Interaction between cosmic strings in higher-dimensional models

Based on *JHEP* 12 (2024) 030 [arXiv:2409.18754]

Yukihiro Kanda (ICRR, University of Tokyo)

Institute for Cosmic Ray Research
University of Tokyo

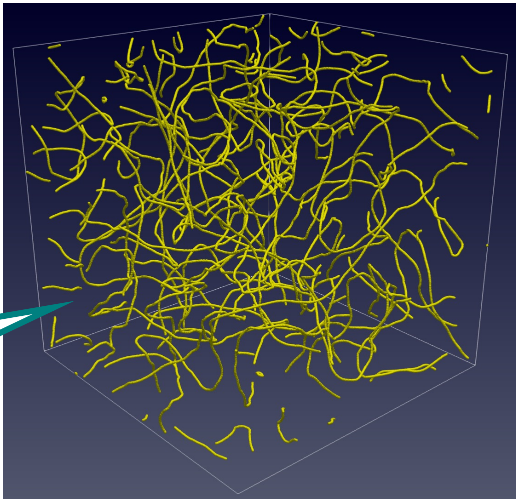
In collaboration with Takuya Hirose (Kyushu Sangyo University)

Summer Institute 2025 at Yeosu, Korea on 21st August 2025

Cosmic string

Cosmic strings [Kibble (1976), Vilenkin, Shellard (2000)]

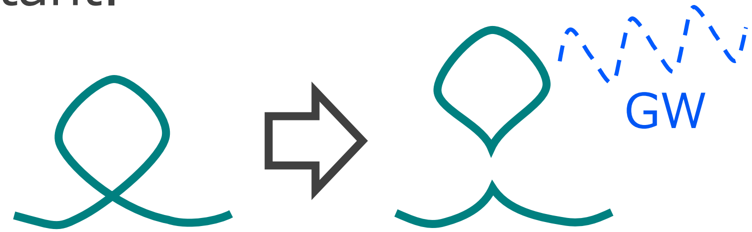
- Linearly excited region of particles.
- Produced after the phase transition ($\cancel{U(1)}$).
- Behaving as cosmological scale string.
- Source of the gravitational waves.

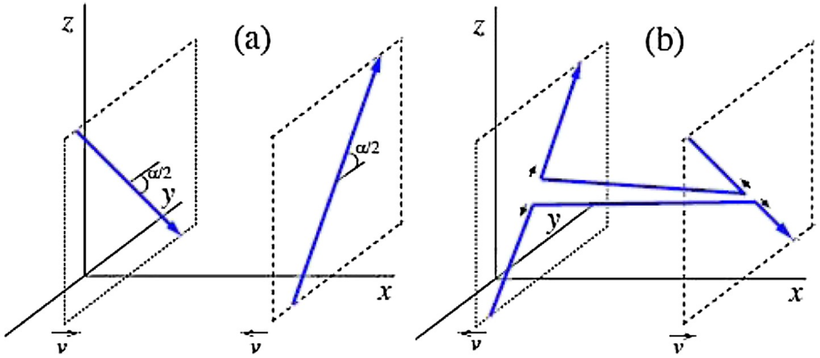


Cosmic strings are not produced in SM.

➡ New physics detected by GW!

To estimate GW spectrum precisely, interaction between strings may be important.

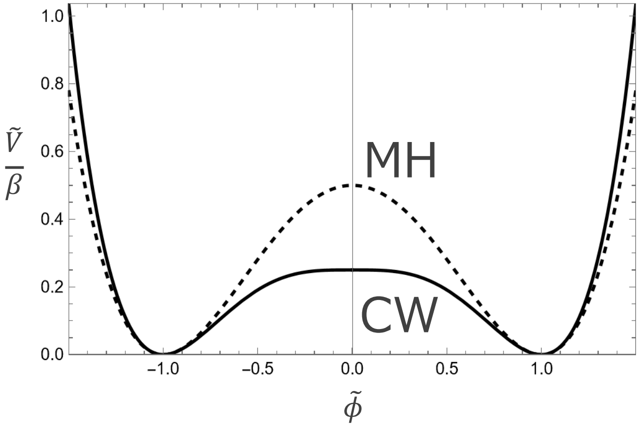




[Verbiest, Achucarro (2011)]

Interaction between strings

The scalar potential in Abelian-Higgs model affects the interaction between the strings.



Notations

$\tilde{\phi} \equiv \phi/v$, $\tilde{V}(\tilde{\phi}) \equiv V/(g^2 v^4)$, $\beta \equiv m_\phi^2/m_A^2$,

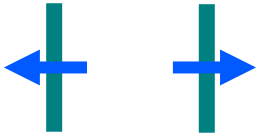
v: a vacuum expectation value of ϕ

g: gauge coupling

The Mexican hat potential

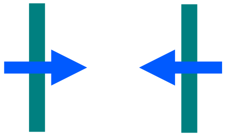
$$\tilde{V}(\tilde{\phi}) = \frac{\beta}{2}\left(|\tilde{\phi}|^2 - 1\right)^2$$

- $\beta > 1$



Repulsive

- $\beta < 1$



Attractive

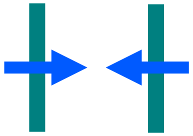
- $\beta = 1$ No force (BPS state)

[Goodband, Hindmarsh (1995)]

The Coleman-Weinberg potential

$$\tilde{V}(\tilde{\phi}) = \frac{\beta}{2}\left[\left(\log|\tilde{\phi}|^2 - \frac{1}{2}\right)|\tilde{\phi}|^4 + \frac{1}{2}\right]$$

$\beta \sim 2.0$ New property!!



Attractive at small distances

Repulsive at large distances

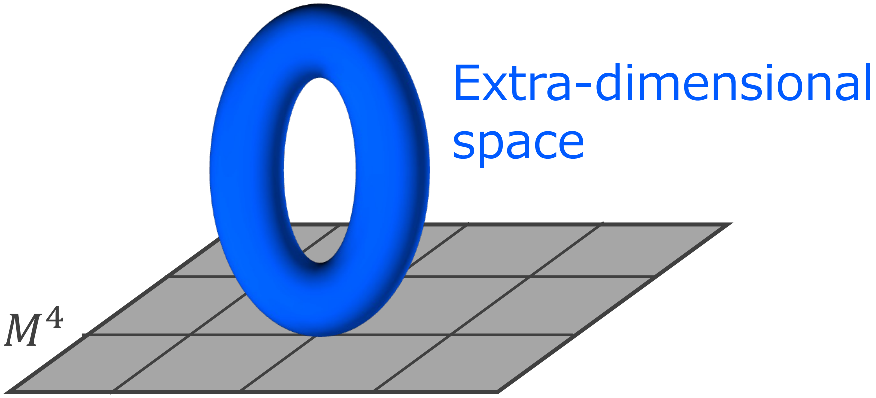
[Eto, Hamada, Jinno, Nitta, Yamada (2022)]

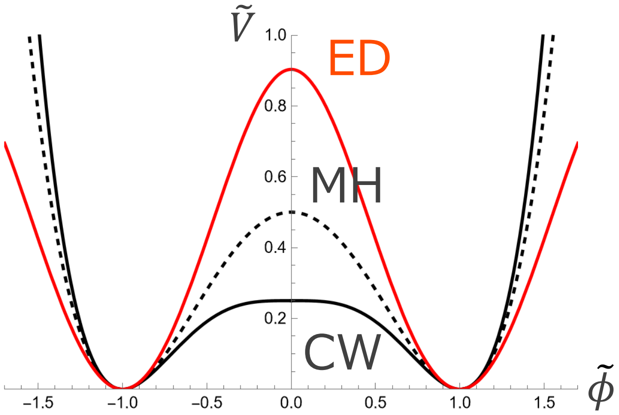
Motivation of our work

Other mechanism (or scalar potential) for ~~$U(1)$~~ ?

Hosotani mechanism in higher-dimensional models

[Hosotani (1983)]

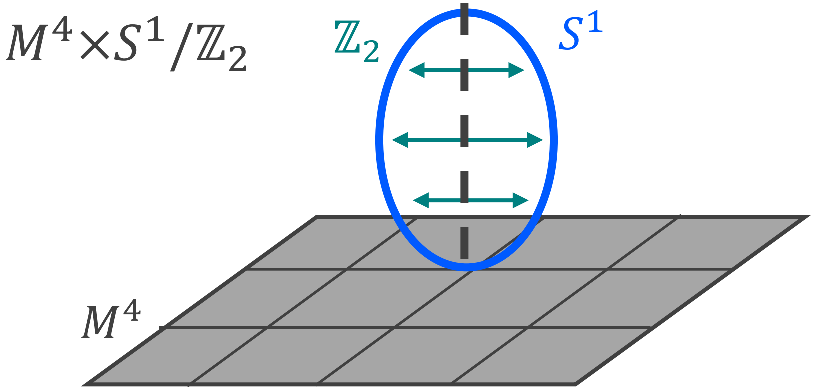




Scalar potential is different from the Mexican hat potential.

How is the inter-string interaction changed?

Higher-dimensional model



- Gauge field contains scalar field

$$A_M = \left(A_\mu, A_y\right) \qquad (M = 0,1,2,3,5)$$

Real scalar field

$$-\frac{1}{4}F_{MN}F^{MN} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\left|D_\mu A_y\right|^2$$

- Many massive modes exists (Kaluza-Klein modes)

$$\phi(x,y) = \frac{1}{\sqrt{\pi R}}\sum_{k=0}^{\infty}\phi_k(x)\cos\left(\frac{ky}{R}\right) \xrightarrow{\partial_y\partial_y\phi^2} \frac{k^2}{R^2}\phi^2 \equiv m_k^2\phi^2$$

Field in M^4

Only $k = 0$ is massless

- Residual symmetry due to the $\mathbb{Z}_2$ charge

$$A_\mu^{(-)}(x,y) = \frac{1}{\sqrt{\pi R}}\sum_{k=1}^{\infty}\phi_k(x)\sin\left(\frac{ky}{R}\right) \Leftarrow$$

No massless mode

⇔ Not symmetry in M^4

Our setup

$SU(2)$ gauge theory on $M^4 \times S^1/\mathbb{Z}_2$ [Kubo, Lim, Yamashita (2002)]

$$S = \int d^4x \int_0^{\pi R} dy \left[-\frac{1}{4} F^a_{MN} F^{a,MN} + \bar{\psi} i \gamma^M D_M \psi \right] \quad \left(\begin{array}{ll} M = 0,1,2,3,5 & x^5 = y \\ a = 1,2,3 & \end{array} \right)$$

Due to the $\mathbb{Z}_2$ parity, only A^3_μ, A^1_y and A^2_y have Kaluza-Klein 0 modes.

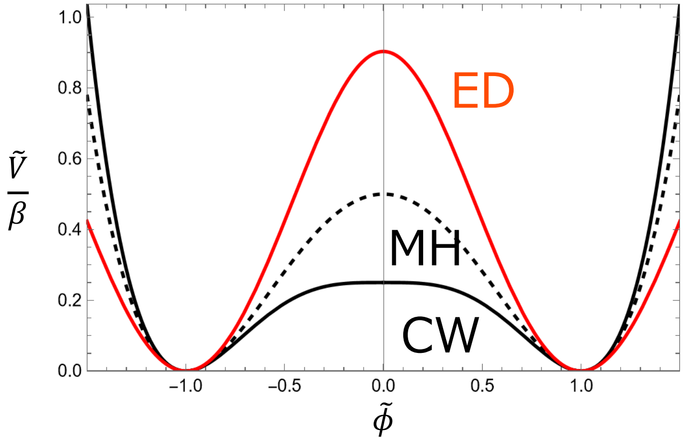
$$S = \int d^4x \left[-\frac{1}{4} F^3_{\mu\nu} F^{3,\mu\nu} - \left| D_\mu \left(\underbrace{\frac{A^1_y - i A^2_y}{\sqrt{2}}}_{\equiv \phi} \right) \right|^2 + V_{\text{eff}}(A^1_y, A^2_y) \right]$$

Abelian-Higgs model

e.g. $SU(2)$ adjoint fermion

$$\tilde{V}(\phi) = \beta \frac{16}{3\zeta(3)\pi^2} \left[\sum_{k=1}^{\infty} \frac{1}{k^5} \cos\left(k\pi|\tilde{\phi}|\right) + \frac{15}{16}\zeta(5) \right]$$

$$\left(\beta \equiv m_\phi^2 / m_A^2\right)$$



We consider cosmic strings in this model.

Numerical method

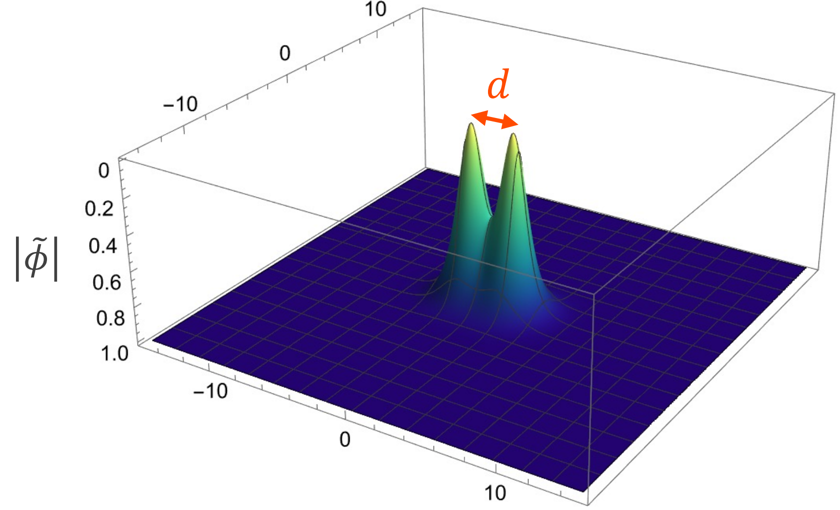
Cosmic string = Linear excited region of the scalar and gauge field

[Nielsen, Olesen (1973)]

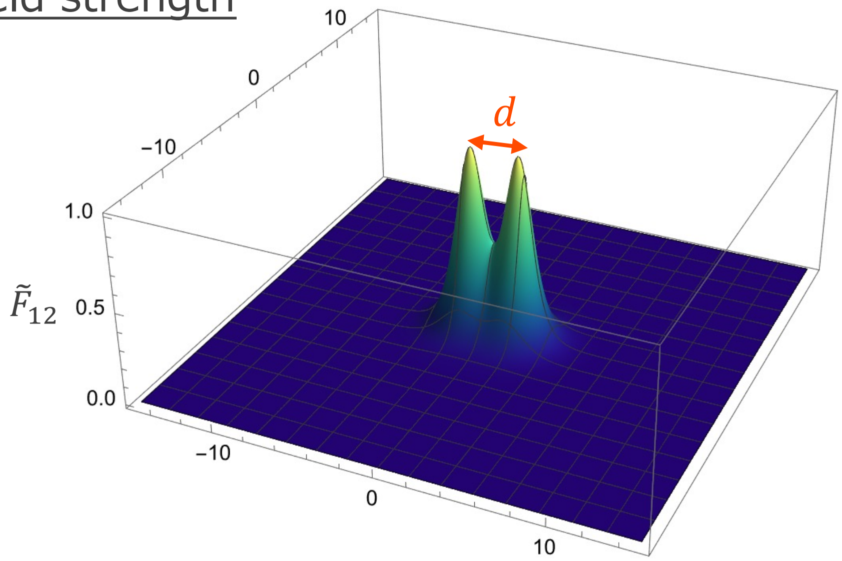
We have investigated the field configurations of two parallel strings separated by a distance d and evaluated the interaction energy.

$(d \equiv gv\Delta r)$

Scalar field



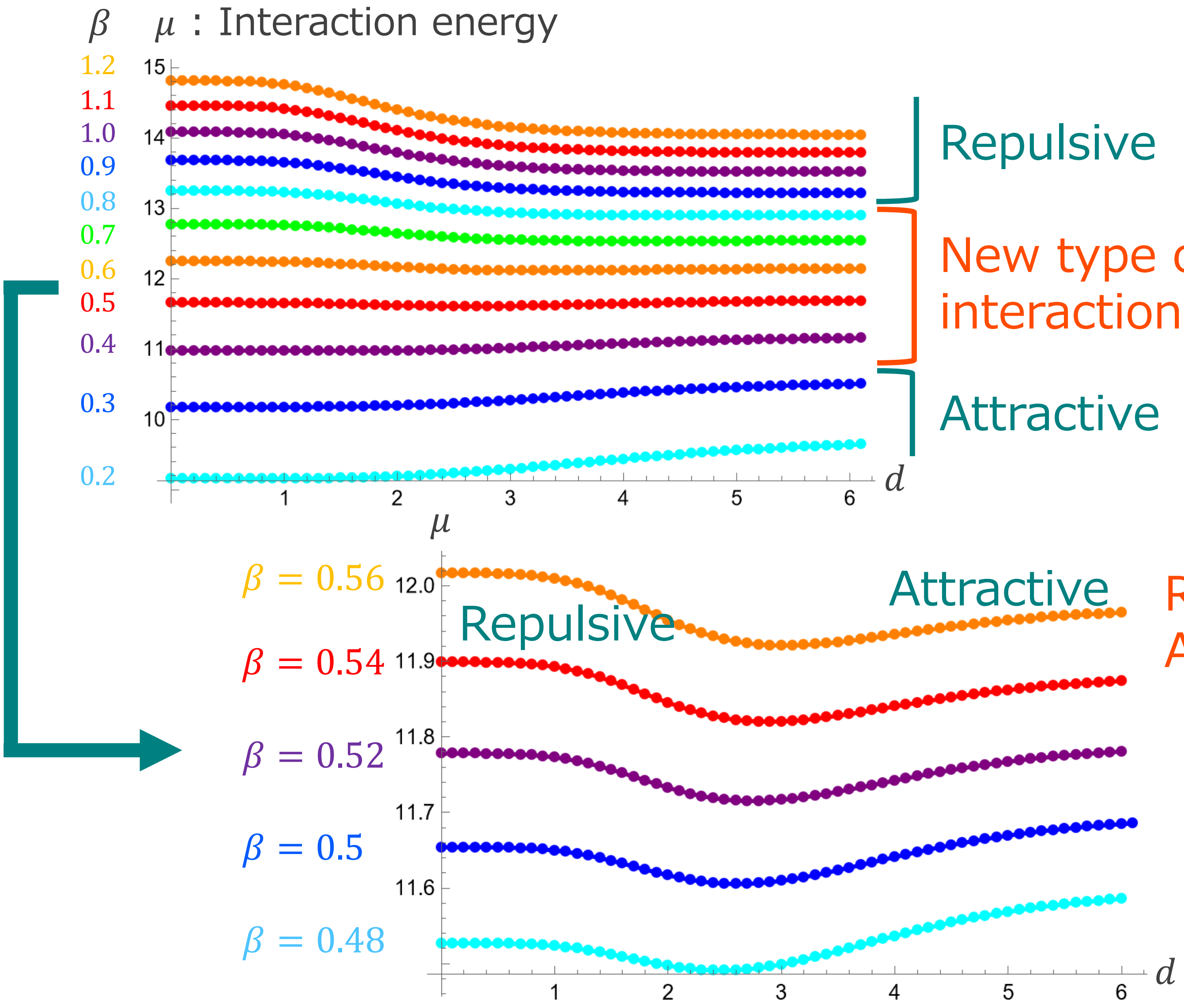
Field strength



Numerical calculations are performed on 2d lattice space ($N = 600^2$).

Result

$(\beta \equiv m_\phi^2/m_A^2)$



Repulsive

New type of interaction!!

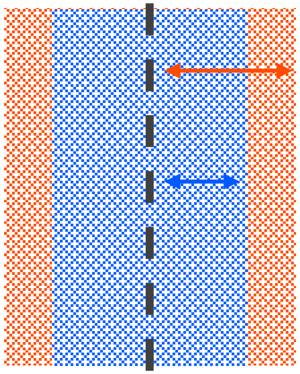
Attractive

Repulsive at small d
Attractive at large d

We analyze at
- Large distance
- Small distance

Interaction at large distance

Large d: Scalar field (attractive) vs. gauge field (repulsive)



$\sim m_\phi^{-1}$: Cause attractive force ($\because$ minimize potential energy)

$\sim m_A^{-1}$: Cause repulsive force ($\because$ minimize magnetic pressure)

Interaction at large d: which fields strongly affect there.

Point source formalism [Speight (1997)]

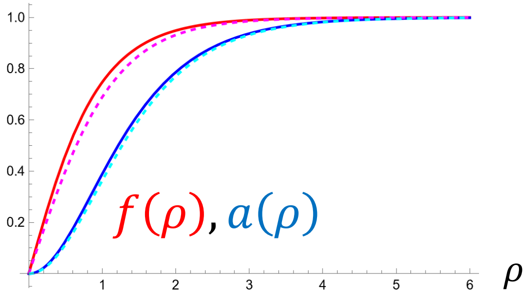
Scalar field: $f(\rho) \sim 1 - c_\phi K_0\left(\sqrt{2\beta}\rho\right)$

Gauge field: $a(\rho) \sim 1 - c_A \rho K_1\left(\sqrt{2}\rho\right)$

at $\rho \gg 1$

$\left(c_\phi, c_A \in \mathbb{R}\right)$

K_i: modified Bessel function



$$\Rightarrow \frac{d\mu}{dd} \propto c_\phi^2 \sqrt{\beta} K_1\left(\sqrt{2\beta}d\right) - c_A^2 K_1\left(\sqrt{2}d\right) \longleftarrow$$

It indicates

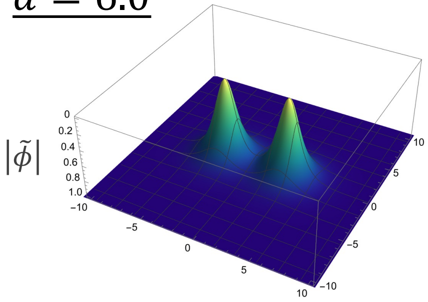
- Repulsive at high β
- Attractive at low β

Consistent with our numerical result.

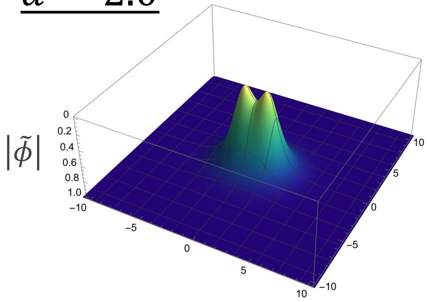
Interaction at small distance

Small d: Increase of the excited region of the scalar field

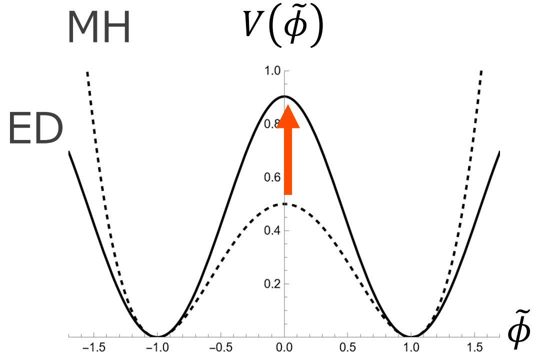
$d = 6.0$



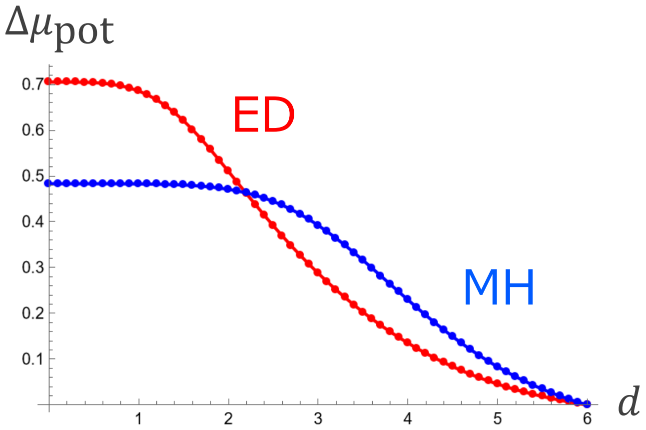
$d = 2.0$



The region between two strings is excited



The value of $V(0)$ must affects.



Comparing potential energy, we found that the potential energy significantly changes at small distances ($d \sim 2$-3).

This effect is not significant at low β.
$(\because V \propto \beta\ )$

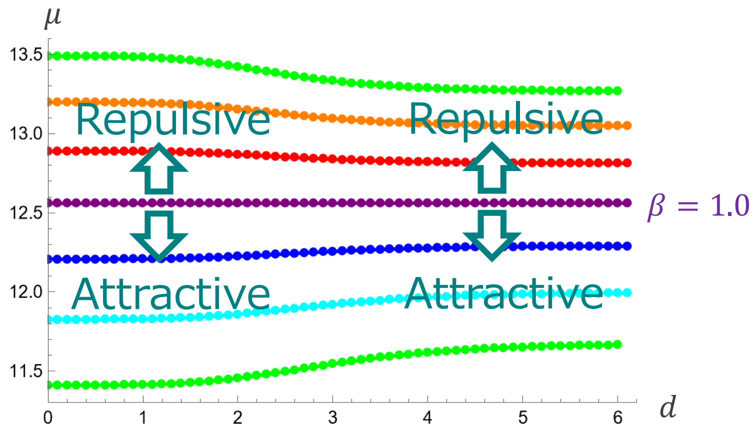
➡ Repulsive at high β, Attractive at low β

Mechanism emerging d-dependence

- ✓ Interaction at large and small distances determined differently.
- ✓ Both of them cause repulsion at large β and attraction at small β.

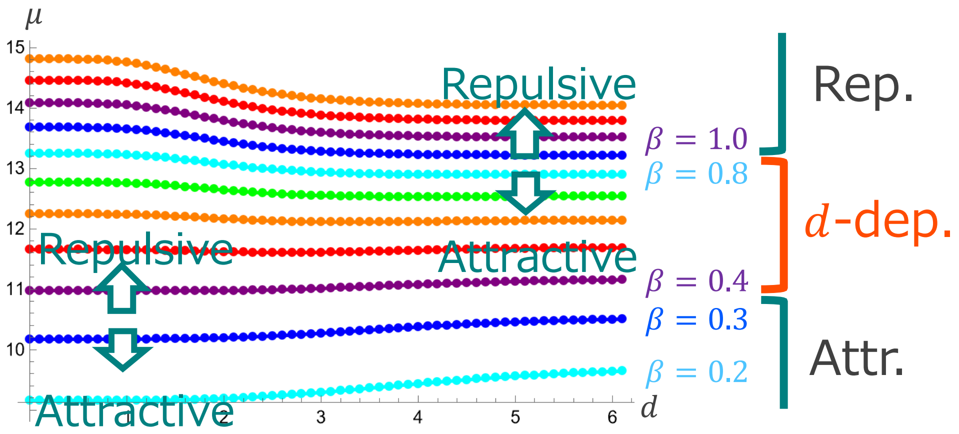
➡ Deviation of critical β at large and small distances can cause distance-dependent interaction.

The Mexican hat case



Guaranteed by BPS state.

The extra-dimensional case



The d-dependence emerges between Repulsive and Attractive.

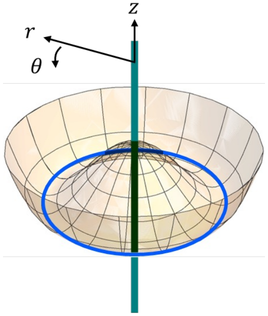
Summary

- We consider the cosmic string with the scalar potential emerging in the $SU(2)$ gauge theory on $M^4 \times S^1/\mathbb{Z}_2$.
- In such the model, the two strings represent the distance dependent interaction.
- We analyze our results in large ad small distances separately. We find the different factors for each case and that it causes the d-dependence of the interaction. This interpretation is consistent with the Coleman-Weinberg potential case.

Back up

Local string

Let’s consider cosmic strings in Abelian-Higgs model



$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \left|D_\mu\phi\right|^2 - \lambda(|\phi|^2 - v^2)^2$$

$$(D_\mu\phi = (\partial_\mu - igA_\mu)\phi)$$

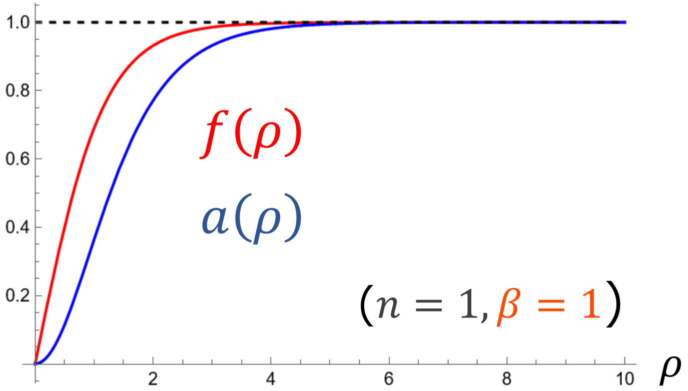
String solution [Abrikosov (1957), Nielsen, Olesen (1973)]

$$\phi(x) = f(r)ve^{in\theta},\ \vec{A}(x) = \frac{na(r)}{gr}\ \vec{e}_\theta,\ (\text{others}) = 0$$

(n: winding number)

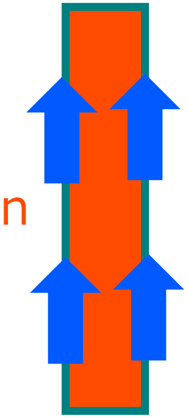
$$(f(0) = a(0) = 0, f(\infty) = a(\infty) = 1)$$

in the cylindrical coordinate (r, θ, z)



$$(\rho \equiv gvr = m_A r/\sqrt{2}, \beta \equiv m_\phi^2/m_A^2 = 2\lambda/g^2)$$

Schematic picture

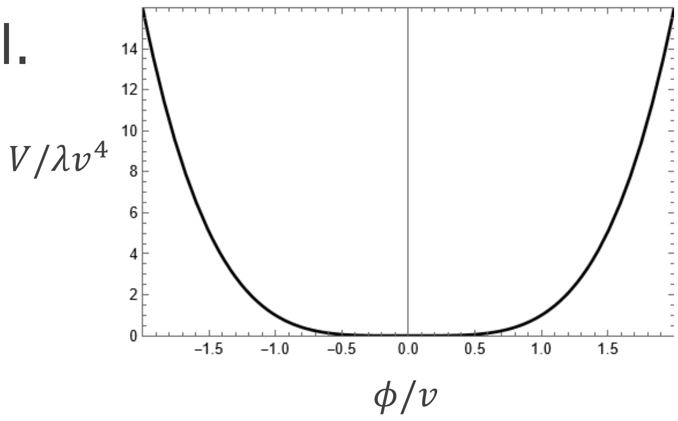


Excited region of ϕ

$$\vec{B} = \frac{na'}{gr}\vec{e}_z$$

Coleman-Weinberg potential

Let’s consider QED theory with ϕ^4 potential.



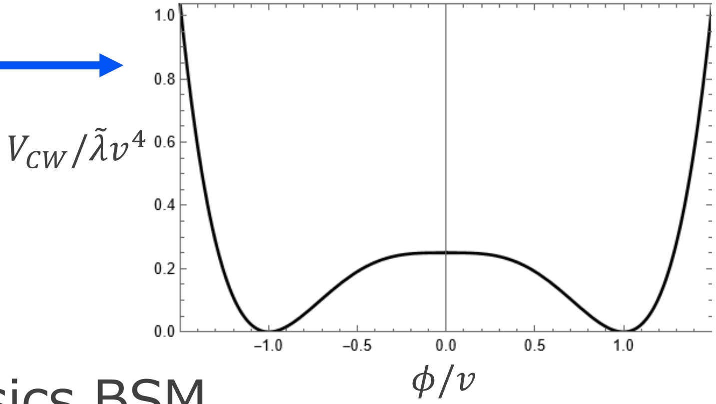
$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \left|D_\mu\phi\right|^2 - \lambda|\phi|^4$$

$$\Big\downarrow$$

Quantum correction (1-loop level)

$$V_{CW}(\phi) = \lambda|\phi|^4 + \frac{3\lambda}{11}|\phi|^4\left(\log\frac{|\phi|^2}{v^2} - \frac{25}{6}\right) = \tilde{\lambda}|\phi|^4\left(\log\frac{|\phi|^2}{v^2} - \frac{1}{2}\right)$$

$U(1)$ is broken! →



Coleman-Weinberg potential

[Coleman, Weinberg (1973)]

CW potential is assumed to describe physics BSM.

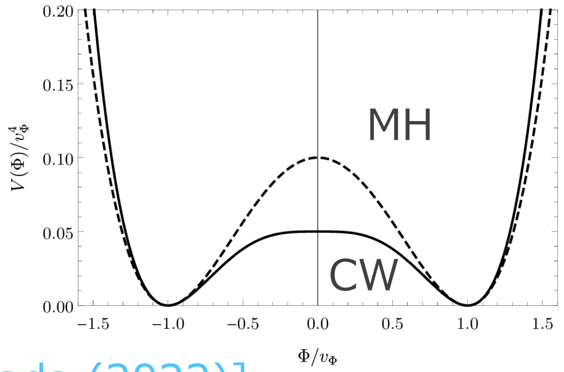
[Iso, Okada, Orikasa (2009),⋯]

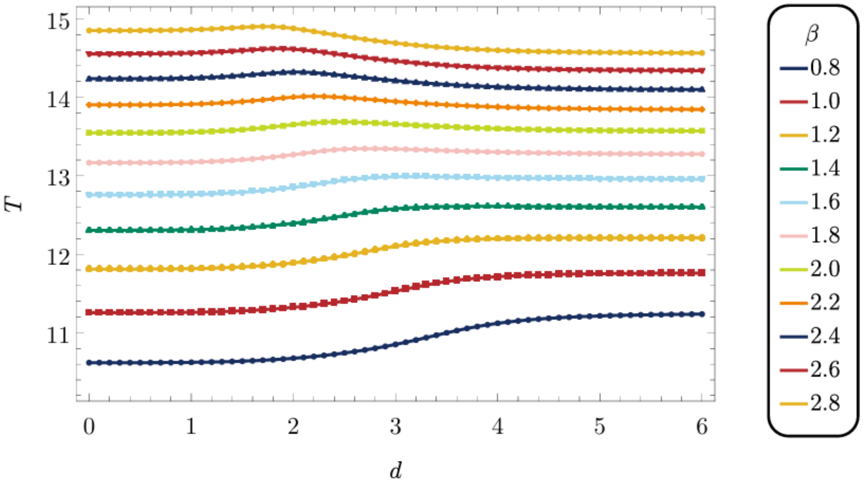
Local string w/ CW potential

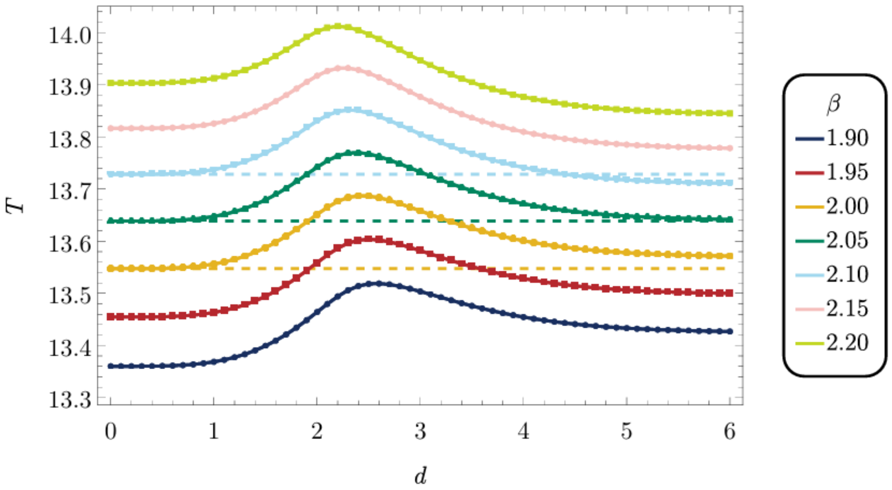
Consider local strings in CW potential

$$\phi(x) = f(r)e^{in\theta}, \vec{A}(x) = \frac{na(r)}{gr}\vec{e}_\theta$$

Property of interaction depends on inter-string distance. [Eto, Hamada, Jinno, Nitta, Yamada (2022)]







✓ No BPS state.

✓ Repulsive at large d, Attractive at small d $(\beta \sim 2.0)$

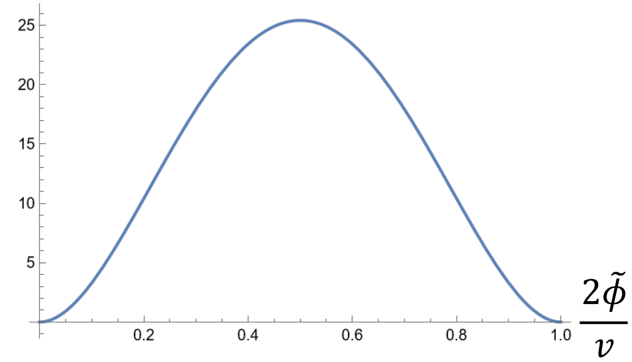
Finite temperature effect

$$\tilde{V}(\phi,T) = \beta\frac{16}{3\zeta(3)\pi^2}\sum_{l=1}^{\infty}\sum_{k=1}^{\infty}\left[\frac{1}{k^5}+\frac{-3+(-1)^l 4}{(k+l^2/(2\pi RT)^2)^{5/2}}\right]\cos\left(\frac{\pi|\phi|}{v}\right)+const.$$

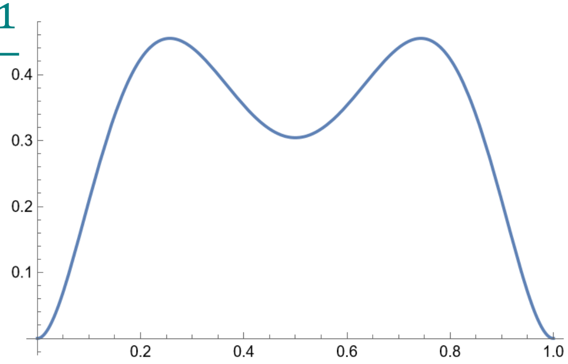
[Dienes, Dudas, Gherghetta, Riotto (1999),
Panico, Serone (2005), Maru, Takenaga (2005)]

$T = 0.5R^{-1}$

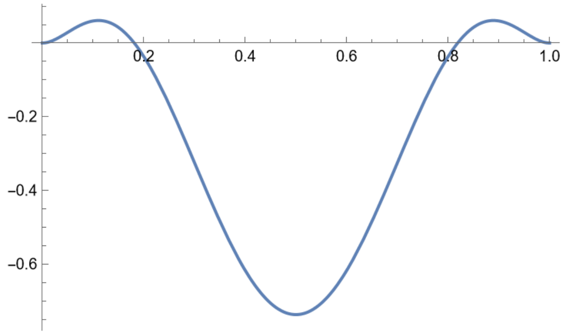
$\tilde{V}\times\frac{3\zeta(3)\pi^2}{16\beta}$



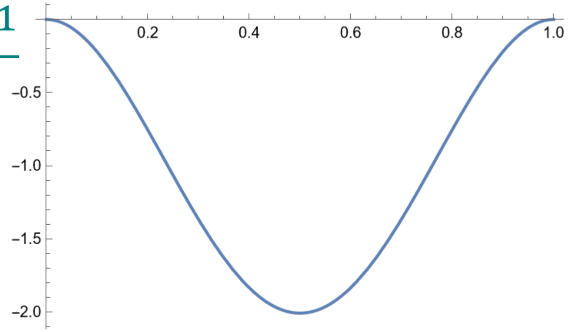
$T = 0.12R^{-1}$



$T = 0.1R^{-1}$

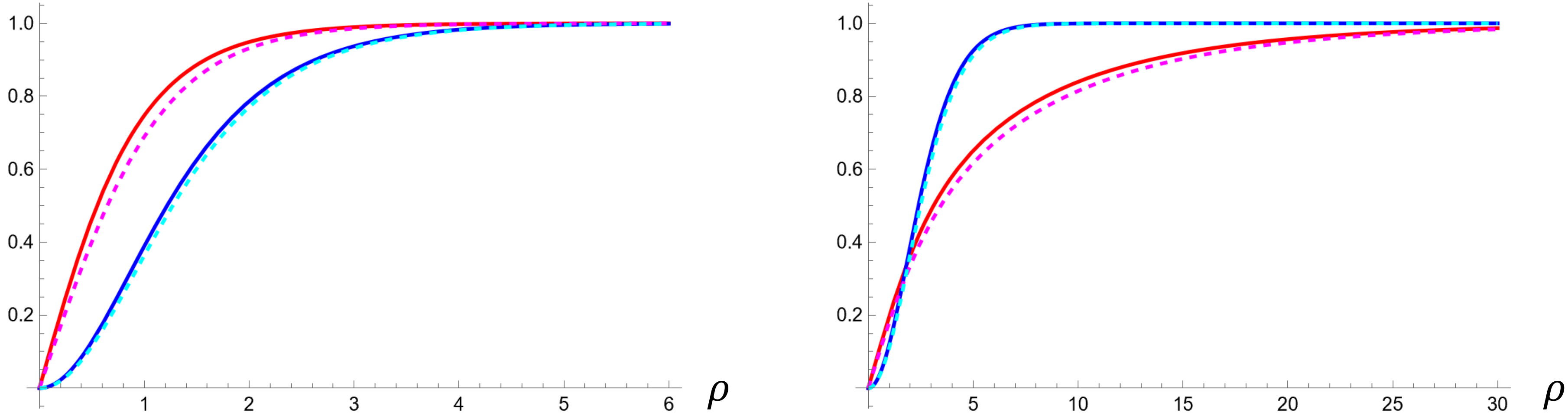


$T = 0.01R^{-1}$



Behavior of string solution

$$\phi(x) = f(r)e^{i\theta}, \vec{A}(x) = \frac{a(r)}{gr}\vec{e}_\theta$$



$\beta = 1, n = 1$

$\beta = 0.005, n = 1$

(Solid lines: $f(\rho), a(\rho)$ in the ex.-dim. model, dashed lines: $f(\rho), a(\rho)$ w/ MH potential)

Half width

$f(\rho)$: 85% of that of MH
$a(\rho)$: 95% of that of MH

$f(\rho)$: 90% of that of MH
$a(\rho)$: 96% of that of MH

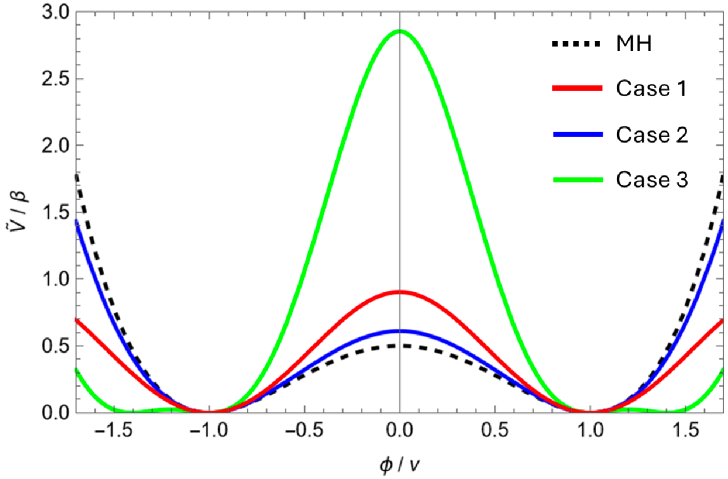
f, a becomes narrower, but they do not change significantly.

Other potentials

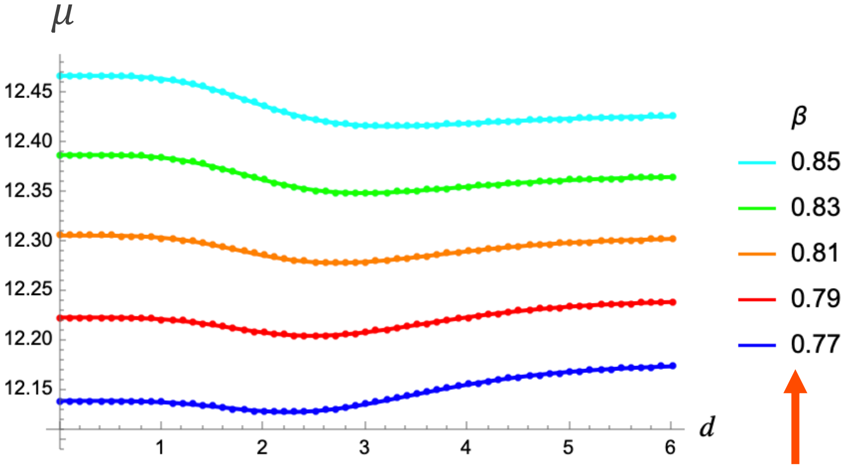
Contents of fermions emerge various effective potentials.

- Case1: $N_{ad}^{(+)} = 1$ (previous one)
- Case2: $N_{ad}^{(+)} = 1, N_{f}^{(-)} = 1$
- Case3: $N_{ad}^{(+)} = 1, N_{f}^{(+)} = 1$

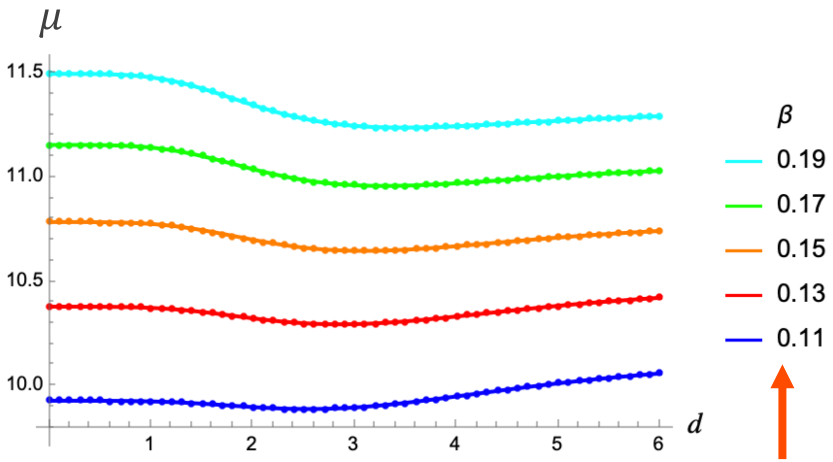
($SU(2)$ adjoint or fundamental, $\mathbb{Z}_2$ parity)



- Case2



- Case3



β at which the distance-dependent interaction emerges changes

Type-1.5 string

Strings having similar distance-dependent interaction are known as "type-1.5" string in condensed matter physics.

[Babaev, Speight (2005)] [Moshchalkov, *et. al* (2009)]

However, situation is completely different from our case.

- ✓ They introduce two scalar fields with the Mexican hat potential.

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \left|D_\mu\phi_1\right|^2 - \left|D_\mu\phi_2\right|^2 - \lambda_1(|\phi_1|^2 - v^2)^2 - \lambda_2(|\phi_2|^2 - v^2)^2$$

⇨ $$\phi_1 = f_1(r)ve^{in\theta}, \phi_2 = f_2(2)ve^{in\theta}, A_\theta = \frac{na(r)}{gr}$$

- ✓ They introduce two different couplings.

$$\beta_1 \equiv \frac{4\lambda_1}{g^2} > 1, \qquad \beta_2 \equiv \frac{4\lambda_2}{g^2} < 1$$

⇨ Attractive at large distances.
Repulsive at small distances.

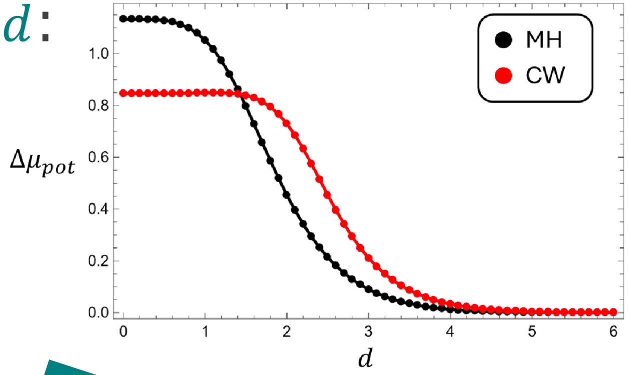
Type-1.5 string in our study is due to the potential shape.

Coleman-Weinberg case

Our interpretation is consistent with the CW case.

Focusing on $\beta = 2.0$,

Small d:

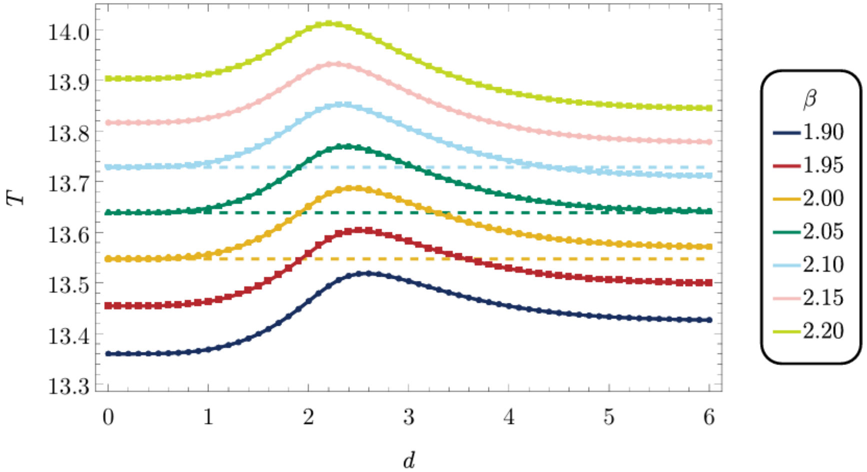


Large d:

Point source formalism shows repulsive force at $d = 6.0$.

[Eto, Hamada, Jinno, Nitta, Yamada (2022)]

Attractive force



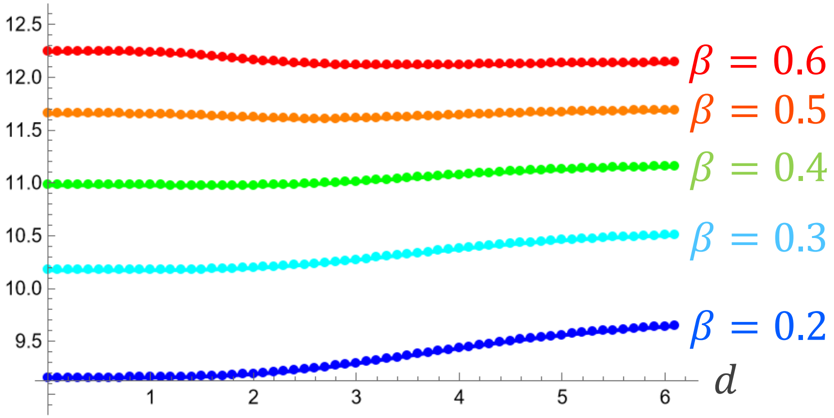
Repulsive force

“type-$\overline{1.5}$”

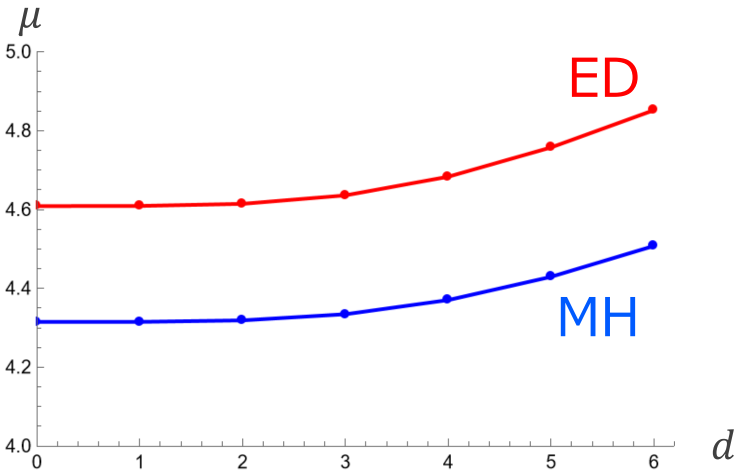
Result on small β

Higher-dimensional theory has only g as a dimensionless parameter.

In our setup, $\beta \sim g^2 \times 0.0075$, but calculation is difficult on this value.



$\beta = 0.01$



However, the interaction tends to be attractive at small β.

The interactions are almost the same.

(Low β is predicted?)