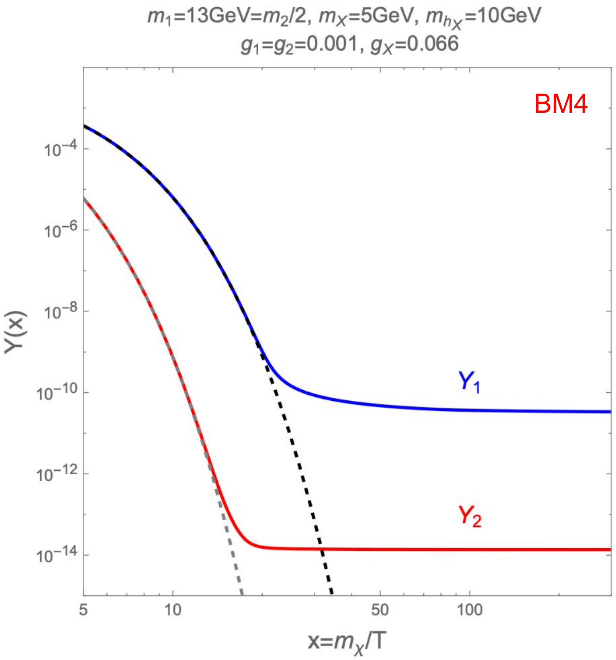
Abundance benchmarks





Abundance benchmarks





Abundance benchmarks: multi-GeV dark matter

	$m_2 \simeq 2m_1$ [GeV]	m_X [GeV]	m_{h_1} [GeV]	$g_{1,2}$	g_X	$\langle\sigma v\rangle^0_{\phi_1^\dagger\phi_2^\dagger\to\phi_1 X}$ [cm^3/s]	$\langle\sigma v\rangle^0_{\phi_1^\dagger\phi_2^\dagger\to\phi_1 h_1}$ [cm^3/s]	$r_1 = \frac{\Omega_1}{\Omega_{\rm DM}}$	$S_0 \times(\mathrm{BR}_{X,h_1}(e^+e^-))$
B1	100	50	80	0.052	0.001	3.3×10^{-30}	8.8×10^{-27}	0.141	$1.98\times10^{6}(X), 752(h_1)$
B2	100	50	80	0.01	0.18	4.0×10^{-27}	3.9×10^{-27}	0.998	$1.17\times10^{5}(X), 1.21\times10^{5}(h_1)$
B3	26	5	10	0.062	0.025	3.1×10^{-25}	2.9×10^{-25}	0.940	$6.07(X), 6.44(h_1)$
B4	26	5	10	0.001	0.066	5.5×10^{-28}	5.2×10^{-28}	0.999	$2.34\times10^{5}(X), 2.48\times10^{5}(h_1)$

We are using the limits from the CMB recombination to constrain the SE factor times branching ratio to e+e-:

$$\langle\sigma v\rangle_{\phi_1^\dagger\phi_2^{(\dagger)}\to\phi_1 X} < 4\times10^{-25}\,\mathrm{cm}^3/\mathrm{s}\left(\frac{f_{\rm eff}}{0.1}\right)^{-1}\cdot\frac{1}{r_1(1-r_1)}\cdot\left(\frac{m_2}{100\,\mathrm{GeV}}\right)$$

$$f_{\rm eff}(m_2) = \frac{\int_0^{m_2/2} dE_e\, E_e\, 2 f_{\rm eff}^{e^+e^-}\,\frac{dN_e}{dE_e}}{m_2}$$

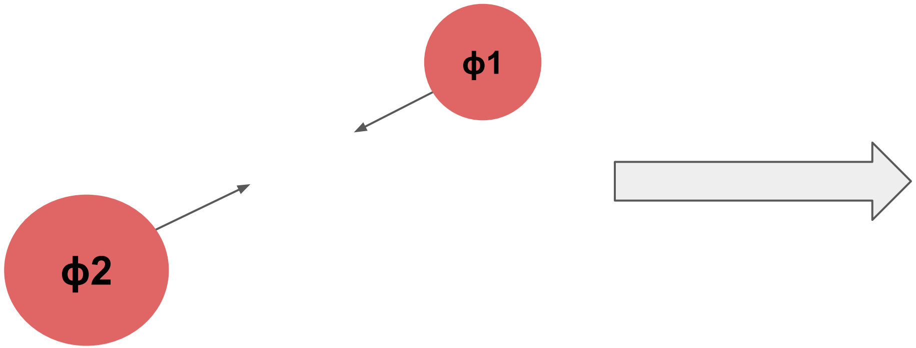
Abundance benchmarks: (sub-) GeV

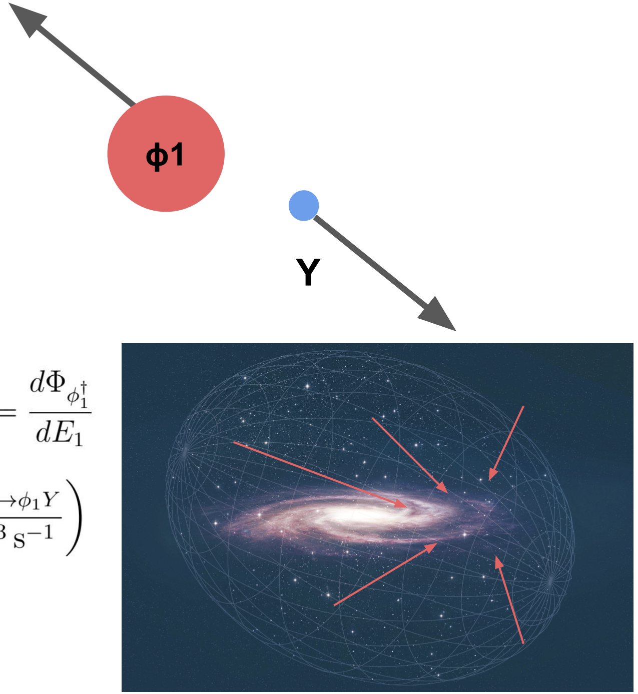
	$m_2 \simeq 2m_1$ [GeV]	m_X [GeV]	m_{h_1} [GeV]	$g_{1,2}$	g_X	$\langle \sigma v \rangle^0_{\phi_1^\dagger \phi_2^\dagger \to \phi_1 X}$ [cm^3/s]	$\langle \sigma v \rangle^0_{\phi_1^\dagger \phi_2^\dagger \to \phi_1 h_1}$ [cm^3/s]	$r_1 = \frac{\Omega_1}{\Omega_{\rm DM}}$	$S_0 \times (\mathrm{BR}_{X,h_1}(e^+e^-))$
B5	6	2	5	0.035	0.03	8.8×10^{-25}	4.7×10^{-25}	0.939	$0.51(X), 0.90(h_1)$
B6	1	1.5	3	0.035	2.7	–	–	0.336	–
B7	1	1.5	0.8	0.014	1.8	–	4.9×10^{-22}	0.9999	$0.09(h_1)$
B8	1	1.5	0.4	0.012	0.8	–	1.3×10^{-22}	0.9998	$0.15(h_1)$
B9	1	0.8	1.5	0.017	0.016	6.6×10^{-25}	–	0.913	$0.050(X)$
B10	1	0.4	1.5	0.012	0.03	5.4×10^{-25}	–	0.817	$0.076(X)$
B11	0.2	0.6	0.04	0.0039	0.5	–	3.4×10^{-23}	0.9998	$0.12(h_1)$
B12	0.2	0.04	0.6	0.008	0.0022	6.1×10^{-25}	–	0.679	$0.006(X)$

Boosted Dark Matter

Dark mediators DM and the SM: production through semi-annihilation of DM -> boost for the final states

$$T_1' = \frac{m_2^2 - m_Y^2}{2(m_1 + m_2)}$$





$$\frac{d\Phi_{\phi_1}}{dE_1} = \frac{1}{8\pi m_1 m_2} \langle \sigma v \rangle_{\phi_1^\dagger \phi_2^{(\dagger)} \to \phi_1 Y} \cdot \frac{dN_{\phi_1}}{dE_1} \frac{1}{2} r_1 (1 - r_1) \int d\Omega \int_{\mathrm{l.o.s.}} ds\, \rho_{\mathrm{DM}}^2(r) = \frac{d\Phi_{\phi_1^\dagger}}{dE_1}$$

$$\Phi_{\mathrm{BSM}} = \frac{m_1}{2m_2} r_1(1 - r_1) \cdot \left(3.2 \times 10^{-3}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}\right) \left(\frac{m_1}{100\,\mathrm{MeV}}\right)^{-2} \left(\frac{\langle \sigma v \rangle_{\phi_1^\dagger \phi_2^{(\dagger)} \to \phi_1 Y}}{10^{-26}\,\mathrm{cm}^3\,\mathrm{s}^{-1}}\right)$$

Boosted dark matter particles can travel through the galaxy and be detected by direct detection or neutrinos experiments!

Nucleus-boosted DM scattering

- Considering the **relativistic** nucleus-DM scattering, the DM wavelength can be of the size or smaller than the nucleus size -> **both coherent and incoherent scattering can be present!**

$$
\begin{aligned}
\sigma_{\rm SI}^{\rm coh} &\equiv \frac{2}{r_1}\,\sigma^{\rm coh}_{\phi_1,\phi_1^\dagger-A} \\
&= \frac{\mu_{A,1}^2}{2\pi m_1^2}\left[\left(Zc_p^{(1)}f_p+(A-Z)c_n^{(1)}f_n\right)^2+Z^2(g_p^{(1)})^2\right], \\
\sigma_{\rm SI}^{\rm inc} &\equiv 2\,\sigma^{\rm inc}_{\phi_1,\phi_1^\dagger-A} \\
&= \frac{\mu_{A,1}^2}{2\pi m_1^2}\left[Z\left(c_p^{(1)}f_p\right)^2+Z(g_p^{(1)})^2+(A-Z)\left(c_n^{(1)}f_n\right)^2\right],
\end{aligned}
$$

$$
\begin{aligned}
\left(\frac{d\sigma_{\rm SI}}{dT_A}\right)_{\rm coh} &= \frac{\sigma_{\rm SI}^{\rm coh}}{T_{A,\rm max}}|F_{\rm SI}(q)|^2, \\
\left(\frac{d\sigma_{\rm SI}}{dT_A}\right)_{\rm inc} &= \frac{\sigma_{\rm SI}^{\rm inc}}{T_{A,\rm max}}\left(1-|F_{\rm SI}(q)|^2\right)
\end{aligned}
$$

$$
\begin{aligned}
c_N^{(i)} &\equiv \frac{m_N}{m_q}\left(\frac{\lambda_{q2}y_{h_2\phi_i^\dagger\phi_i}}{m_{h_2}^2}+\frac{\lambda_{q1}y_{h_1\phi_i^\dagger\phi_i}}{m_{h_1}^2}\right), \\
f_p &\equiv \sum_{q=u,d,s} f_{Tq}^{(p)}+\frac{2}{9}f_{TG}^{(p)} \simeq 0.28\,, \\
f_n &\equiv \sum_{q=u,d,s} f_{Tq}^{(n)}+\frac{2}{9}f_{TG}^{(n)} \simeq 0.28\,, \\
g_p^{(i)} &= -\frac{2eq_i g_X \varepsilon m_i}{m_X^2}, \\
g_n^{(i)} &\approx 0\,.
\end{aligned}
$$

Nucleus-boosted DM scattering

- The differential event rate will be given by

$$\frac{dR_A}{dT_A} = \frac{1}{m_A}\int_{T_{1,\mathrm{min}}}^{\infty} dT_1 \, \frac{d\sigma_{\mathrm{SI}}}{dT_A} \, \frac{d\Phi_{\phi_1}}{dT_1}$$

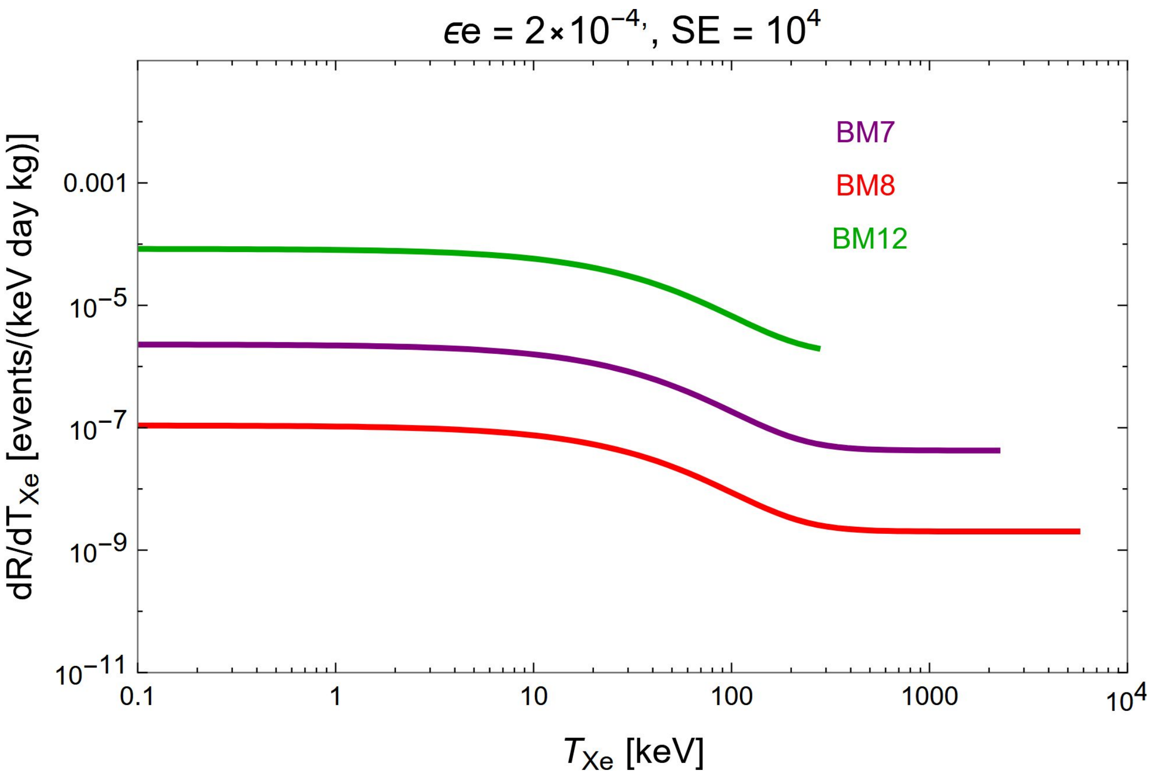
- We have to consider that a given recoil energy is produced minimum DM kinetic energy given by

$$T_{1,\mathrm{min}} = \left(\frac{T_A}{2} - m_1\right)\left[1 \pm \sqrt{1 + \frac{2T_A}{m_A}\frac{(m_1 + m_A)^2}{(T_A - 2m_1)^2}}\right]$$

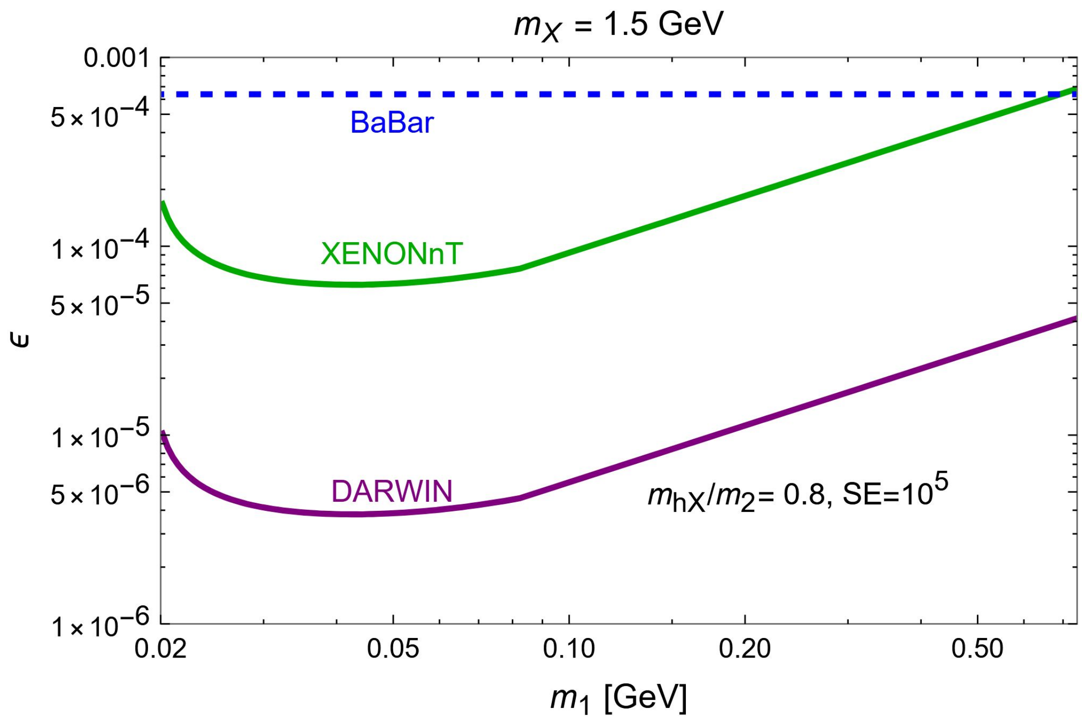
- Reciprocally, a given DM energy can produce, at most, a given recoil energy:

$$T_{A,\mathrm{max}} = \frac{T_1(T_1 + 2m_\chi)}{(m_1 + m_A)^2/(2m_A) + T_1}$$

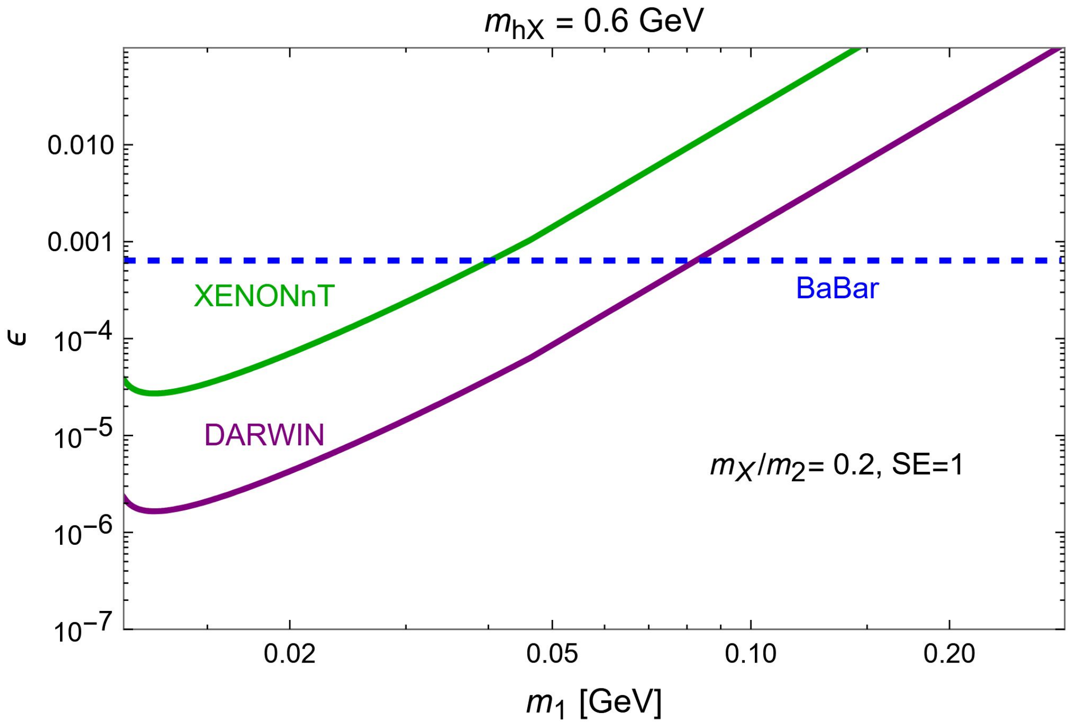
DD for boosted DM: Recoil energy



DD for boosted DM: Limits



DD for boosted DM: Recoil energy

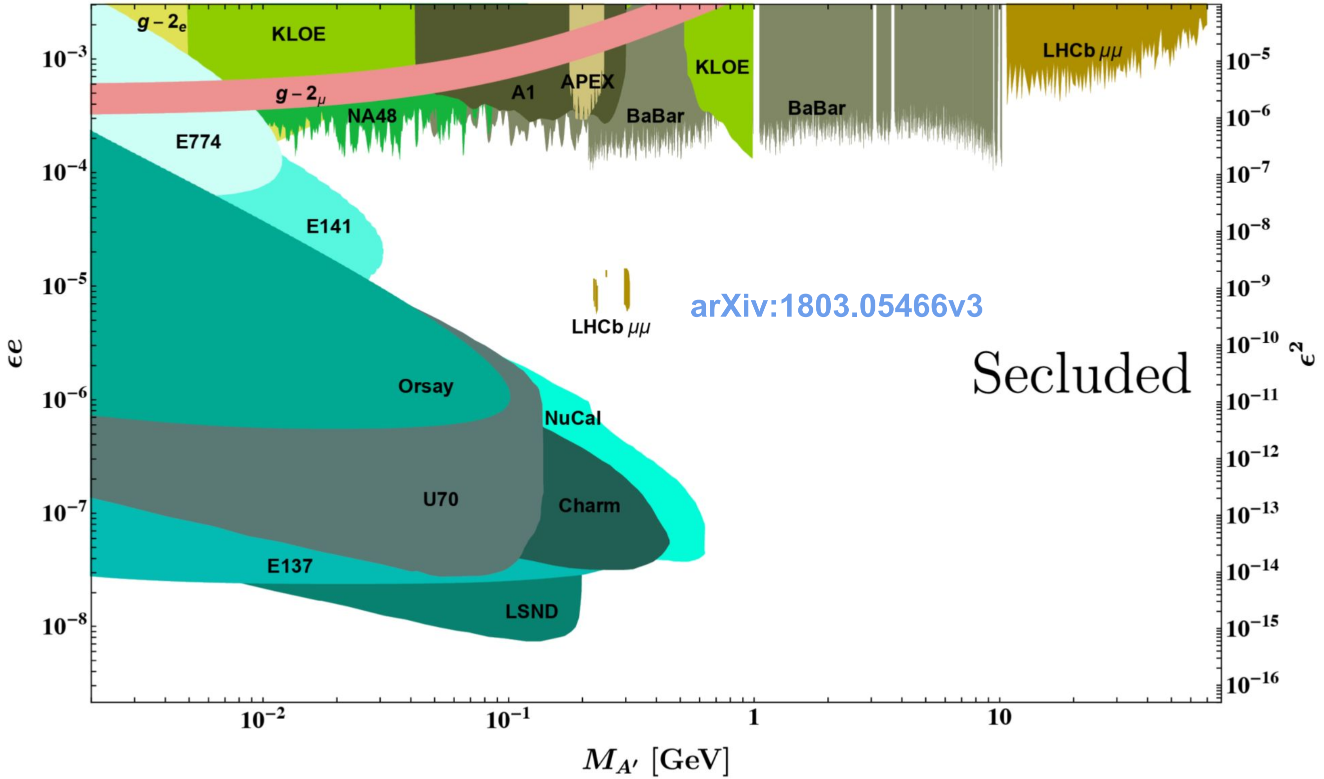


Conclusions and next steps

- The Z4 model provides a viable candidate of SRDM to explain **Galaxy scale data**.
- DM abundance can be provided by the thermal freeze-out in both GeV and multi-GeV (WIMP) limits.
- This model implies in a **boosted DM** components in addition to the halo DM and is constrained by current (XENONnT) and future experiments (DARWIN, DUNE).

Backup

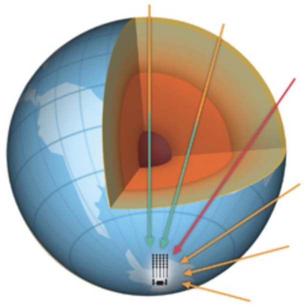
Beam dump Experiments



Lower limits: Earth's internal structure attenuation effect

If the proton-DM cross-section is too high, the boosted DM particles will scatter and lose kinetic energy below the detection threshold through their propagation inside Earth's internal structure, where the dense rock implies a high nuclear density.

$$\frac{dT_\chi}{dz} = -\sum_{\mathcal{T}} n_{\mathcal{T}} \int_0^{T_{\mathcal{T}}^{\max}} dT_{\mathcal{T}}\, T_{\mathcal{T}}\, \frac{d\sigma_{\chi\mathcal{T}}}{dT_{\mathcal{T}}}(T_\chi, T_{\mathcal{T}}),$$

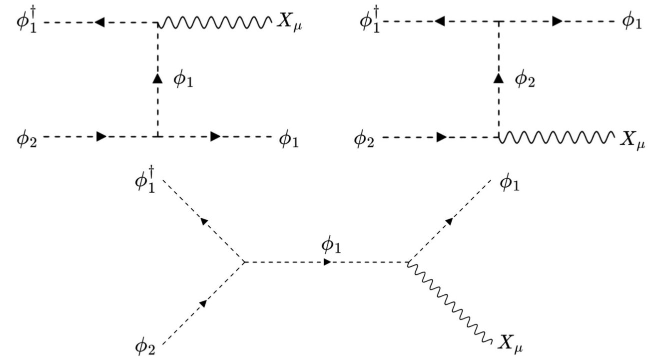


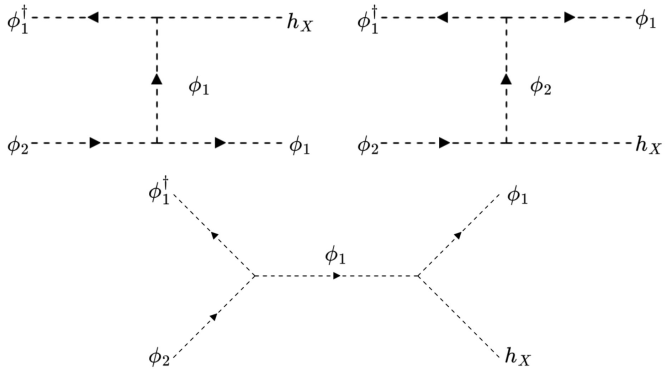
Interaction with the dark sector

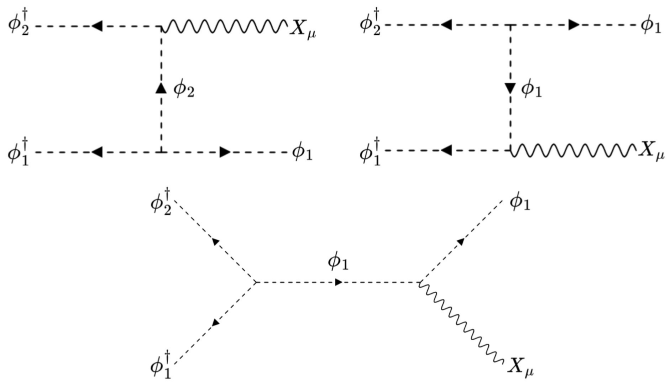
$$\mathcal{L}_{h_X-\phi_1,s,a} = -\lambda_\chi v_\chi h_\chi^3 - \frac{1}{4}\lambda_\chi h_\chi^4 - \frac{1}{2}\lambda_{\chi 1}(2v_\chi h_X + h_X^2)|\phi_1|^2 - \frac{1}{4}\lambda_{\chi 2}(2v_\chi h_X + h_X^2)(s^2+a^2)$$
$$-\frac{1}{\sqrt{2}}\kappa_1 h_X(s^2-a^2) - \frac{1}{2}\kappa_2 h_X s(\phi_1^2+\phi_1^{\dagger 2}) - \frac{1}{2}i\kappa_2 h_X a(\phi_1^2-\phi_1^{\dagger 2}). \quad (11)$$

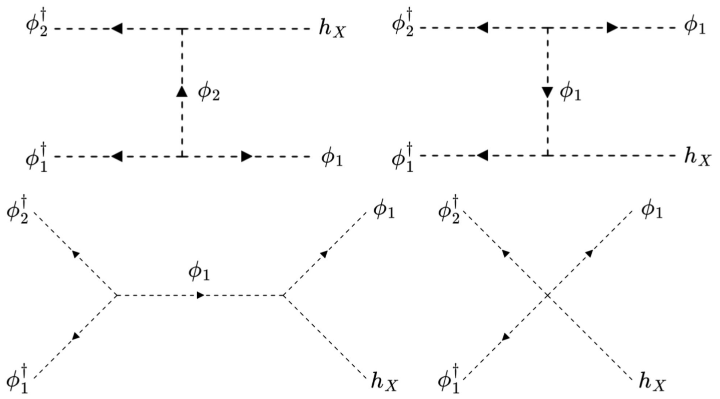
$$\begin{aligned}
\mathcal{L}_X &= \frac{m_X^2}{v_\chi} h_X X_\mu X^\mu + 8g_X^2 h_X^2 X_\mu X^\mu \\
&\quad + ig_X X_\mu(\phi_1^\dagger \partial^\mu \phi_1 - \phi_1 \partial^\mu \phi_1^\dagger) + g_X^2 X_\mu X^\mu \phi_1^\dagger \phi_1 \\
&\quad + 2ig_X X_\mu(\phi_2^\dagger \partial^\mu \phi_2 - \phi_2 \partial^\mu \phi_2^\dagger) + 4g_X^2 X_\mu X^\mu \phi_2^\dagger \phi_2 \\
&= \frac{m_X^2}{v_\chi} h_X X_\mu X^\mu + 8g_X^2 h_X^2 X_\mu X^\mu \\
&\quad + ig_X X_\mu(\phi_1^\dagger \partial^\mu \phi_1 - \phi_1 \partial^\mu \phi_1^\dagger) + g_X^2 X_\mu X^\mu \phi_1^\dagger \phi_1 \\
&\quad - 2g_X X_\mu(s\partial^\mu a - a\partial^\mu s) + 2g_X^2 X_\mu X^\mu (s^2+a^2).
\end{aligned}$$

Z4 Self-Resonant DM: Semi-annihilation diagrams









Interaction with the SM sector

$$\mathcal{L}_{\rm NC} \simeq A_\mu J^\mu_{\rm EM} + Z_\mu \left(J^\mu_Z + \varepsilon\, t_W J^\mu_X \right) + X_\mu \left(-\varepsilon J^\mu_{\rm EM} + J^\mu_X \right)$$

$$\begin{aligned}
J^\mu_{\rm EM} &= e \bar{f} \gamma^\mu Q_f f, \\
J^\mu_Z &= \frac{e}{2 s_W c_W} \bar{f} \gamma^\mu (\tau^3 - 2 s_W^2 Q_f) f, \\
J^\mu_X &= i g_X (\phi_1^\dagger \partial^\mu \phi_1 - \phi_1 \partial^\mu \phi_1^\dagger) - 2 g_X (s \partial^\mu a - a \partial^\mu s).
\end{aligned}$$

$$\mathcal{L}_H \supset -\lambda_{\chi H} |H|^2 |\chi|^2 - \lambda_{H1} |H|^2 |\phi_1|^2 - \lambda_{H2} |H|^2 |\phi_2|^2 - \lambda_H |H|^4 - m_H^2 |H|^2$$

$$\begin{aligned}
\mathcal{L}_{h_1,h_2} &= -y_{h_1 \phi_1^\dagger \phi_1} h_1 |\phi_1|^2 - y_{h_2 \phi_1^\dagger \phi_1} h_2 |\phi_1|^2 \\
&\quad -\frac{1}{2} y_{h_1 ss} h_1 (s^2 + a^2) - \frac{1}{2} y_{h_2 ss} h_2 (s^2 + a^2) - (\lambda_{h_1} h_1 + \lambda_{h_2} h_2) \bar{f} f
\end{aligned}$$