# Leptogenesis in the PQ pole inflation

Jun Ho Song, Chung Ang University

**PQ genesis** Eung Jin Chun, Hyun Min Lee, JH Song, (work in progress)

1. [Inflation models with Peccei-Quinn symmetry and axion kinetic misalignment, H.M Lee, A.G. Menkara, M.J Seong, J.H Song August 30, 2024]
2. [Peccei-Quinn Inflation at the Pole and Axion Kinetic Misalignment, H.M Lee, A.G. Menkara, M.J Seong, J.H Song JHEP 05 Oct 26, 2023]

2025 SI Workshop

# Motivation

**Axiogenesis** (arXiv:1910.02080v2):

Mechanism of spontaneous baryogenesis through the axion kinetic misalignment

**PQ genesis:**

Mechanism of spontaneous baryogenesis through the axion kinetic misalignment
**including Seesaw mechanism**

# Outline

- **Leptogenesis driven by majoron (arXiv.2311.09005v2)**

**PQ pole inflation (arXiv.2310.17710v2)**

# Outline

Before SSB $$S' - S = -\int d^4x\ \epsilon(x)\ \partial_\mu j^\mu_{\text{tot}} + \mathcal{O}(\epsilon^2) = 0$$

After SSB $$S' - S = -\int d^4x\ \epsilon(x)\partial_\mu(\partial^\mu\theta + j^\mu_{\text{B-L}}) + \mathcal{O}(\epsilon^2) = 0$$

- **Leptogenesis driven by majoron (arXiv.2311.09005v2)**

$$-\mathcal{L}_{\text{int}} = \frac{1}{2}\sum_I y_{N_I}\Phi\bar{N}_I^c N_I + \sum_{\alpha,\, I} Y_{N,\alpha I}\bar{l}_\alpha\tilde{H}N_I + h.c.$$

After Spontaneous symmetry breaking $U(1)_{B-L}$, B-L charge stored in the axial rotation convert to the B-L charge for lepton (lepton asymmetry)

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## PQ genesis

$$-\mathcal{L}_{\text{int}} = y_Q \Phi Q Q^c + \frac{1}{2}\sum_I y_{N_I}\Phi N N + \sum_{\alpha, I} y_{v,\alpha I}\bar{l}_\alpha \tilde{H} N_I + h.c.$$ (For KSVZ)

After Spontaneous symmetry breaking $U(1)_{PQ}$, PQ charge stored in the rotation of axion convert to the PQ charge for heavy quark and Right handed neutrino

Difference: For B-L charge, PQ complex field is neutral, so we just imposed B-L explicit breaking by considering Seesaw mechanism.

# Outline

"PQ mechanism is the theory solves the Strong CP problem by causing axions to settle down into a QCD vaccum"

## PQ pole inflation (arXiv.2310.17710v2)

$$\frac{\mathcal{L}_J}{\sqrt{-g_J}} = -\frac{1}{2}M_P^2\,\Omega(\Phi)R(g_J) + |\partial_\mu\Phi|^2 - \Omega^2(\Phi)V_E(\Phi)$$

$$V_E(\Phi) = V_0' + \frac{\beta_m}{M_P^{2m-4}}|\Phi|^{2m} - m_\Phi^2|\Phi|^2 + \left(\sum_{k=0}^{[n/2]}\frac{c_k}{2M_P^{n-4}}|\Phi|^{2k}\Phi^{n-2k} + \text{h.c.}\right)$$

Inflation can be driven by PQ field in the Einstein frame if **PQ field is conformally coupled to gravity**

Large **non zero axial velocity** (=PQ charge) was produced from PQ explicitly breaking potential originated by Quantum gravity effect during inflation.

# Cogenesis requirements

$$\mu_{l_i} + \mu_{e_i^c} - \mu_H = 0 \qquad \mu_{q_i} + \mu_{u_i^c} + \mu_H = c_S \frac{m_d^2}{m_u^2+m_d^2}\dot{\theta}\,\delta_{i1}$$

$$\mu_{l_i} + \mu_H = x_l \dot{\theta} \qquad \mu_{q_i} + \mu_{d_i^c} - \mu_H = c_S \frac{m_u^2}{m_u^2+m_d^2}\dot{\theta}\,\delta_{i1}$$

$$\textstyle\sum_i (3\mu_{q_i} + \mu_{l_i}) = c_W \dot{\theta} \qquad \sum_i (2\mu_{q_i} + \mu_{u_i^c} + \mu_{d_i^c}) = c_S \dot{\theta}$$

For $T_{B-L} \lesssim 10^5$GeV,

$$\Gamma_{y_{u_1,d_1}} \ll \Gamma_{y_{u_{2,3},d_{2,3}}}, \Gamma_{SS,WS}$$

Given non-zero $Y_\theta \equiv \frac{n_\theta}{s} = \frac{n_\theta(T_{\rm RH})}{s(T_{\rm RH})}$

$$Y_{B-L} \equiv \frac{n_{\text{B-L}}}{s} = \frac{n_{B-L}(T_{B-L})}{s(T_{B-L})}$$

$$Y_B = \frac{28}{79} Y_{B-L} = \frac{28}{79}\left(\frac{\mu_{B-L}}{6\dot{\theta}}\right) Y_\theta \left(\frac{T_{B-L}}{v_a}\right)^2$$

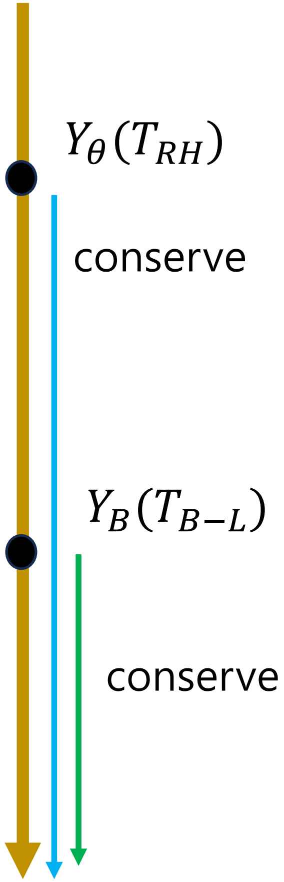
$$Y_B \simeq 0.87 \times 10^{-10}$$

$T_{B-L}$ is decoupled temperature for B-L violation chemical interaction.

$T_{B-L} = \frac{M_N}{7.8}$ when $K = \frac{\Gamma(T\to 0)}{H(T=M_N)} = 50$ $\qquad \sum_{\alpha,I} y_{v,\alpha I} \bar{l}_\alpha \tilde{H} N_I + h.c.$

B-L violation interaction which is inverse $N_R$ decay occurs actively in **wash-out regime**

→ It depends on chemical equilibrium relations (for $\frac{\mu_{B-L}}{6\dot{\theta}}$)



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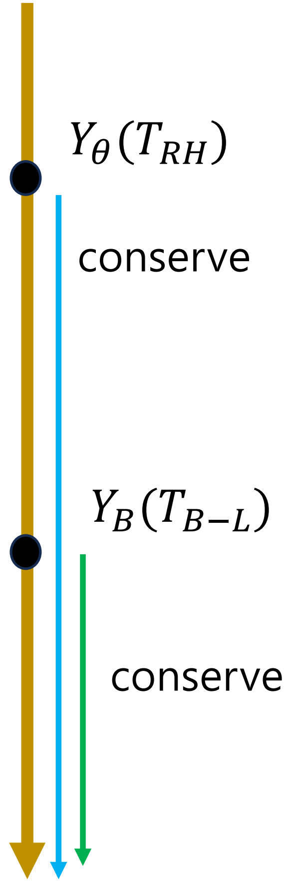
Post-inflationary dynamics

$n_\theta^{end} = n_\theta^{end}(\zeta, n)$ ------------------------------------> $Y_\theta = Y_\theta(T_{RH}, \zeta, n)$



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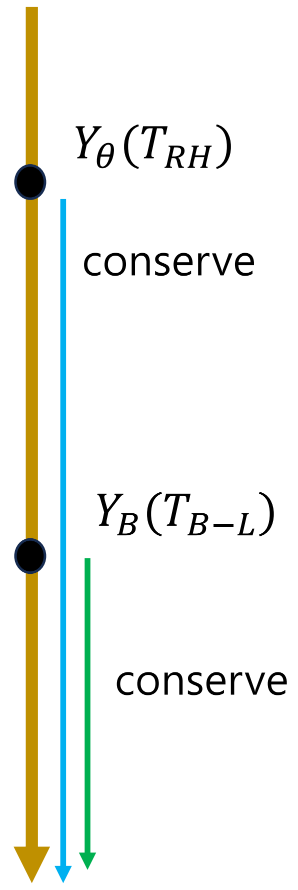
$n_\theta^{end} = n_\theta^{end}(\zeta, n)$ --- Post-inflationary dynamics ---> $Y_\theta = Y_\theta(T_{RH}, \zeta, n) = Y_{DM}(f_a)$

If axion is wave dark matter, then $Y_\theta$ is **uniquely fixed**

**(axion dark matter)** $\quad 2m_a Y_\theta \simeq 0.44 \text{ eV} \qquad m_a \approx 5.7\text{meV}\left(\frac{10^9\text{GeV}}{f_a}\right)$

By considering the above cogenesis conditions, we get the observed baryon asymmetry

$M_N \sim 10^3$GeV for $10^8 GeV \leq f_a \leq 10^9 GeV$



If **Cogenesis requirements** are satisfied
in the **PQ pole inflation scenario**,

We can explain 4 problems!

1. **horizon(flatness) problem**
2. **Baryon asymmetry**
3. **DM abundance**
4. **Strong CP problem**

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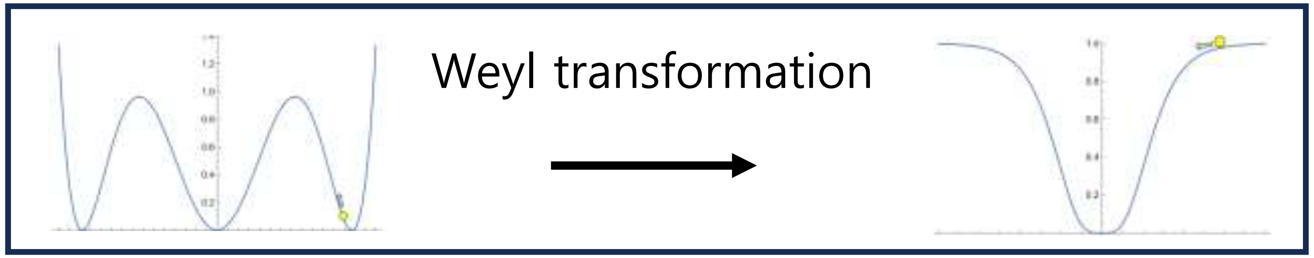
# Set up




- We set the PQ fields with non-conformal gravitational interactions in the Jordan frame.

$$\frac{\mathcal{L}_J}{\sqrt{-g_J}} = -\frac{1}{2}(1-2\zeta|\Phi|^2)R + |\partial_\mu \Phi|^2 - V_J(\Phi)$$

For the conformal coupling,

$$\zeta = \frac{1}{6}$$

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$$V_E = \frac{V_J\left(\frac{\rho}{\sqrt{2}}\right)}{\Omega^2} \qquad V_E = V_{PQ} + V_{PQV}$$

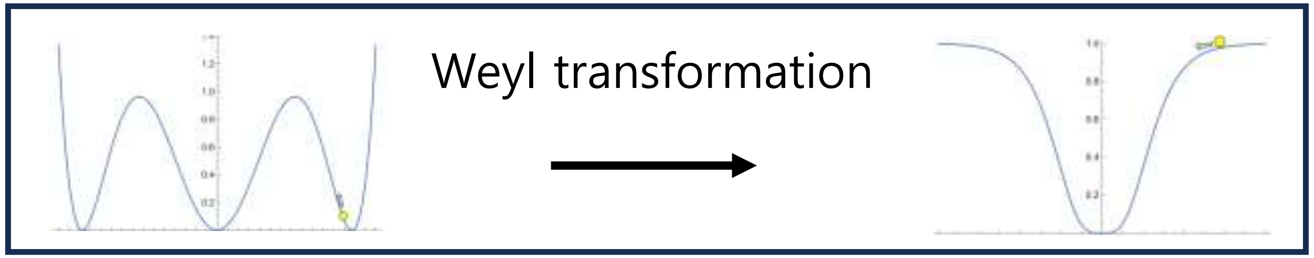
- In the Einstein frame, we consider the dynamics of PQ field

$$\begin{aligned}\frac{\mathcal{L}_E}{\sqrt{-g_E}} &= -\frac{1}{2}R + \frac{1}{\Omega}|\partial_\mu \Phi|^2 + \frac{3}{4}\frac{(\partial_\mu \Omega)^2}{\Omega^2} - V_E \\ &= -\frac{1}{2}R + \frac{1+\zeta(6\zeta-1)\rho^2}{2(1-\zeta\rho^2)^2}(\partial_\mu \rho)^2 + \frac{\rho^2(\partial_\mu \theta)^2}{2(1-\zeta\rho^2)} - V_E(\rho)\end{aligned}$$

$$\Phi = \frac{1}{\sqrt{2}}\rho\, e^{i\theta} \qquad g_{J,\mu\nu} = \frac{g_{E,\mu\nu}}{\Omega}$$

$$\Omega = 1 - 2\zeta|\Phi|^2 = 1 - \zeta\rho^2$$

# Set up




$$V_E = V_{PQ} + V_{PQV} \qquad V_{PQ} = -\mu_\Phi |\Phi|^2 + \lambda_\Phi |\Phi|^4 \qquad V_{\rm PQV} = \lambda_n \frac{\Phi^n}{M_P^{n-4}} + {\rm h.c.} \qquad \Phi = \frac{1}{\sqrt{2}}\rho\, e^{i\theta}$$

**Setting:** Inflation is driven by $\boldsymbol{V_{PQ}}$, **PQ conserving potential** ➡ $V_{PQ} > V_{PQV}$, $\frac{dV_{\rm PQ}}{d\phi} > \frac{dV_{\rm PQV}}{d\phi}$

If $\zeta \gtrsim 1$, Canonical normalization: $\rho \simeq \frac{1}{\sqrt{\zeta}} \tanh\left(\frac{\phi}{\sqrt{6}}\right)$, $\quad \rho \sim \frac{1}{\sqrt{\zeta}}$

$$V_{\rm PQ} \simeq \lambda_\Phi |\Phi|^4 \simeq \frac{\lambda_\Phi}{4} \frac{M_P^4}{\zeta^2} \underbrace{\tanh^4\left(\frac{\phi}{\sqrt{6}M_P}\right)}_{\simeq 1} \qquad |\Phi| = \frac{\rho}{\sqrt{2}} \to \frac{M_P}{\sqrt{2\zeta}}$$

Inflation occurs at the pole

---

$$V_{\rm PQV} = \lambda_n \frac{\Phi^n}{M_P^{n-4}} + {\rm c.c} = \frac{|\lambda_n|}{\sqrt{2^n}} \frac{M_P^4}{\zeta^{\frac{n}{2}}} \underbrace{\tanh^n\left(\frac{\phi}{\sqrt{6}M_P}\right)}_{\simeq 1} 2\cos\left(n\theta + \alpha\right)$$

➡ During inflation,
It breaks $U(1)_{PQ}$ explicitly, and it gives large **Axial velocity**

↕

**Net non zero PQ charge**

# Inflation model constraints

We consider the case $\zeta \gtrsim 1$

**Slow-roll inflation**

At the end of inflation,

$$\rho_e \simeq (\sqrt{3}-1)/\sqrt{\zeta}$$

$$\phi_{\rm end} \simeq M_P$$

$$\epsilon_* \simeq \frac{3}{4N^2},$$

$$\eta_* \simeq -\frac{1}{N} + \frac{3}{16\zeta N^2}$$

$$N \simeq \frac{3}{4}\frac{1}{1-\zeta\rho_*^2}$$

$$\begin{aligned} n_s &= 1 - 6\epsilon_* + 2\eta_* \\ &= 1 - \frac{2}{N} - \frac{9}{2N^2} + \frac{3}{8\zeta N^2} \\ r &= \frac{12}{N^2}. \end{aligned}$$

When N=60,

$$n_s \cong 0.9654 + 0.0001\,\zeta^{-1}$$

$$r \cong 0.00333$$

**CMB normalization**

Inflation occurs at the pole

$$\rho \to \frac{1}{\sqrt{\zeta}}$$

$$V_E \simeq \frac{1}{4}\lambda_\Phi \rho^4 \simeq \frac{\lambda_\Phi}{4\zeta^2} \equiv V_I$$

$$A_s = \frac{1}{24\pi^2}\frac{V_I}{\epsilon_*} = 2.1 \times 10^{-9}$$

$$\boxed{\lambda_\Phi = 4 \times 10^{-10} \cdot \zeta^2}$$

**$\lambda_\phi$ depends on $\zeta$**

# Axion quality problem

$U(1)_{PQ}$ shift symmetry, $\bar{\theta} \to \bar{\theta} + \frac{a}{2f_a}$

$$\mathcal{L}_{\rm int} \supset -\frac{y_Q}{\sqrt{2}} f_a e^{i\frac{a}{f_a}} QQ^c + \frac{g_s^2}{32\pi^2}\bar{\theta} G^a_{\mu\nu}\tilde{G}^{a\mu\nu} \longrightarrow \mathcal{L}_{\rm int} \supset -\frac{y_Q}{\sqrt{2}} f_a QQ^c + \frac{g_s^2}{32\pi^2}\left(\bar{\theta} + \xi\frac{a}{f_a}\right) G^a_{\mu\nu}\tilde{G}^{a\mu\nu} + \frac{\partial_\mu a}{2f_a} J^\mu_{\rm PQ}$$

For **KSVZ** model, there is only one heavy quark charged under $U(1)_{PQ}$ $\xi = 1$

$$|\theta_{\rm eff}| = \left|\bar{\theta} + \xi\frac{\langle a\rangle}{f_a}\right| < 10^{-10}$$ (EDM bound)

$V_{PQV}(\phi, \theta)$ may be able to shift QCD vacuum ➡

$$\left(\frac{f_a}{M_P}\right)^n < \frac{\sqrt{12^n}\xi \cdot \zeta^{\frac{n}{2}}}{2n|\lambda_n|}\left(\frac{\Lambda_{\rm QCD}}{M_P}\right)^4 \cdot 10^{-10}$$

# Upper bound of initial velocity

At the end of inflation, axial velocity is

$$\dot{\theta}_{\rm end} \simeq -\frac{1}{3H}\frac{\partial V_{\rm PQV}}{\partial\theta}\frac{\zeta}{M_P^2}\frac{e^{-\frac{2\phi}{\sqrt{6}M_P}}}{1-e^{-\frac{2\phi}{\sqrt{6}M_P}}}\Bigg|_{\phi=\phi_{\rm end}} \quad \propto \quad |\lambda_n|\zeta^{1-\frac{n}{2}}$$

If $\lambda_n$ is too large, it may violate **EDM bound** or inflation from **PQ conserving potential**

$$|\lambda_n| < \frac{\sqrt{2^n}}{n}\lambda_\Phi\zeta^{\frac{n}{2}-2}$$

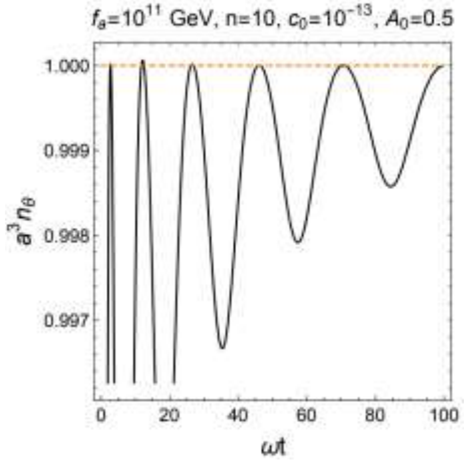
Condition from Inflation dominantly driven by $\boldsymbol{V_{PQ}}$
It is stronger than EDM bound

# Post-inflationary evolution

Canonical normalization

$$\rho \simeq \begin{cases} \frac{M_P}{\sqrt{\zeta}} \tanh\left(\frac{\phi}{\sqrt{6}M_P}\right) & \rho \sim \frac{M_P}{\sqrt{\zeta}} \\ \frac{M_P}{\sqrt{\zeta}} \sqrt{1 - e^{-\frac{2\phi}{\sqrt{6}M_P}}} & \frac{M_P}{\zeta} \lesssim \rho \lesssim \frac{M_P}{\sqrt{\zeta}} \\ \phi\left(1 - \zeta^2 \left(\frac{\phi}{M_P}\right)^2\right) & \rho \lesssim \frac{M_P}{\zeta} \\ \phi & \rho \ll \frac{M_P}{\zeta} \end{cases}$$

$$V_{PQ} = \frac{\lambda_\Phi}{4}\left(\rho^2 - f_a^2\right)^2$$




$$a^3 n_\theta = a^3 \phi^2 \dot{\theta} \simeq \text{const}$$

PQ violation terms are suppressed after end of inflation $\phi \ll M_p$

Potential with canonical normalized field

$$V(\phi) \simeq \begin{cases} \frac{\lambda_\Phi M_P^4}{4\zeta^2} \tanh^4\left(\frac{\phi}{\sqrt{6}M_P}\right) & \rho \sim \frac{M_P}{\sqrt{\zeta}} \\ \frac{\lambda_\Phi \phi^2 M_P^2}{6\zeta^2} & \frac{M_P}{\zeta} \lesssim \rho \lesssim \frac{M_P}{\sqrt{\zeta}} \\ \frac{1}{4}\lambda_\Phi \phi^4 & 2f_a \lesssim \rho \lesssim \frac{M_P}{\zeta} \\ \lambda_\Phi f_a^2 \phi^2 & \phi \lesssim 2f_a \end{cases}$$

$$w_\phi \simeq \begin{cases} -1 \\ 0 \\ \frac{1}{3} \\ 0 \end{cases}$$

$$w_\phi \equiv \frac{P_\phi}{\rho_\phi}$$

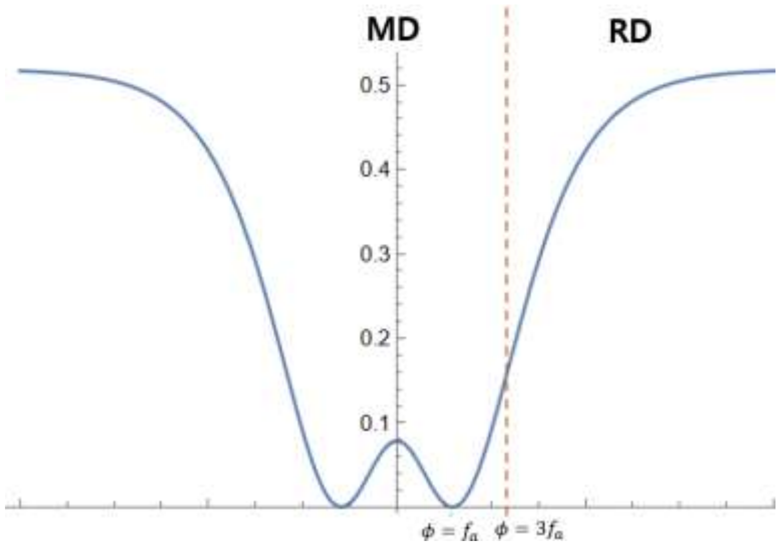
# Inflaton condensation

$$\phi = \phi_0(t)\mathcal{P}(t)$$

$\phi_0(t)$ The amplitude of the inflaton oscillation

$$\mathcal{P}(t) = \sum_{n=-\infty}^{\infty} \mathcal{P}_n \, e^{-in\omega t}$$

Periodic function




$$E_\phi = nw$$

**Inflaton energy**

$$\omega = m_\phi \sqrt{\frac{\pi m}{2m-1}} \frac{\Gamma\left(\frac{1}{2} + \frac{1}{2m}\right)}{\Gamma\left(\frac{1}{2m}\right)}$$

$$m_\phi^2 = V_E''(\phi_0) = 2\alpha_m m(2m-1)\phi_0^{2m-2}$$

$$V_{PQ} = \frac{\lambda_\phi}{4}(\phi^2 - f_a^2)^2$$

$$V_E(\phi) \simeq \alpha_m \phi^{2m}$$

m=1 => quadratic potential

m=2 => quartic potential

# Reheating

$$\dot{\rho}_\phi + 3(1+w_\phi)H\rho_\phi = -\Gamma_\phi(1+w_\phi)\rho_\phi$$
$$\dot{\rho}_R + 4H\rho_R = \Gamma_\phi(1+w_\phi)\rho_\phi$$

PQ pole inflation in **KSVZ** model:

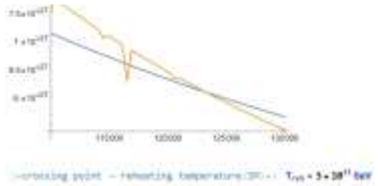
$$\Gamma_\phi = \Gamma(\phi \to \bar{Q}Q) + \Gamma(\phi \to \bar{N}N)$$

K=4

$$T_{\rm RH} \simeq 1.5 \times 10^6 \text{ GeV} \left(\frac{y_{f,\rm eff}}{10^{-4}}\right)^2 \left(\frac{\lambda_\Phi}{10^{-11}}\right)^{1/4}$$

K=2

$$T_{\rm RH}^{\rm k=2} \approx 7.9 \times 10^3 \text{GeV} \left(\frac{y_Q}{10^{-6}}\right) \left(\frac{f_a}{10^9 \text{ GeV}}\right)^{1/2} \left(\frac{\lambda_\Phi}{10^{-11}}\right)^{1/4}$$

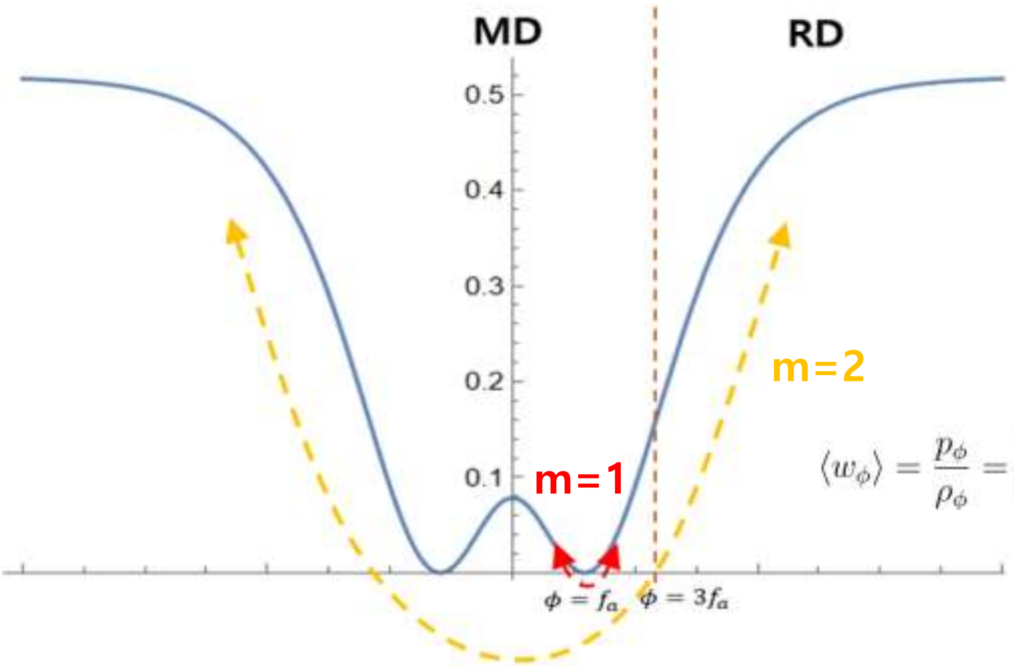


**Reheating condition**: $\rho_\phi \cong \rho_R$

$$m_Q(t) = \frac{y_Q}{\sqrt{2}}\phi(t)$$

Initial condition

$$\phi_{\rm end} \simeq \sqrt{\frac{3}{8}} M_P \ln\left[\frac{1}{2} + \frac{2m}{3}\left(2m + \sqrt{4m^2+3}\right)\right]$$




$$\langle w_\phi \rangle = \frac{p_\phi}{\rho_\phi} = \frac{m-1}{m+1}$$

If $(E_\phi = wn) > (m_Q(t) = \frac{y_Q}{\sqrt{2}}\phi(t))$ during one oscillation,
$\phi \to \bar{Q}Q$ is Kinematically allowed

$y_Q < \sqrt{\lambda_\phi}$ — $E_\phi > m_Q(t)$ is always satisfied

$y_Q > \sqrt{\lambda_\phi}$ — $y_c < y^Q_{\rm eff,k=4}$ — $E_\phi > m_Q(t)$ is only satisfied around $\phi \cong 0$

# Reheating

$m_Q(t)=\frac{y_Q}{\sqrt{2}}\phi(t)$

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$$\dot\rho_\phi + 3(1+w_\phi)H\rho_\phi = -\Gamma_\phi(1+w_\phi)\rho_\phi$$
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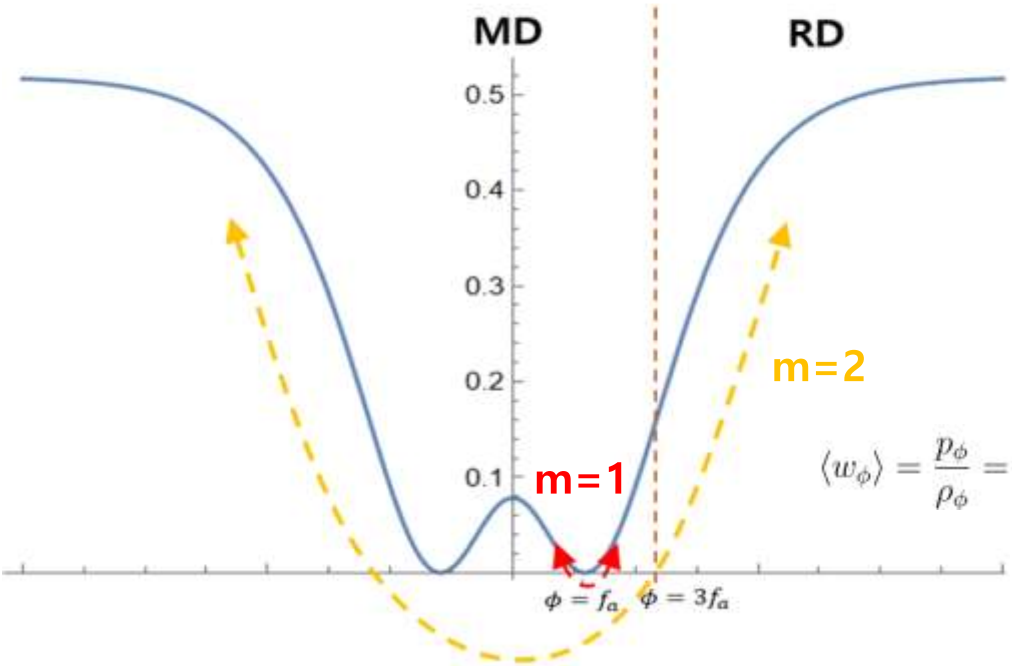


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PQ pole inflation in **KSVZ** model:

$\Gamma_\phi = \Gamma(\phi\to\bar{Q}Q) + \Gamma(\phi\to\bar{N}N)$

$y_Q$ is free parameter

$y_N$ is uniquely determined from cogenesis condition

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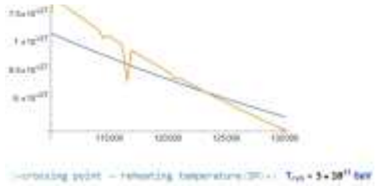
$$T_{\rm RH} \simeq 1.5\times10^6\ {\rm GeV}\left(\frac{y_{f,\rm eff}}{10^{-4}}\right)^2\left(\frac{\lambda_\Phi}{10^{-11}}\right)^{1/4}$$

K=2

$$T_{\rm RH}^{\rm k=2} \approx 7.9\times10^3{\rm GeV}\left(\frac{y_Q}{10^{-6}}\right)\left(\frac{f_a}{10^9\ {\rm GeV}}\right)^{1/2}\left(\frac{\lambda_\Phi}{10^{-11}}\right)^{1/4}$$



**Reheating condition**: $\rho_\phi \cong \rho_R$

If $(E_\phi = wn) > (m_Q(t) = \frac{y_Q}{\sqrt{2}}\phi(t))$ during one oscillation,
$\phi\to\bar{Q}Q$ is Kinematically allowed

$y_Q < \sqrt{\lambda_\phi}$ — $E_\phi > m_Q(t)$ is always satisfied

$y_Q > \sqrt{\lambda_\phi}$ — $y_c < y^Q_{\rm eff,k=4}$ — $E_\phi > m_Q(t)$ is only satisfied around $\phi \cong 0$

# Reheating

$$\dot{\rho}_\phi + 3(1+w_\phi)H\rho_\phi = -\Gamma_\phi(1+w_\phi)\rho_\phi$$
$$\dot{\rho}_R + 4H\rho_R = \Gamma_\phi(1+w_\phi)\rho_\phi$$

PQ pole inflation in **KSVZ** model:

$$\Gamma_\phi = \Gamma(\phi \to \bar{Q}Q) + \Gamma(\phi \to \bar{N}N)$$

$y_Q$ is free parameter

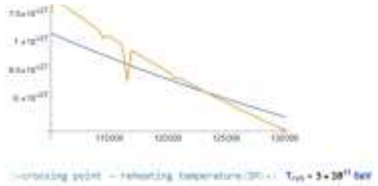
$y_N$ is uniquely determined from cogenesis condition

K=4

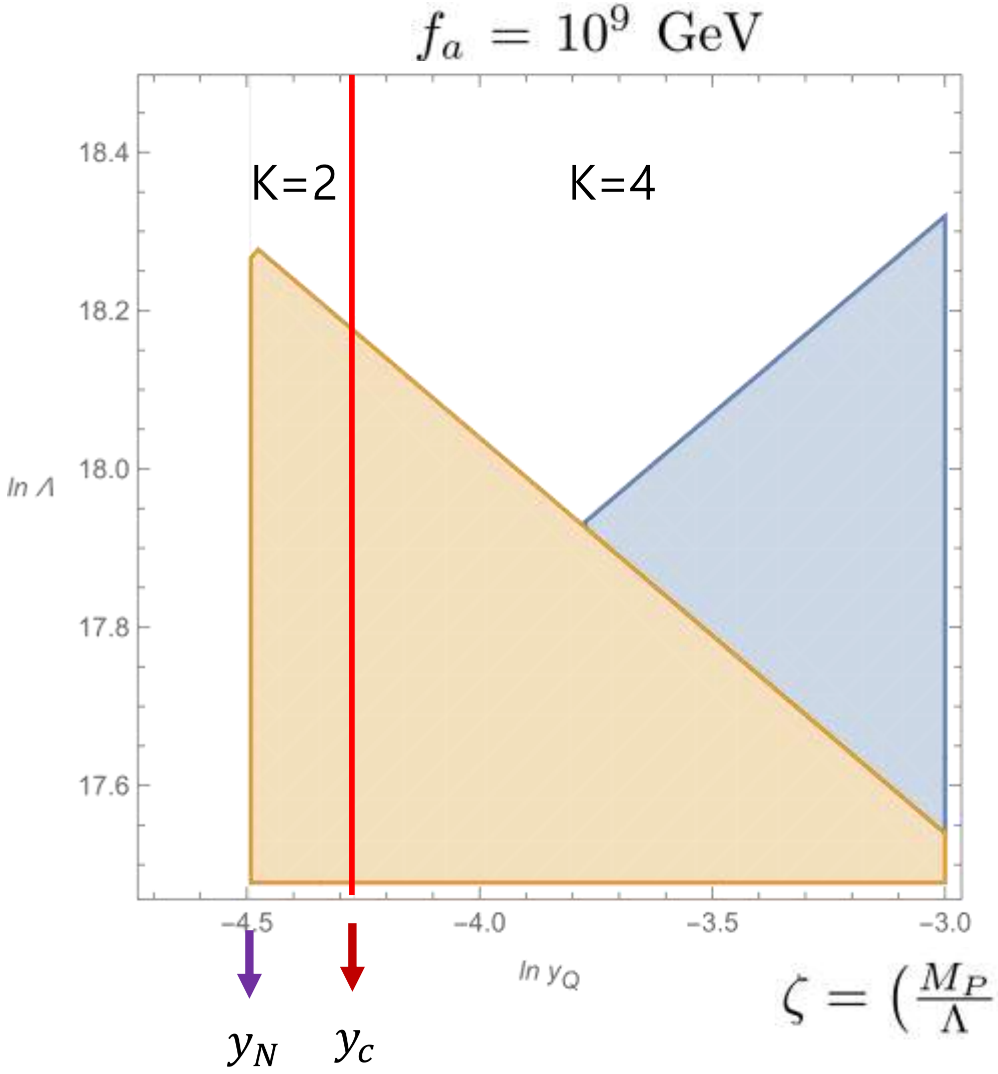
$$T_{\rm RH} \simeq 1.5 \times 10^6 \text{ GeV} \left(\frac{y_{f,\rm eff}}{10^{-4}}\right)^2 \left(\frac{\lambda_\Phi}{10^{-11}}\right)^{1/4}$$

K=2

$$T_{\rm RH}^{\rm k=2} \approx 7.9 \times 10^3 \text{GeV} \left(\frac{y_Q}{10^{-6}}\right)\left(\frac{f_a}{10^9 \text{ GeV}}\right)^{1/2}\left(\frac{\lambda_\Phi}{10^{-11}}\right)^{1/4}$$



**Reheating condition**: $\rho_\phi \cong \rho_R$




$$\zeta = \left(\frac{M_P}{\Lambda}\right)^2$$

| | | |
|---|---|---|
| $y_Q < \sqrt{\lambda_\phi}$ | | Orange Region |
| $y_Q > \sqrt{\lambda_\phi}$ | $y_c < y^Q_{\rm eff,k=4}$ | Blue Region |

# Evolution of axion velocity

$n_{\theta,\text{end}}$ is upper bounded from

1. axion quality problem
2. Inflation is driven by $V_{PQ}$ ( $V_{PQ} > V_{PQV}$, $\frac{dV_{PQ}}{d\phi} > \frac{dV_{PQV}}{d\phi}$ )

$$V(\phi) \simeq \begin{cases} \frac{\lambda_\Phi M_P^4}{4\zeta^2}\tanh^4\left(\frac{\phi}{\sqrt{6}M_P}\right) & \rho \sim \frac{M_P}{\sqrt{\zeta}} \\ \frac{\lambda_\Phi \phi^2 M_P^2}{6\zeta^2} & \frac{M_P}{\zeta} \lesssim \rho \lesssim \frac{M_P}{\sqrt{\zeta}} \\ \frac{1}{4}\lambda_\Phi \phi^4 & 2f_a \lesssim \rho \lesssim \frac{M_P}{\zeta} \\ \lambda_\Phi f_a^2 \phi^2 & \phi \lesssim 2f_a \end{cases} \qquad w_\phi \simeq \begin{cases} -1 \\ 0 \\ \frac{1}{3} \\ 0 \end{cases}$$

$T_{\rm RH} > T_c$

$$n_\theta(T_{\rm RH}) = n_{\theta,\rm end}\left(\frac{a_{\rm end}}{a_{\rm end}^{\rm MD}}\right)^3\left(\frac{a_{\rm end}^{\rm MD}}{a_{\rm RH}}\right)^3$$

$$Y_\theta(T_{\rm RH}) \lesssim 1.2\times 10^2\cdot(0.55)^{\frac{n}{2}-3}\times\zeta^{-\frac{1}{2}}$$

$T_{\rm RH} < T_c$

$$n_\theta(T_{\rm RH}) = n_{\theta,\rm end}\left(\frac{a_{\rm end}}{a_{\rm end}^{\rm MD}}\right)^3\left(\frac{a_{\rm end}^{\rm MD}}{a_c}\right)^3\left(\frac{a_c}{a_{\rm RH}}\right)^3$$

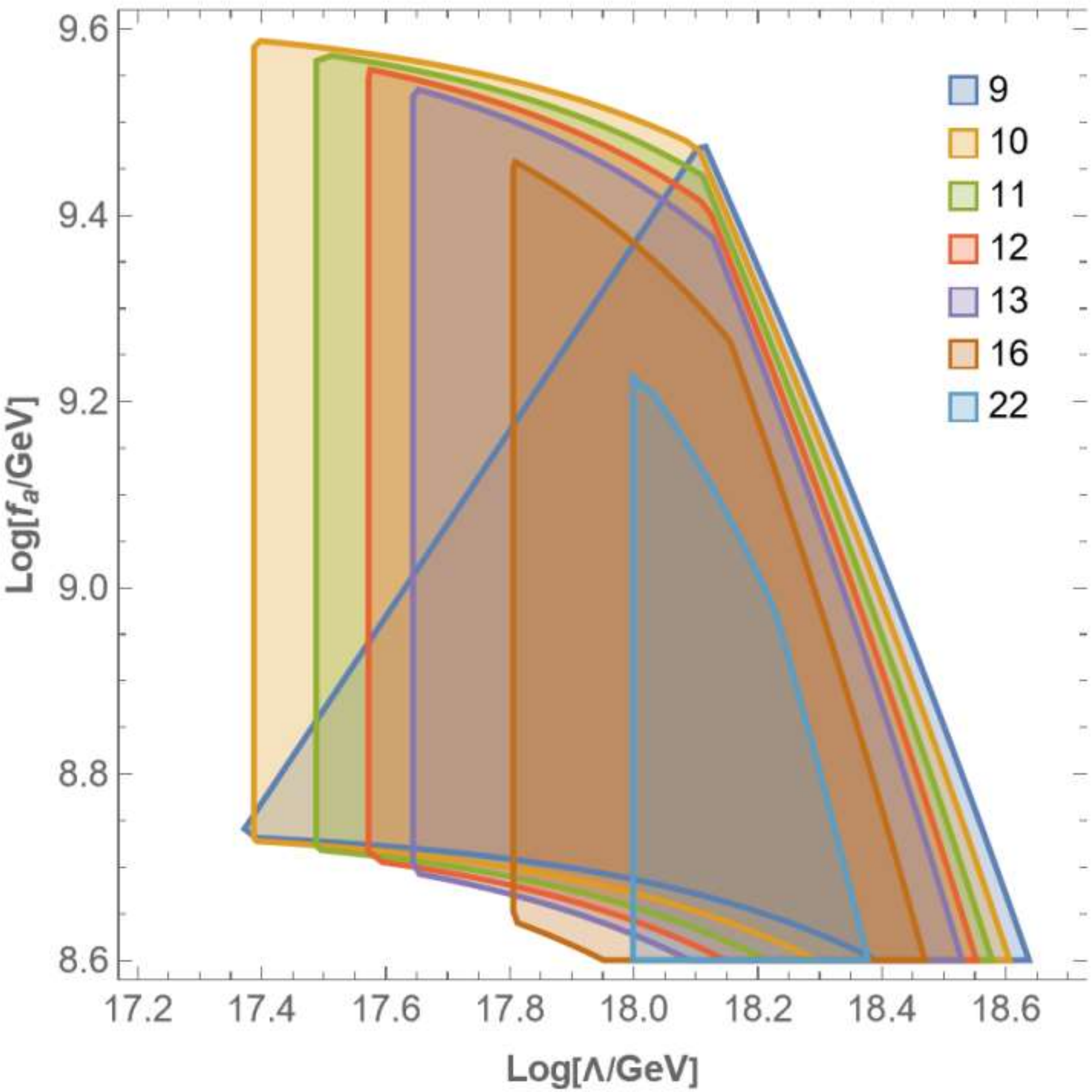
$$Y_\theta(T_{\rm RH}) \lesssim 7.0\times(0.55)^{\frac{n}{2}-3}\zeta^{-\frac{1}{2}}\sqrt{\frac{10^8\ {\rm GeV}}{f_a}\left(\frac{y_Q}{10^{-6}}\right)}$$

**(axion dark matter)**

$$2m_a Y_\theta \simeq 0.44\ {\rm eV}$$

$$m_a \approx 5.7{\rm meV}\left(\frac{10^9{\rm GeV}}{f_a}\right)$$

$$\therefore Y_\theta \cong 40\cdot\left(\frac{f_a}{10^9 GeV}\right)$$




$$\zeta = \left(\frac{M_P}{\Lambda}\right)^2$$

$$\zeta > 1 \quad \leftrightarrow \quad \ln \Lambda < 18.38$$