Quantum Entanglement in $t\bar{t}$ at a Photon Collider

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Quantum Entanglement @ LHC

EUROPEAN ORGANISATION FOR NUCLEAR RESEARCH (CERN)

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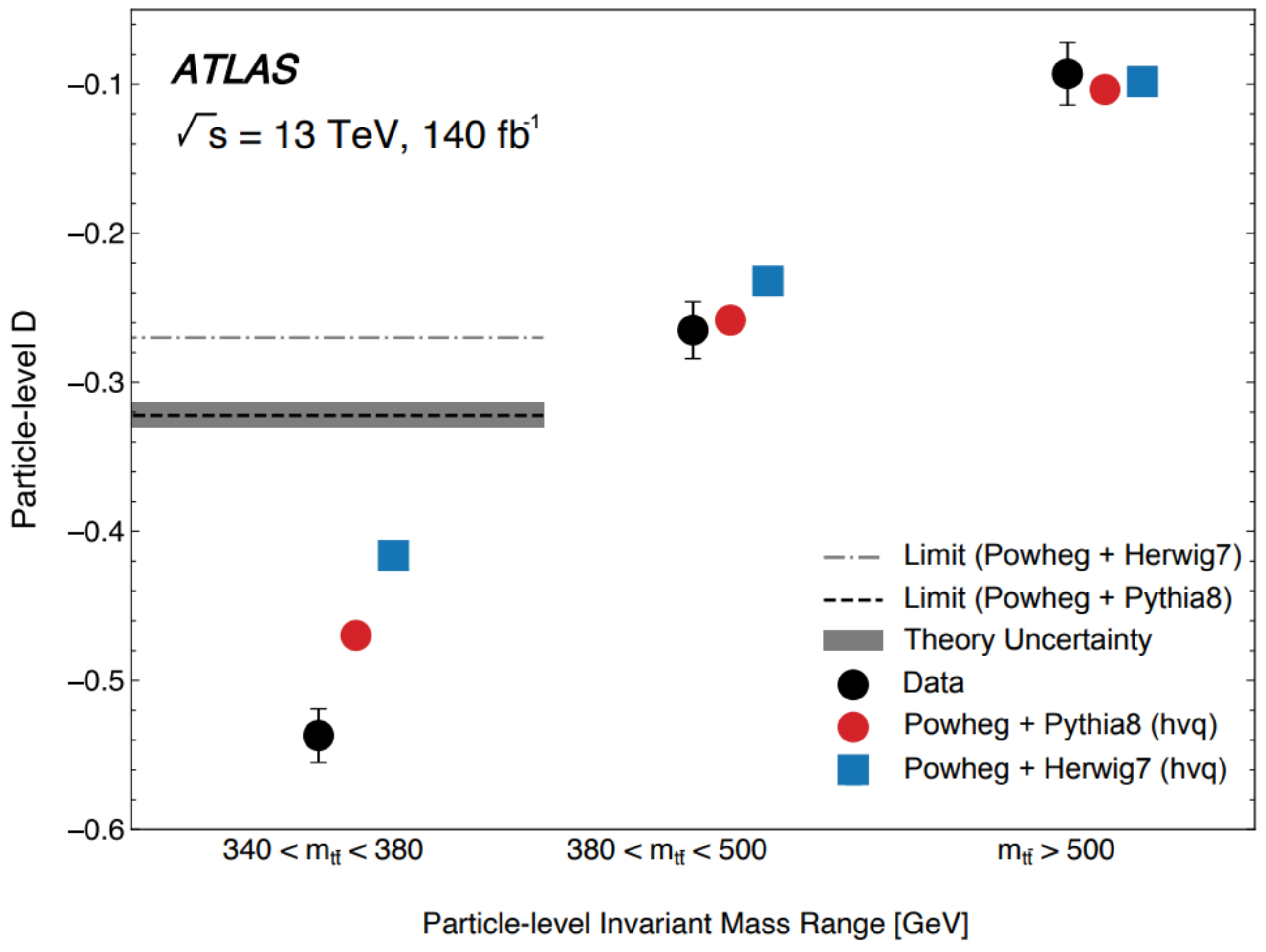
CERN-EP-2023-230
October 3, 2024

Observation of quantum entanglement with top quarks at the ATLAS detector

The ATLAS Collaboration

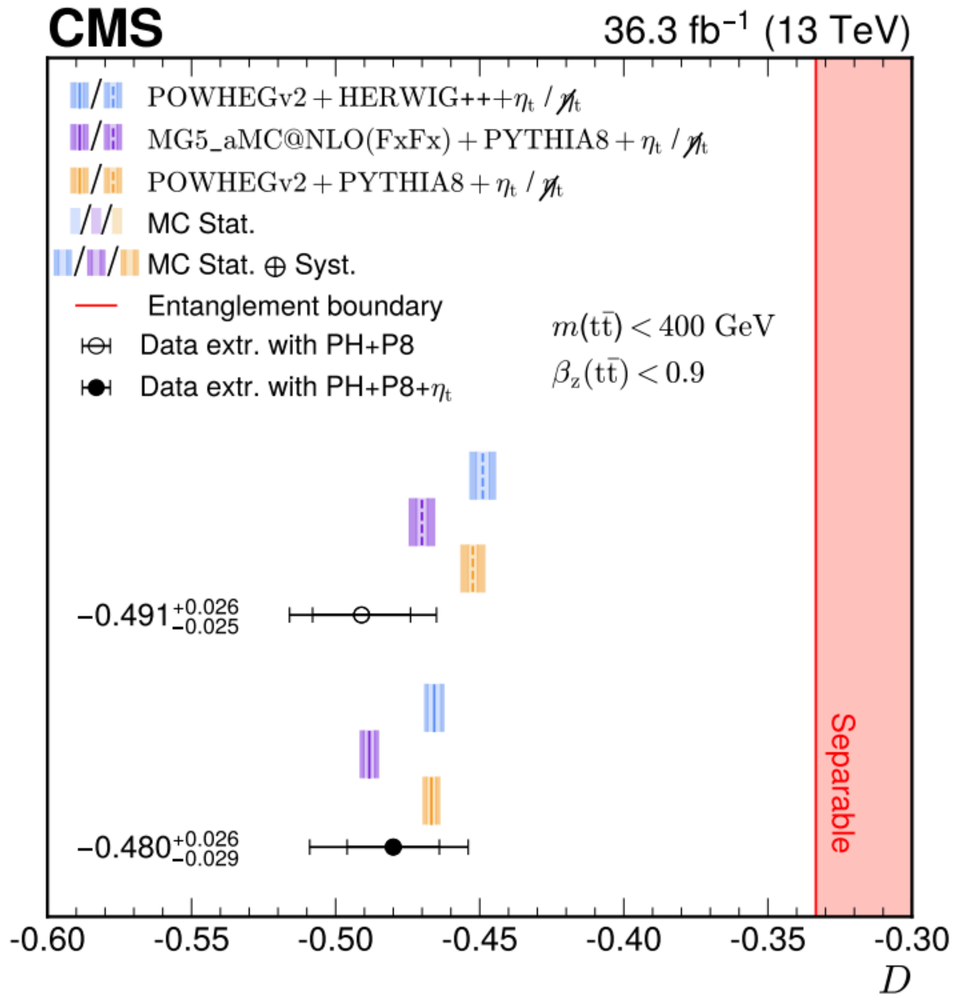


EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH (CERN)

CERN-EP-2024-137
2024/10/30
CMS-TOP-23-001

Observation of quantum entanglement in top quark pair production in proton-proton collisions at $\sqrt{s} = 13$ TeV

The CMS Collaboration*

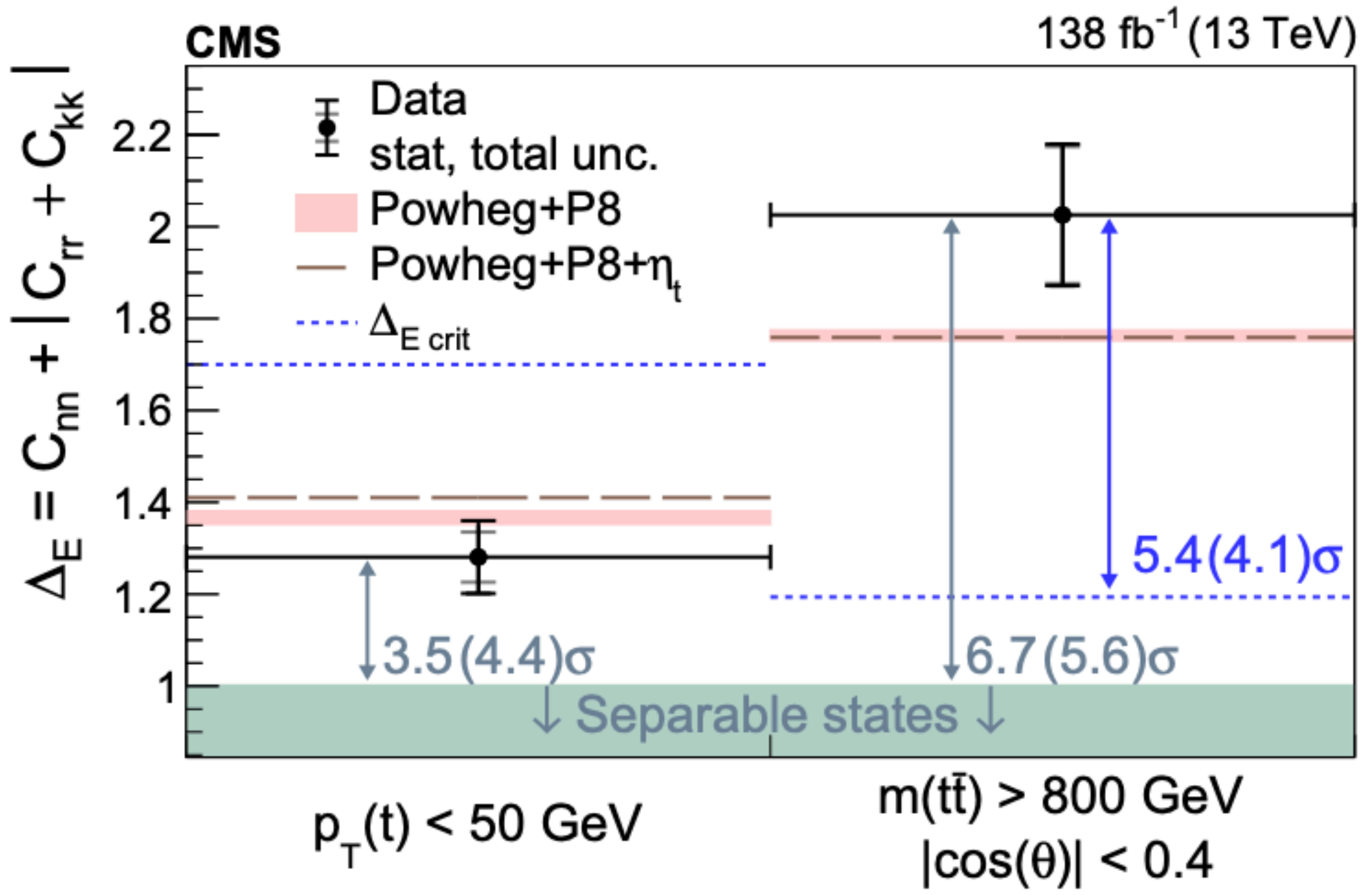


EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH (CERN)

CERN-EP-2024-231
2025/01/09
CMS-TOP-23-007

Measurements of polarization and spin correlation and observation of entanglement in top quark pairs using lepton+jets events from proton-proton collisions at $\sqrt{s} = 13$ TeV

The CMS Collaboration*



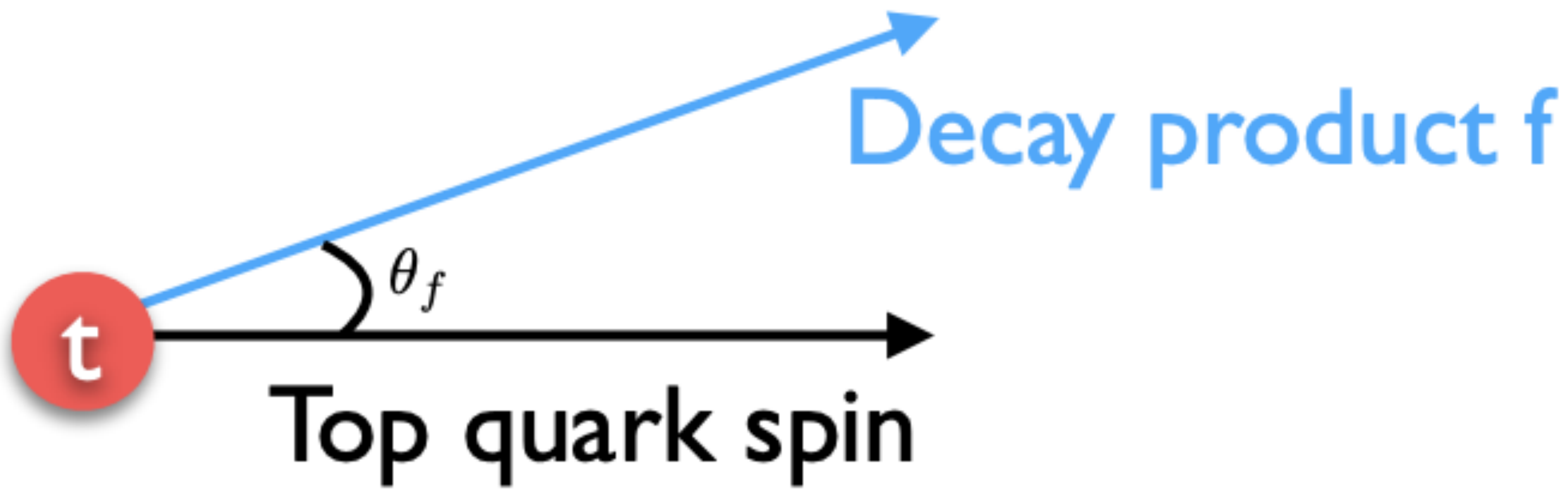
Why Top Quark?

- Heaviest fundamental fermion in the SM
- Decay before it hadronizes

$$\tau_t \sim 5 \times 10^{-25}\,\mathrm{s}\,.$$

$$\tau_{had} \sim 10^{-24}\,\mathrm{s}\,.$$

$$\tau_{\text{spin decorrelation}} \sim 10^{-21}\,\mathrm{s}\,.$$



- $\sigma_{t\bar{t}} = 834\,\mathrm{pb}$
- Top polarization directly observable via angular distribution of its decay products

$$\frac{1}{\Gamma_f}\frac{d\Gamma_f}{d\cos\theta_f} = \frac{1}{2}(1 + \omega_f \cos\theta_f)$$

	$l^+, \bar{d}$	b	$\bar{\nu}, u$
ω_f	1	-0.4	-0.3

Spin analyzing power

Quantum Tomography

- One qubit characterized by 3 parameters

$$\rho = \frac{I_2 + B_i \sigma_i}{2} \qquad\qquad B_i = \langle \sigma_i \rangle = tr(\sigma_i \rho)$$

- Two qubit characterized by 15 parameters

$$\rho = \frac{I_2 \otimes I_2 + B_i \sigma_i \otimes I_2 + \bar{B}_i I_2 \otimes \sigma_i + C_{ij} \sigma_i \otimes \sigma_j}{4}$$

$$B_i = \langle \sigma_i \otimes I_2 \rangle$$

$$\bar{B}_i = \langle I_2 \otimes \sigma_i \rangle$$

Polarizations

$$C_{ij} = \langle \sigma_i \otimes \sigma_j \rangle$$

Spin correlations

- P and CP invariance under $t\bar{t}$ production
  $\rightarrow B_i = \bar{B}_i = 0$ and $C_{ij} = C_{ji}$

Quantum Tomography

- One qubit characterized by 3 parameters

$$\rho = \frac{I_2 + B_i\sigma_i}{2} \qquad B_i = \langle \sigma_i \rangle = tr(\sigma_i \rho)$$

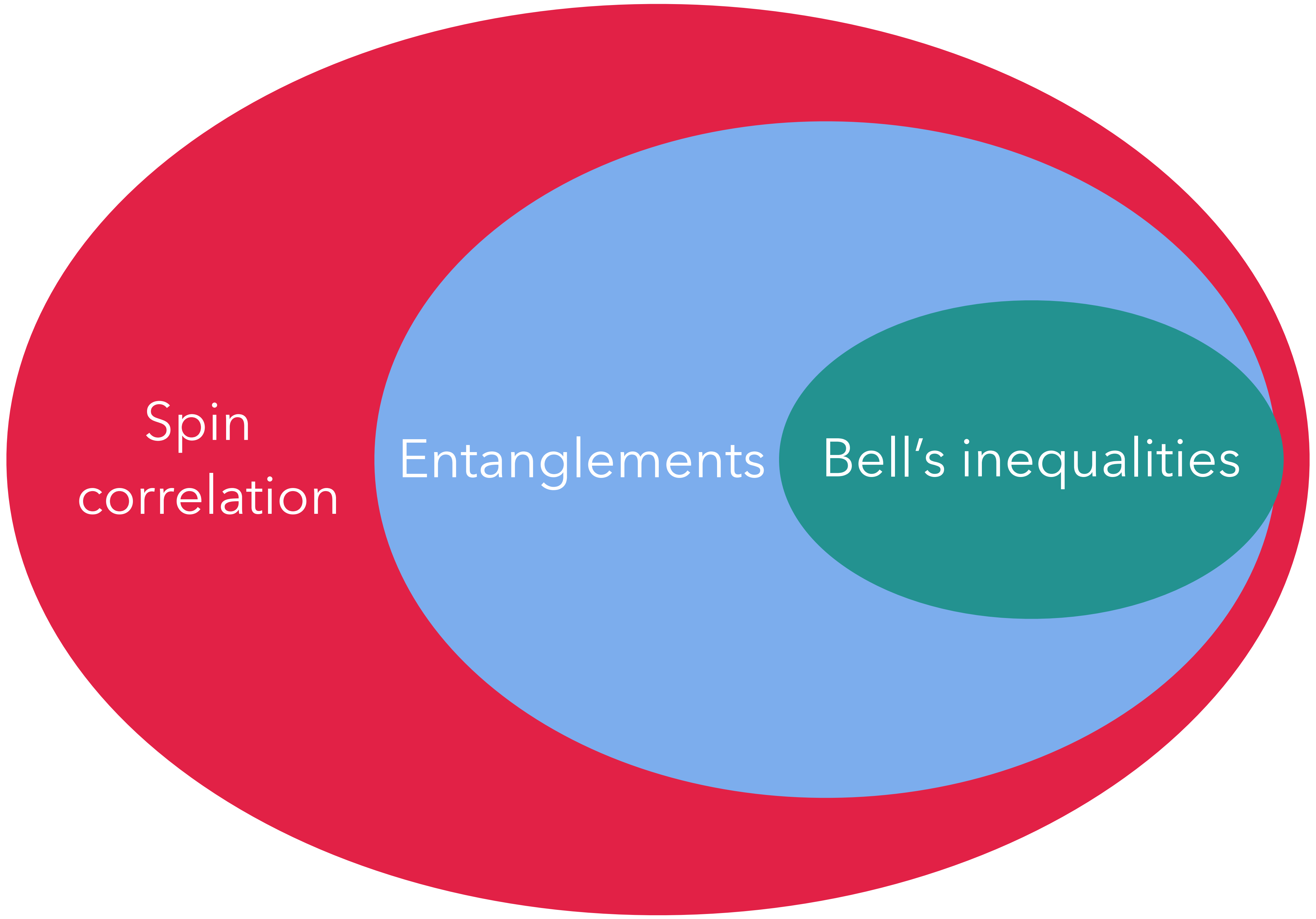
- Two qubit characterized by 15 parameters

$$\rho = \frac{I_2 \otimes I_2 + B_i\sigma_i \otimes I_2 + \bar{B}_i I_2 \otimes \sigma_i + C_{ij}\sigma_i \otimes \sigma_j}{4}$$

$B_i = \langle \sigma_i \otimes I_2 \rangle$

$\bar{B}_i = \langle I_2 \otimes \sigma_i \rangle$ Polarizations

$C_{ij} = \langle \sigma_i \otimes \sigma_j \rangle$ Spin correlations



- P and CP invariance under $t\bar{t}$ production
  $\rightarrow B_i = \bar{B}_i = 0$ and $C_{ij} = C_{ji}$

Quantum Entanglement

- A quantum state of two subsystems A and B is **separable** when its density matrix ρ can be expressed as a convex sum

$$\rho = \sum_i p_i \rho_A^i \otimes \rho_B^i$$

- If the state is not separable, it is named **entangled**

[Peres '96; Horodecki '97]

- The **Peres-Horodecki criterion** providees a necessary and sufficient condition for entanglement in two-qubit systems:

Take the **partial transpose** (transpose indices associated only to Bob (or Alice))

$$\rho^{T2} = \sum_i p_i \rho_A^i \otimes (\rho_B^i)^T$$

For a separable system, ρ^{T2} is non-negative

→ If ρ^{T2} has at least one negative eigenvalue, the system is entangled.

Quantum Entanglement

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- **Concurrence** criterion

For a separable state, the concurrence $C(\rho) = 0$ zero and for an entangled state, the concurrence is positive and $C(\rho) = 1$ is the maximally entangled state.

$$C(\rho) = \inf_{\{p_n, |\psi_n\rangle\}} \sum_n p_n C(|\psi_n\rangle)$$

Quantum Entanglement

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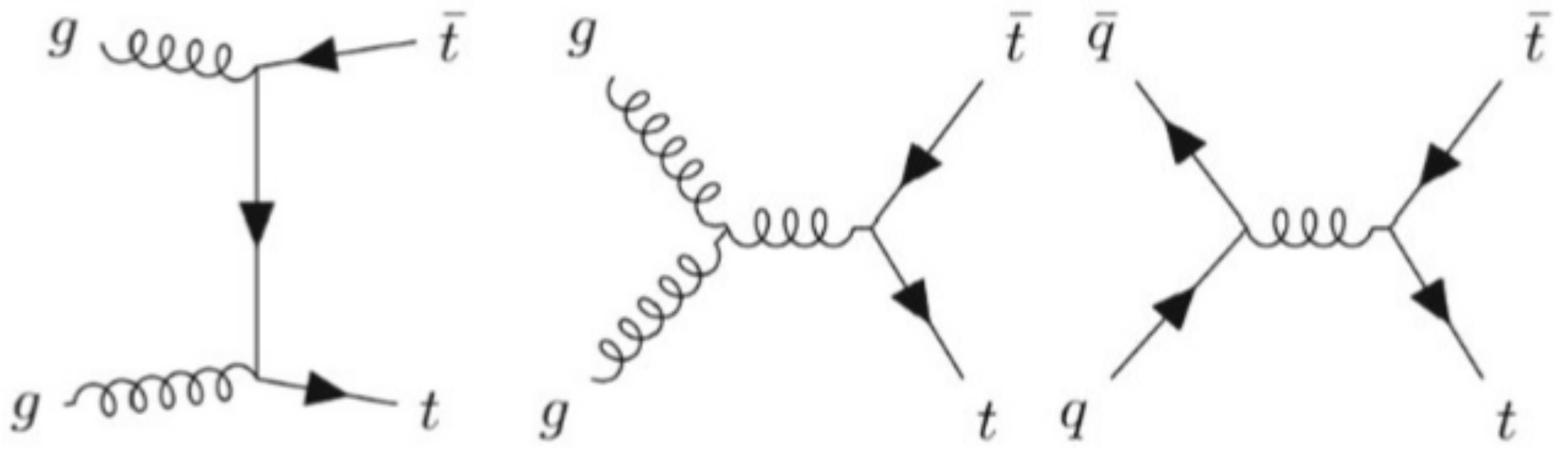
$$C(\rho) = \max\left\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\right\}$$

where λ_i are the eigenvalues of the matrix $\sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$, here $\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$.

Top quark pairs @ LHC



- Spin-density matrix

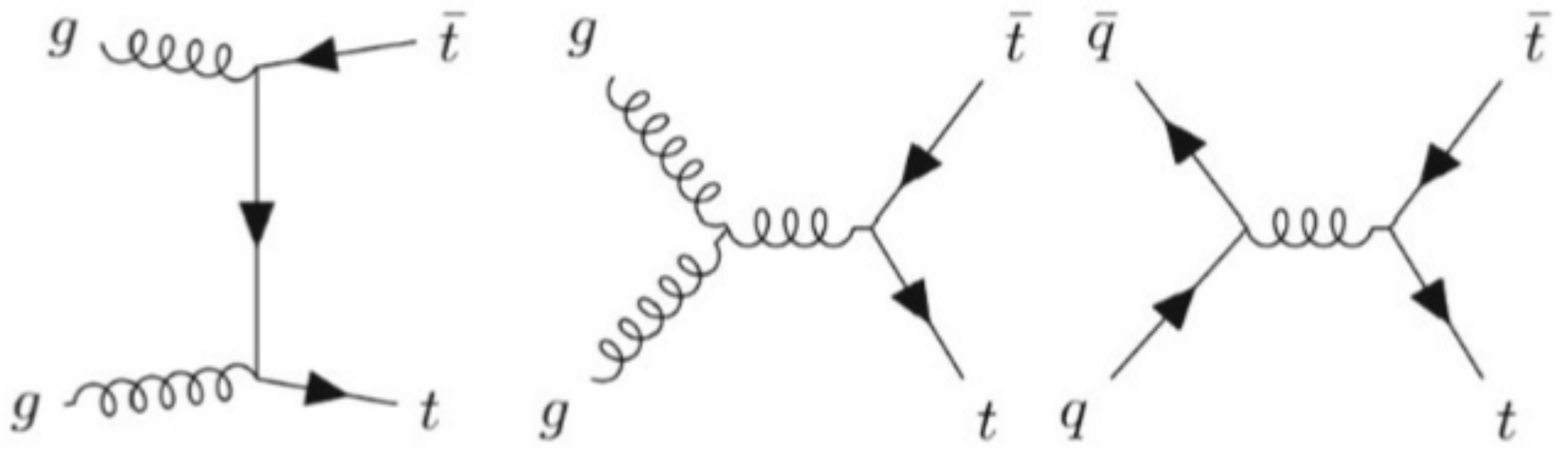
$$\mathscr{R}_{\sigma\sigma'\bar{\sigma}\bar{\sigma}'}(\hat{\beta},\theta) = \frac{1}{4}\sum_{\lambda_1,\lambda_2=\pm 1} \mathscr{M}_{\lambda_1\lambda_2;\sigma\bar{\sigma}}(\theta)\, \mathscr{M}^*_{\lambda_1\lambda_2;\sigma'\bar{\sigma}'}(\theta)$$

- Fano decompositioin

$$\mathscr{R}(\hat{\beta},\theta) = \tilde{A}\; I_2 \otimes I_2 + \tilde{B}_i^+\, \sigma_i \otimes I_2 + \tilde{B}_i^-\, I_2 \otimes \sigma_i + \tilde{C}_{ij}\, \sigma_i \otimes \sigma_j$$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \hat{\beta}}{32\pi\hat{s}}\tilde{A}$$

Top quark pairs @ LHC



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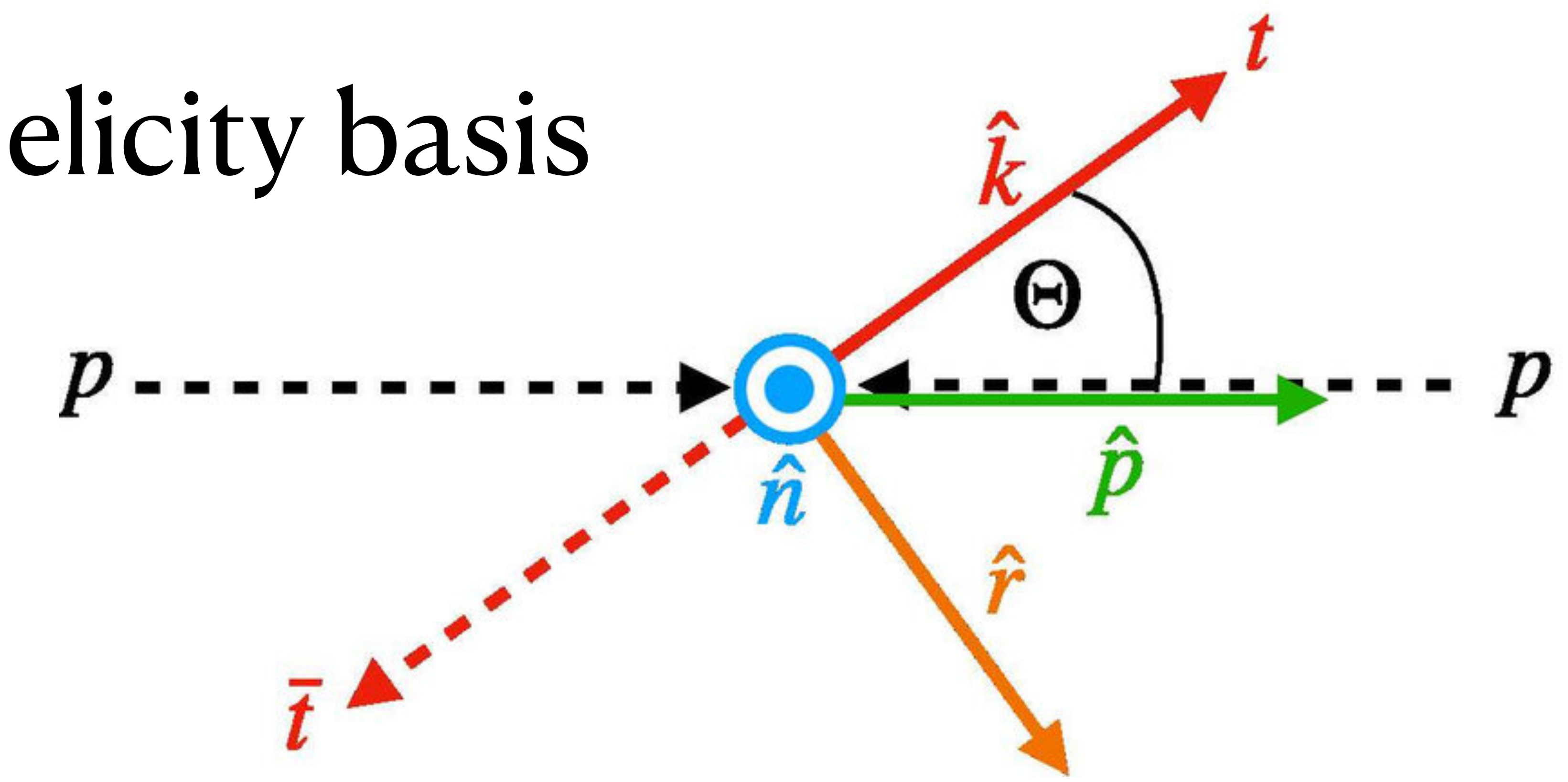
- Fano decompositioin

$$\mathscr{R}(\hat{\beta},\theta) = \tilde{A}\; I_2 \otimes I_2 + \tilde{B}_i^+ \sigma_i \otimes I_2 + \tilde{B}_i^- I_2 \otimes \sigma_i + \tilde{C}_{ij}\sigma_i \otimes \sigma_j$$

$$\rho = \frac{\mathscr{R}}{Tr(\mathscr{R})} = \frac{I_2 \otimes I_2 + B_i^+\sigma_i \otimes I_2 + \bar{B}_i^- I_2 \otimes \sigma_i + C_{ij}\sigma_i \otimes \sigma_j}{4}$$

Top quark pairs @ LHC

- Helicity basis



- Fano coefficients

$$\mathscr{R}(\hat{\beta},\theta) = \tilde{A}\; I_2 \otimes I_2 + \tilde{B}_i^+ \sigma_i \otimes I_2 + \tilde{B}_i^- I_2 \otimes \sigma_i + \tilde{C}_{ij} \sigma_i \otimes \sigma_j$$

$$\beta = \sqrt{1 - \frac{4m_t^2}{s}} \qquad z = \cos\theta$$

[Afik, Nova '20]
[Severi, Boschi, Maltoni, Sioli '21]
[Saavedra, Casas '22]

- $gg \to t\bar{t}$

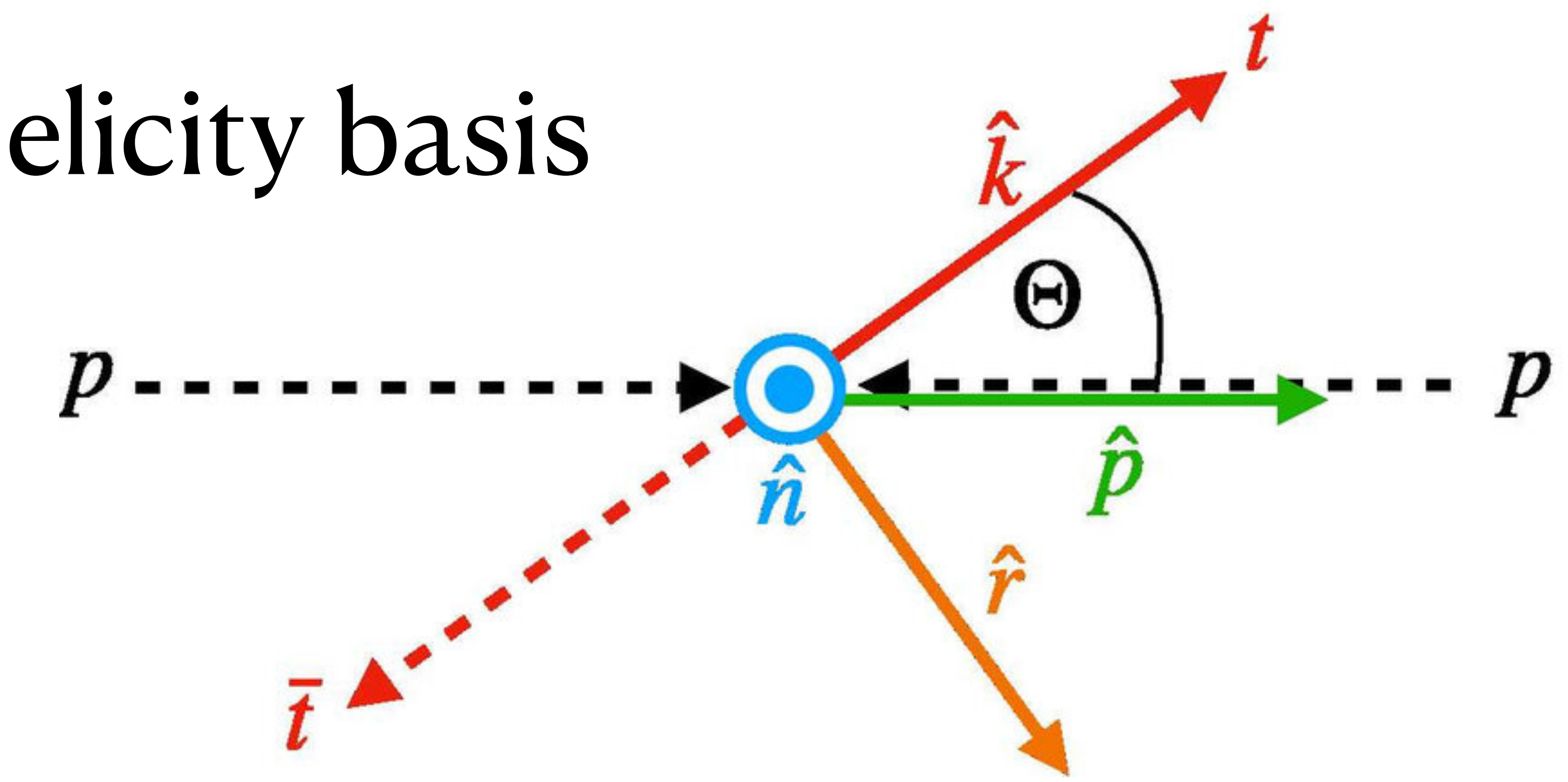
$$\begin{aligned}
F_g(z) &= \frac{7+9\beta^2 z^2}{192(1-\beta^2 z^2)^2},\\
\tilde{A}^{gg} &= F_g(z)(1+2\beta^2(1-z^2)-\beta^4(2-2z^2+z^4)),\\
\tilde{C}^{gg}_{nn} &= -F_g(z)(1-2\beta^2+\beta^4(2-2z^2+z^4)),\\
\tilde{C}^{gg}_{kk} &= -F_g(z)(1-2z^2(1-z^2)\beta^2-\beta^4(2-2z^2+z^4)),\\
\tilde{C}^{gg}_{rr} &= -F_g(z)(1-\beta^2(2-\beta^2)(2-2z^2+z^4)),\\
\tilde{C}^{gg}_{rk} &= \tilde{C}^{gg}_{kr} = F_g(z)2z(1-z^2)^{3/2}\beta^2\sqrt{1-\beta^2},
\end{aligned}$$

- $q\bar{q} \to t\bar{t}$

$$\begin{aligned}
F_q &= \frac{1}{18},\\
\tilde{A}^{q\bar{q}} &= F_q(2-\beta^2(1-z^2)),\\
\tilde{C}^{q\bar{q}}_{nn} &= -F_q\beta^2(1-z^2),\\
\tilde{C}^{q\bar{q}}_{kk} &= F_q(2z^2+\beta^2(1-z^2)),\\
\tilde{C}^{q\bar{q}}_{rr} &= F_q(2-\beta^2)(1-z^2),\\
\tilde{C}^{q\bar{q}}_{rk} &= \tilde{C}^{q\bar{q}}_{kr} = F_q 2z\sqrt{(1-\beta^2)(1-z^2)},
\end{aligned}$$

Top quark pairs @ LHC

- Helicity basis



$$\beta = \sqrt{1 - \frac{4m_t^2}{s}} \qquad z = \cos\theta$$

[Afik, Nova '20]
[Severi, Boschi, Maltoni, Sioli '21]
[Saavedra, Casas '22]

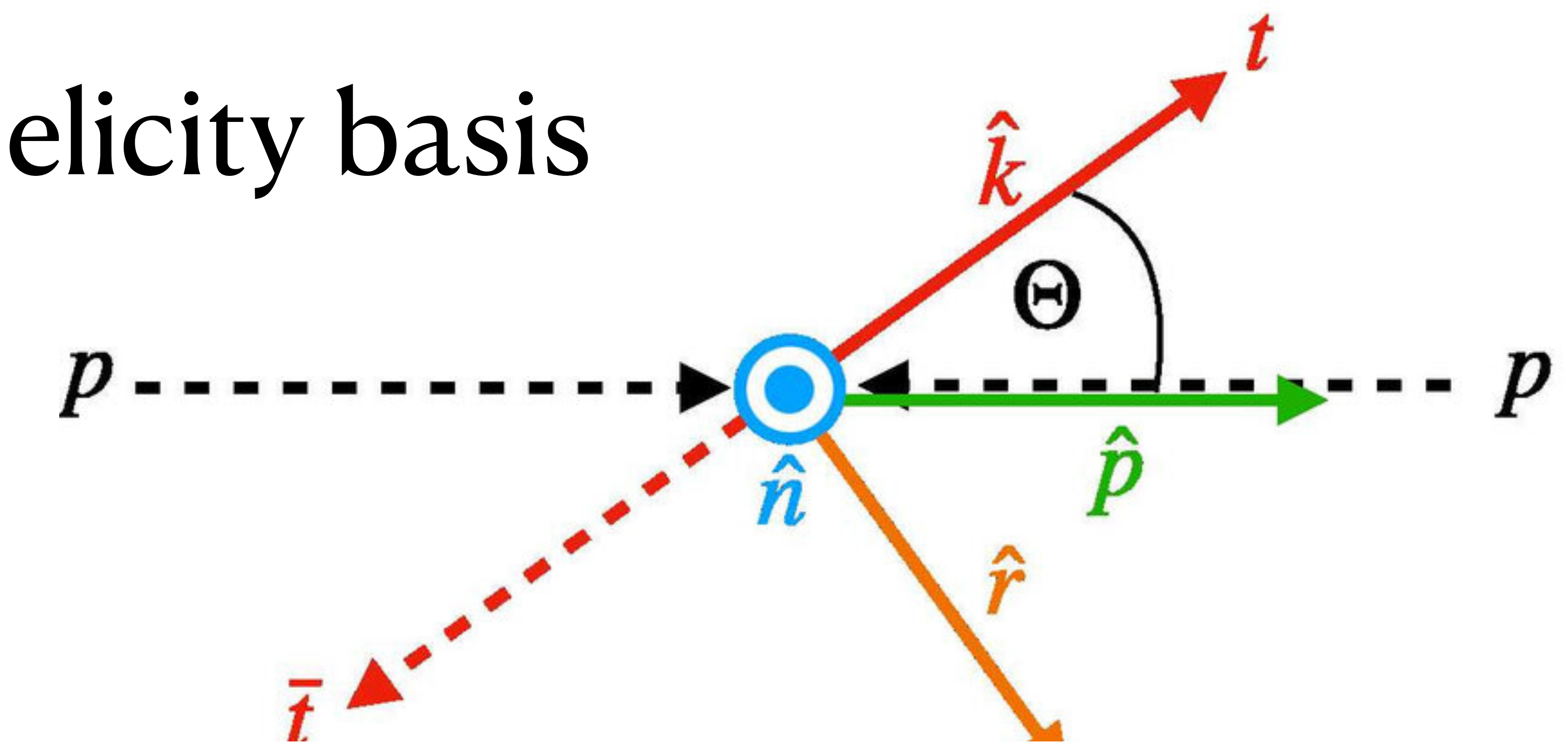
- Angular averaged spin-density matrix

$$R = \frac{1}{4\pi}\int d\Omega \mathscr{R}(\hat{\beta}, z), \qquad \rho = \frac{R}{Tr(R)}$$

$$\frac{1}{\sigma}\frac{d^2\sigma}{d\Omega_1 d\Omega_2} = \frac{1}{4\pi^2}\left(1 + \alpha_1 \vec{B}_1 \cdot \hat{\ell}_1 + \alpha_2 \vec{B}_2 \cdot \hat{\ell}_2 + \alpha_1\alpha_2\, C_{ij}\,\hat{\ell}_{1i}\,\hat{\ell}_{2i}\right)$$

Top quark pairs @ LHC

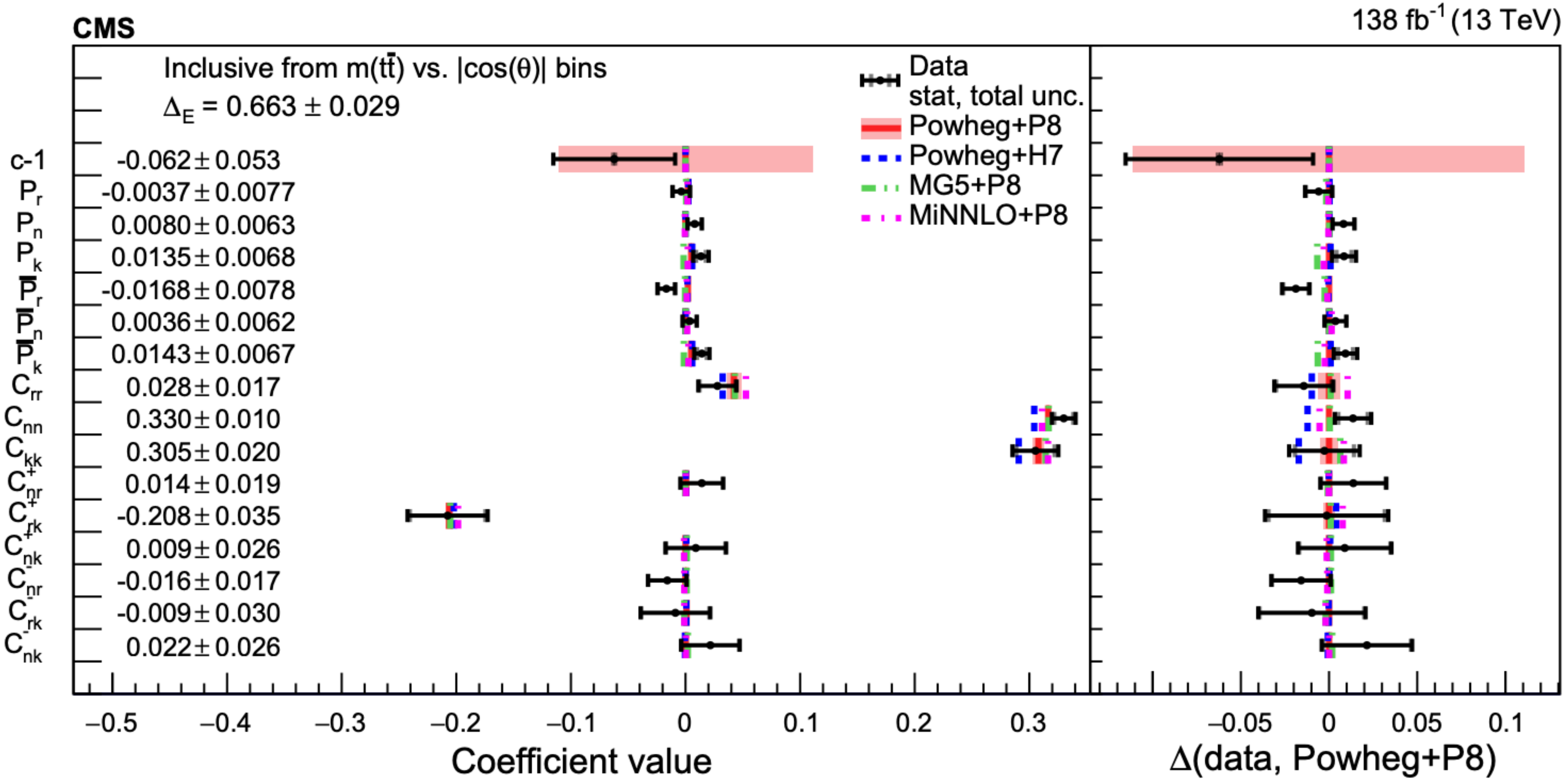
- Helicity basis



- Angular averaged spin-density matrix

$$R = \frac{1}{4\pi}\int d\Omega \mathscr{R}$$

$$\frac{1}{\sigma}\frac{d^2\sigma}{d\Omega_1 d\Omega_2} = \frac{1}{4\pi^2}$$



Peres-Horodecki criterion

[Maltoni, Severi, Tentori, Vryonidoud '24]

$$D^{(1)} = {}^1\!/\!_3(+C_{kk} + C_{rr} + C_{nn}),$$
$$D^{(k)} = {}^1\!/\!_3(+C_{kk} - C_{rr} - C_{nn}),$$
$$D^{(r)} = {}^1\!/\!_3(-C_{kk} + C_{rr} - C_{nn}),$$
$$D^{(n)} = {}^1\!/\!_3(-C_{kk} - C_{rr} + C_{nn}).$$
$$D_{\min} \equiv \min\{D^{(1)}, D^{(k)}, D^{(r)}, D^{(n)}\},$$

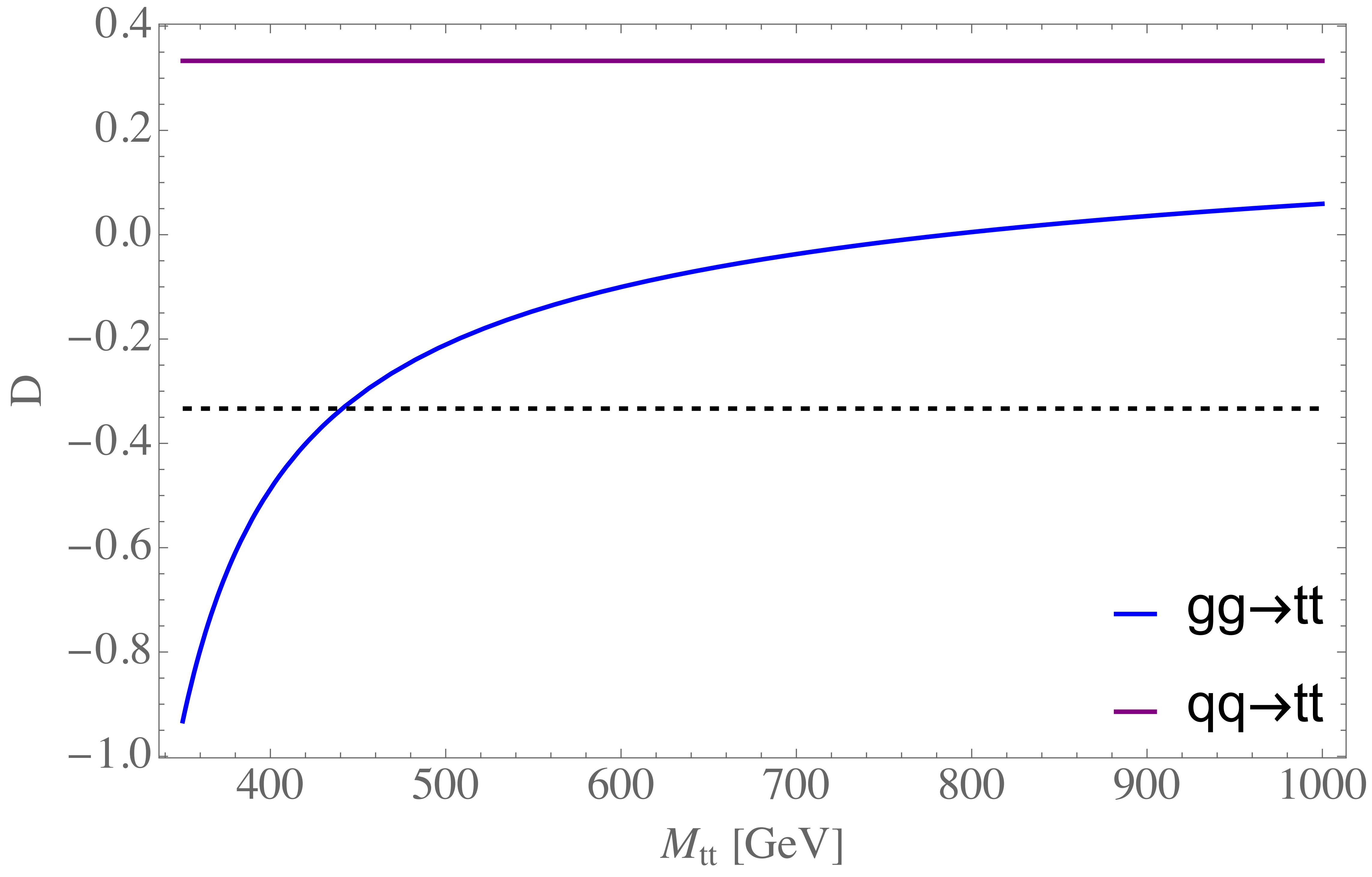
$$D_{\min} < -{}^1\!/\!_3 \qquad \text{(entanglement)}$$

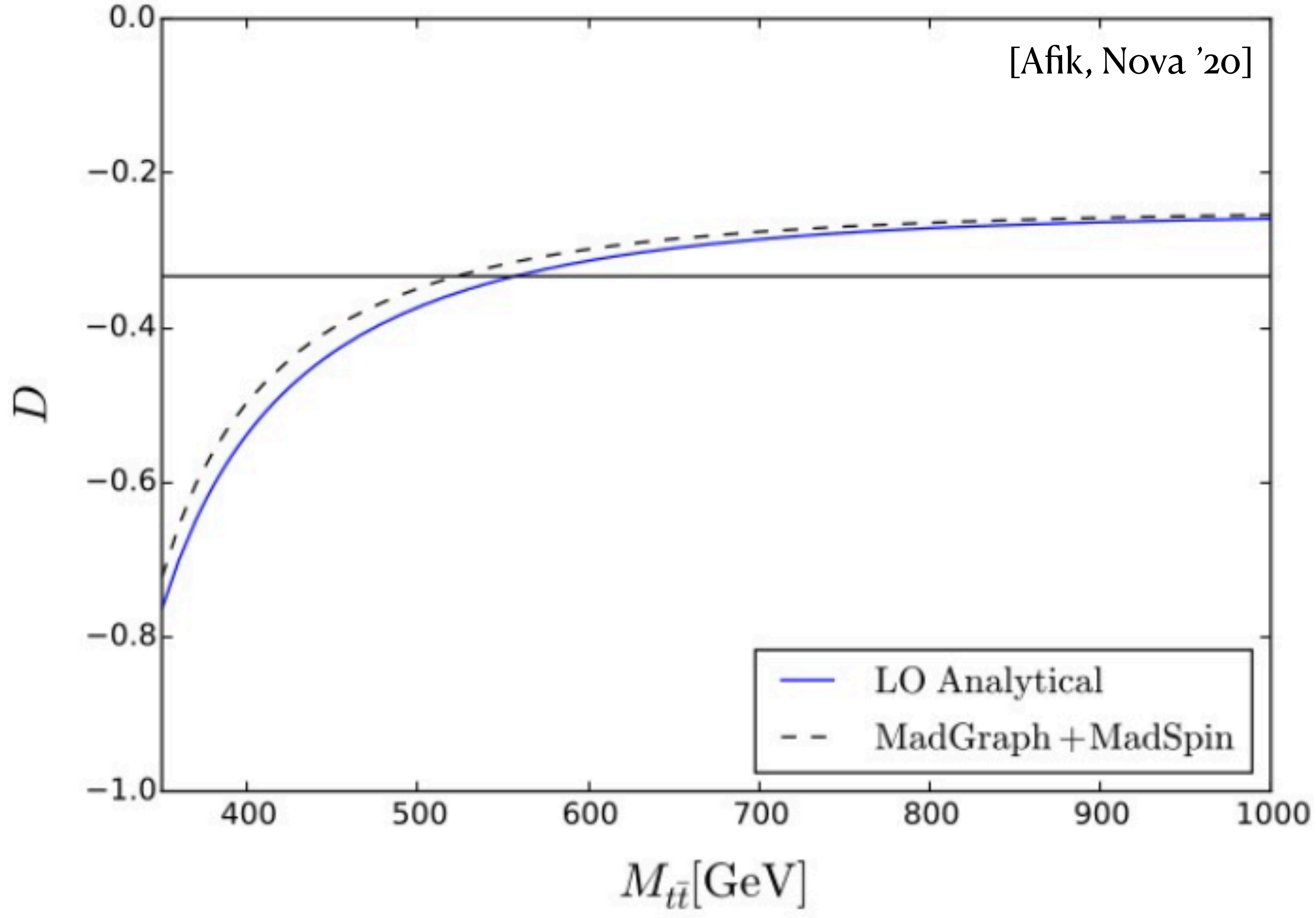
$$\frac{d\sigma}{d\cos\phi} = \frac{1}{2}\left(1 - D\cos\phi\right), \qquad D = \frac{tr[C]}{3}.$$

Peres-Horodecki criterion $D < -1/3$

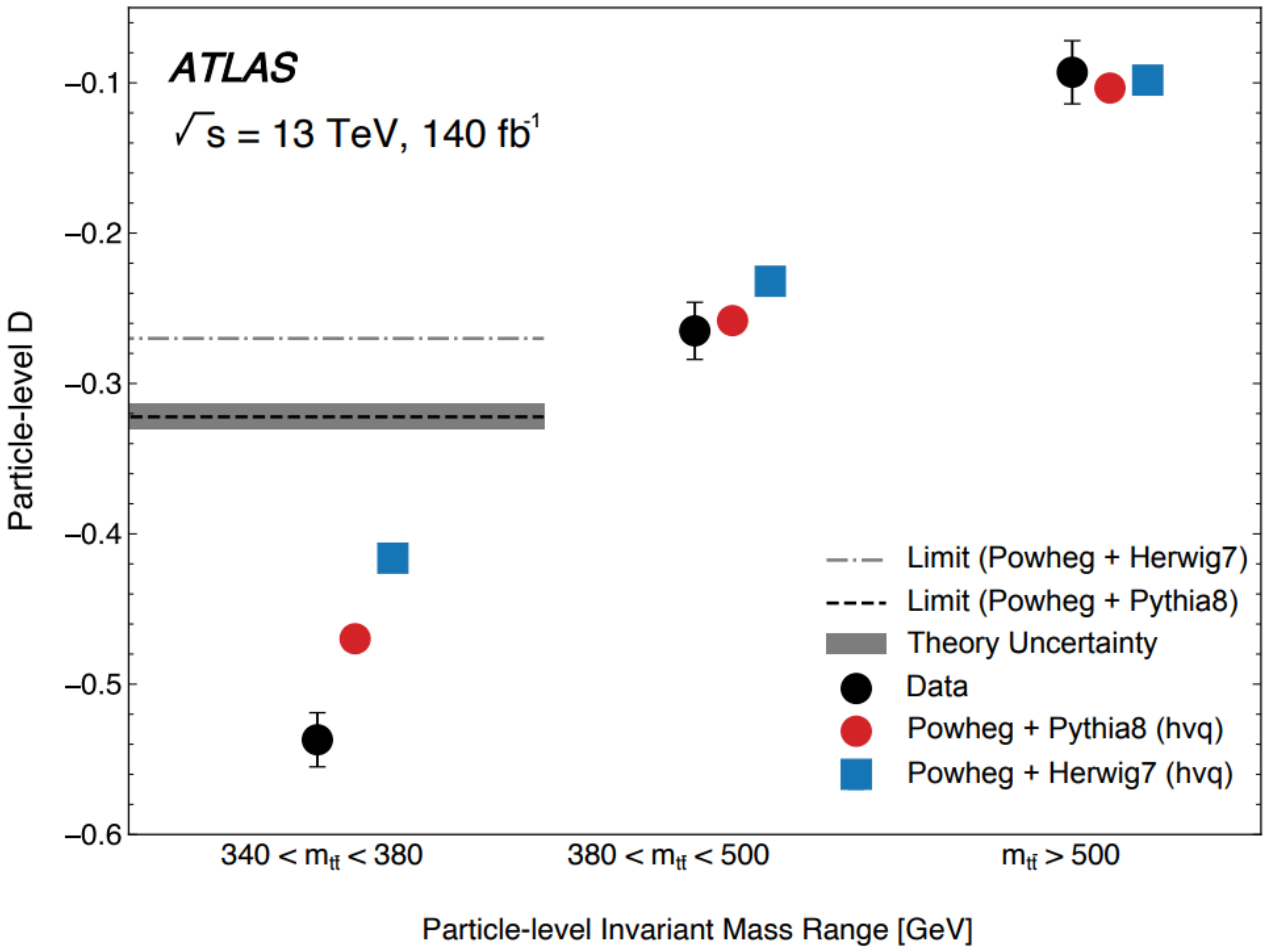
$$\frac{d\sigma}{d\cos\phi} = \frac{1}{2}\left(1 - D\cos\phi\right), \qquad D = \frac{tr[C]}{3}.$$

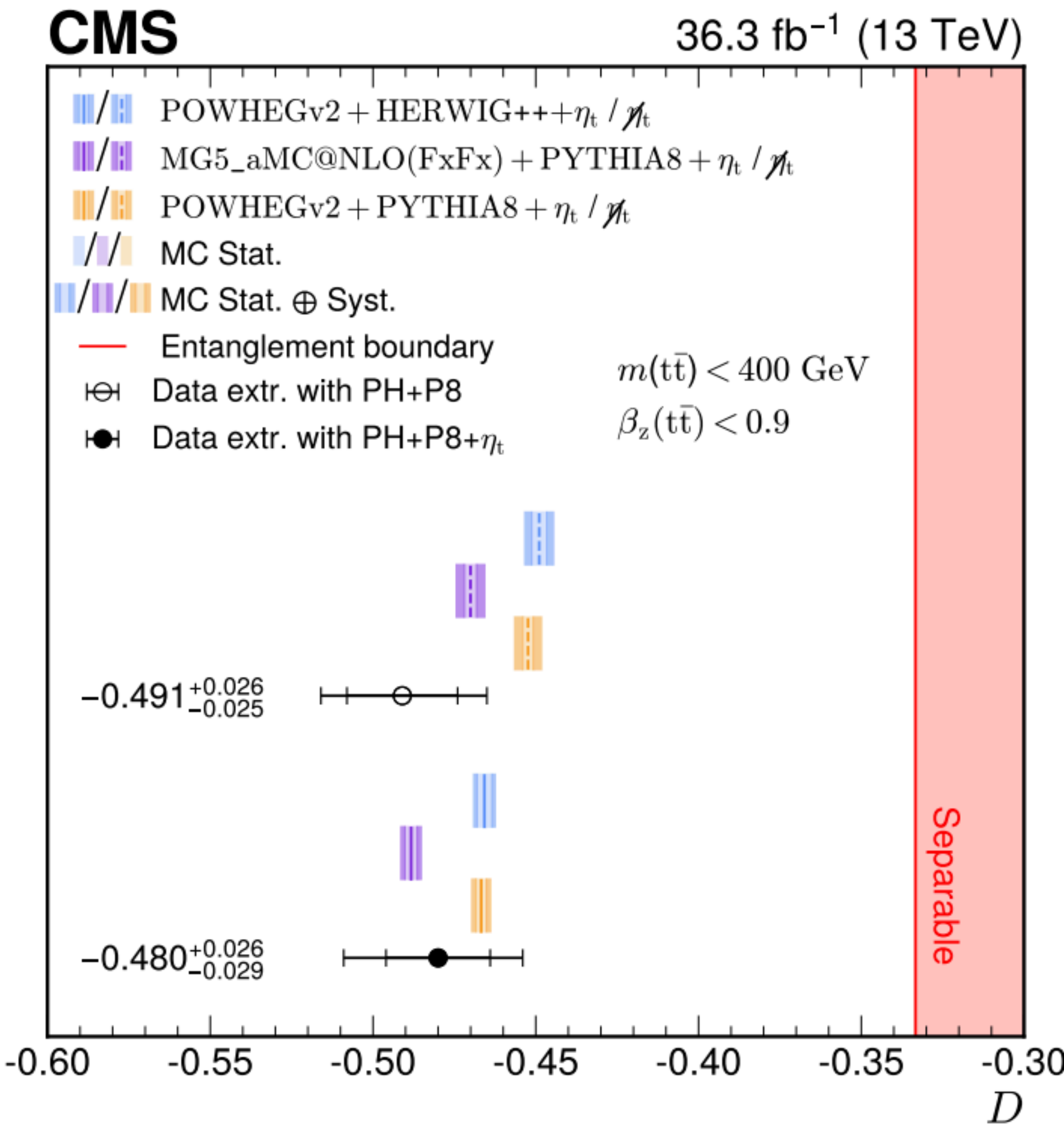
- ϕ is the angle between the leptons measured in the parent top/antitop frames





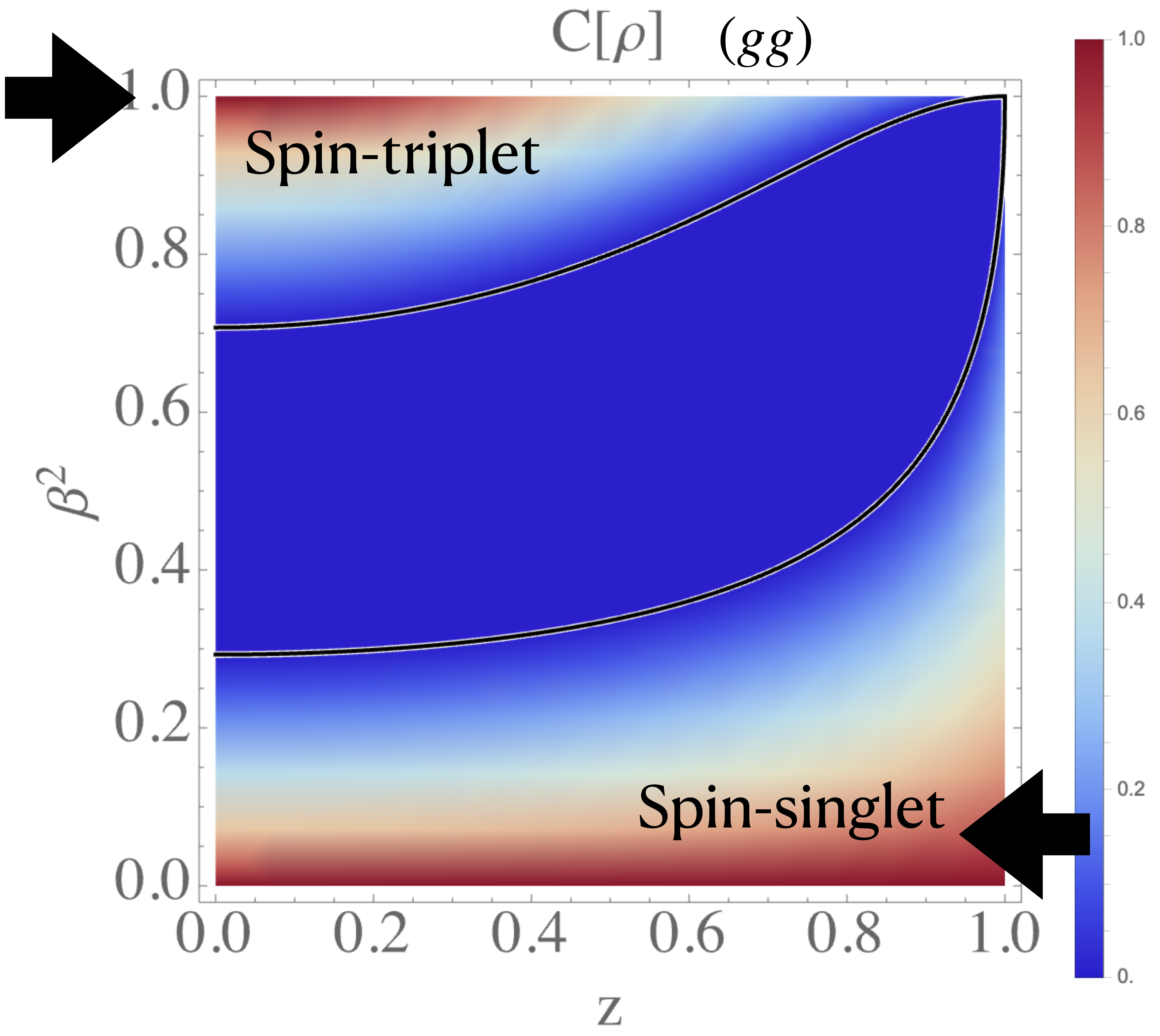
Peres-Horodecki criterion $D < -1/3$

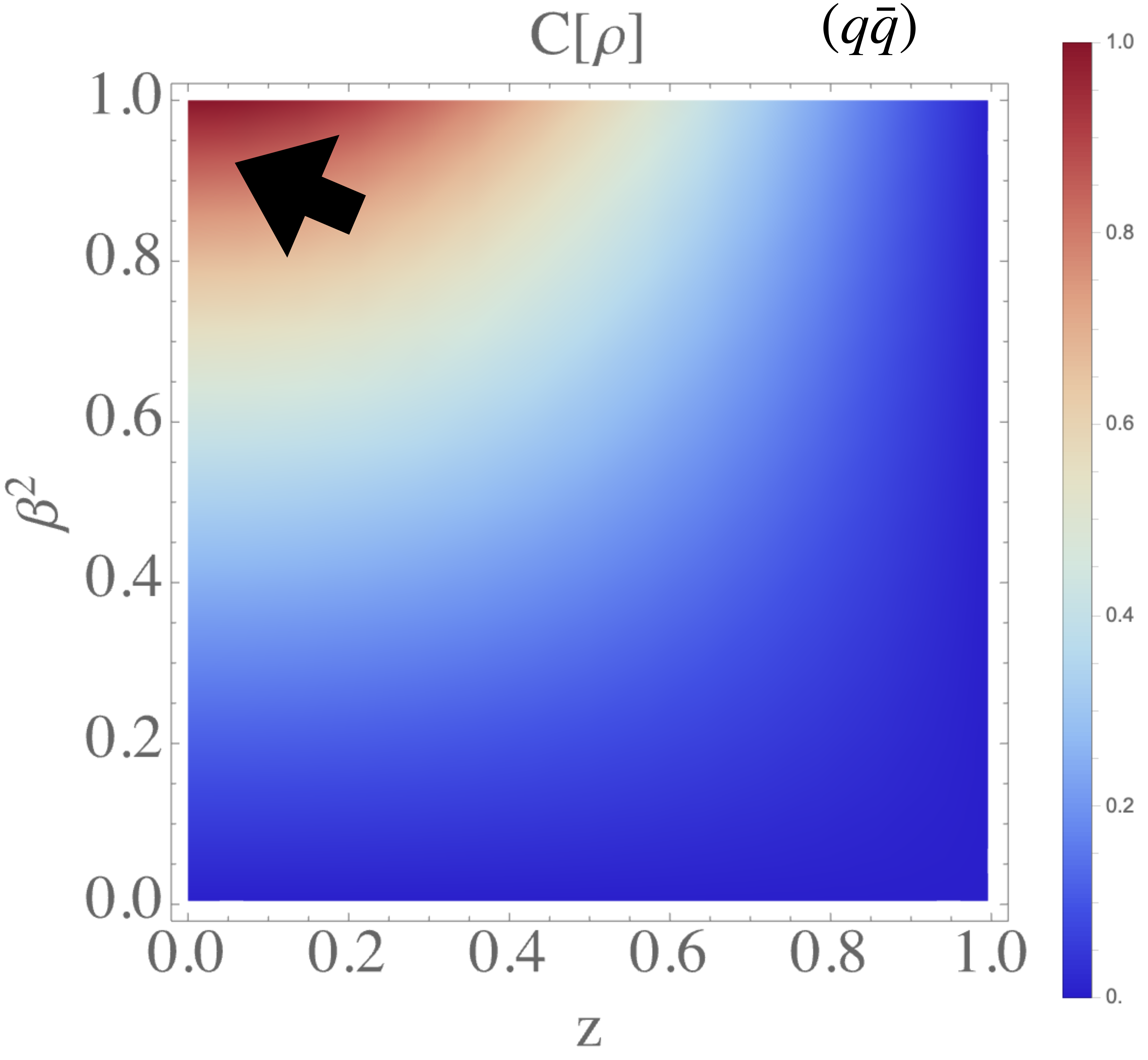




Concurrence

$$C = \frac{1}{2}\max\left(0, -1 - 3D_{\min}\right)$$





Bell's inequality violation

[Maltoni, Severi, Tentori, Vryonidoud '24]

$$\left|\vec{a}_1 \cdot C \cdot (\vec{b}_1 - \vec{b}_2) + \vec{a}_2 \cdot C \cdot (\vec{b}_1 + \vec{b}_2)\right| > 2$$

$$D^{(1)} = {}^1\!/_3(+C_{kk} + C_{rr} + C_{nn}),$$
$$D^{(k)} = {}^1\!/_3(+C_{kk} - C_{rr} - C_{nn}),$$
$$D^{(r)} = {}^1\!/_3(-C_{kk} + C_{rr} - C_{nn}),$$
$$D^{(n)} = {}^1\!/_3(-C_{kk} - C_{rr} + C_{nn}).$$

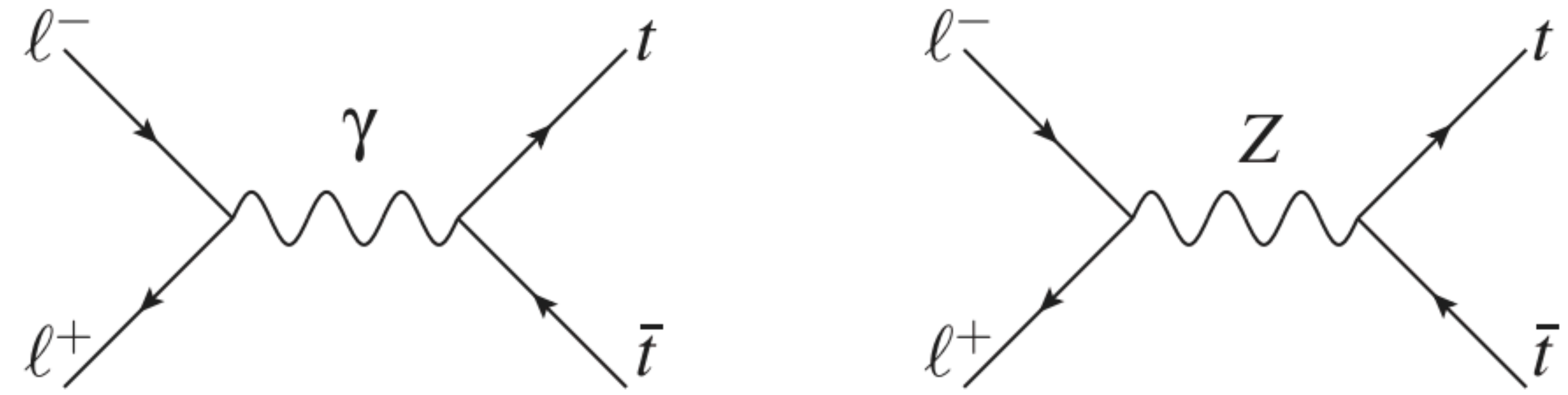
$$D_{\min} \equiv \min\{D^{(1)}, D^{(k)}, D^{(r)}, D^{(n)}\},$$

$$D_{\min} < -{}^1\!/_3 \qquad \text{(entanglement)}$$

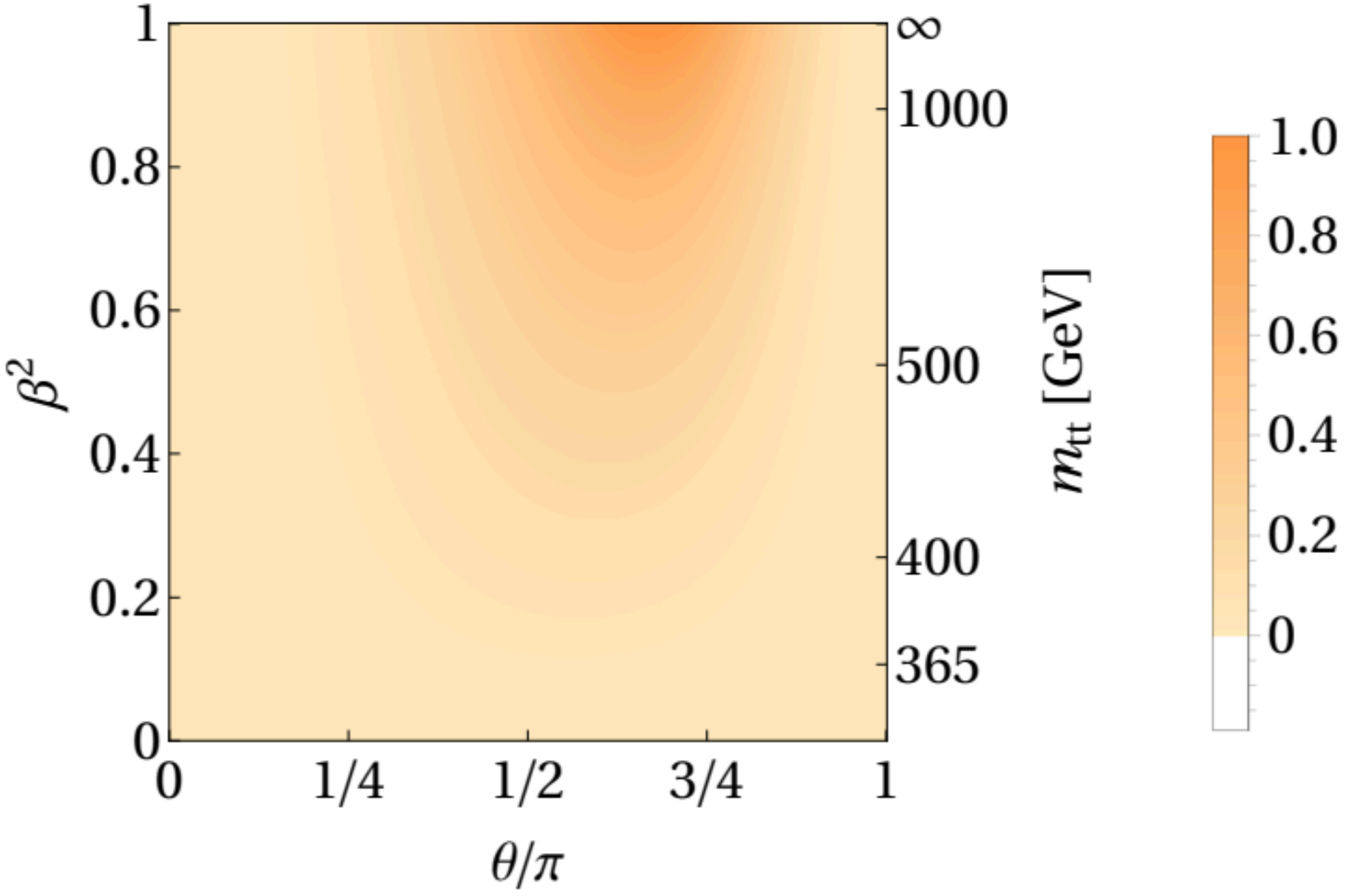
$$D_{\min} < -\frac{1}{\sqrt{2}} \qquad \text{(Bell violation)}$$

Top quark pairs @ Lepton Collider

[Maltoni, Severi, Tentori, Vryonidoud '24]



Concurrence



$$\begin{cases} A^{[0]} &= F^{[0]}\left(\beta^2 c_\theta^2 - \beta^2 + 2\right), \\ \widetilde{C}_{kk}^{[0]} &= F^{[0]}\left[\beta^2 - \left(\beta^2 - 2\right) c_\theta^2\right], \\ \widetilde{C}_{kr}^{[0]} &= F^{[0]}\, 2\sqrt{1-\beta^2} c_\theta s_\theta, \\ \widetilde{C}_{rr}^{[0]} &= F^{[0]}\, s_\theta^2 (2-\beta^2), \\ \widetilde{C}_{nn}^{[0]} &= -F^{[0]}\, \beta^2 s_\theta^2, \end{cases} \quad \begin{cases} A^{[1]} = 2\, F^{[1]}\, c_\theta, \\ \widetilde{C}_{kk}^{[1]} = 2\, F^{[1]}\, c_\theta, \\ \widetilde{C}_{kr}^{[1]} = F^{[1]}\, \sqrt{1-\beta^2}\, s_\theta, \\ \widetilde{C}_{rr}^{[1]} = 0, \\ \widetilde{C}_{nn}^{[1]} = 0, \end{cases} \quad \begin{cases} A^{[2]} = F^{[2]}\, \left(1 + c_\theta^2\right), \\ \widetilde{C}_{kk}^{[2]} = F^{[2]}\, \left(1 + c_\theta^2\right), \\ \widetilde{C}_{kr}^{[2]} = 0, \\ \widetilde{C}_{rr}^{[2]} = -F^{[2]}\, s_\theta^2, \\ \widetilde{C}_{nn}^{[2]} = F^{[2]}\, s_\theta^2 . \end{cases}$$

$$F^{[0]} = 48 e^4 \Big(Q_{\mathrm{t}}^2 Q_\ell^2 + 2\,\mathrm{Re}\, \frac{4 Q_{\mathrm{t}} Q_\ell g_{\mathrm{Vt}} g_{\mathrm{V}\ell} m_{\mathrm{t}}^2}{D_Z} + \frac{16 g_{\mathrm{Vt}}^2 m_{\mathrm{t}}^4 (g_{\mathrm{V}\ell}^2 + g_{\mathrm{A}\ell}^2)}{|D_Z|^2} \Big),$$

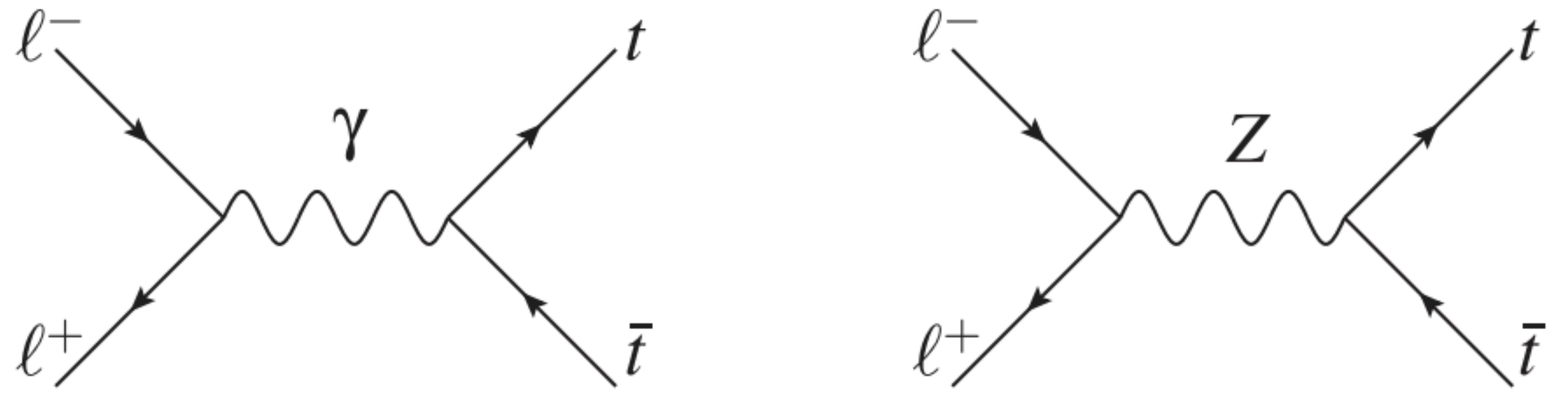
$$F^{[1]} = 192 e^4 g_{\mathrm{At}} g_{\mathrm{A}\ell} m_{\mathrm{t}}^2 \beta \Big(\frac{16 g_{\mathrm{Vt}} g_{\mathrm{V}\ell} m_{\mathrm{t}}^2}{|D_Z|^2} + 2\,\mathrm{Re}\, \frac{Q_{\mathrm{t}} Q_\ell}{D_Z} \Big),$$

$$F^{[2]} = \frac{768 e^4 g_{\mathrm{At}}^2 m_{\mathrm{t}}^4 \beta^2 (g_{\mathrm{V}\ell}^2 + g_{\mathrm{A}\ell}^2)}{|D_Z|^2},$$

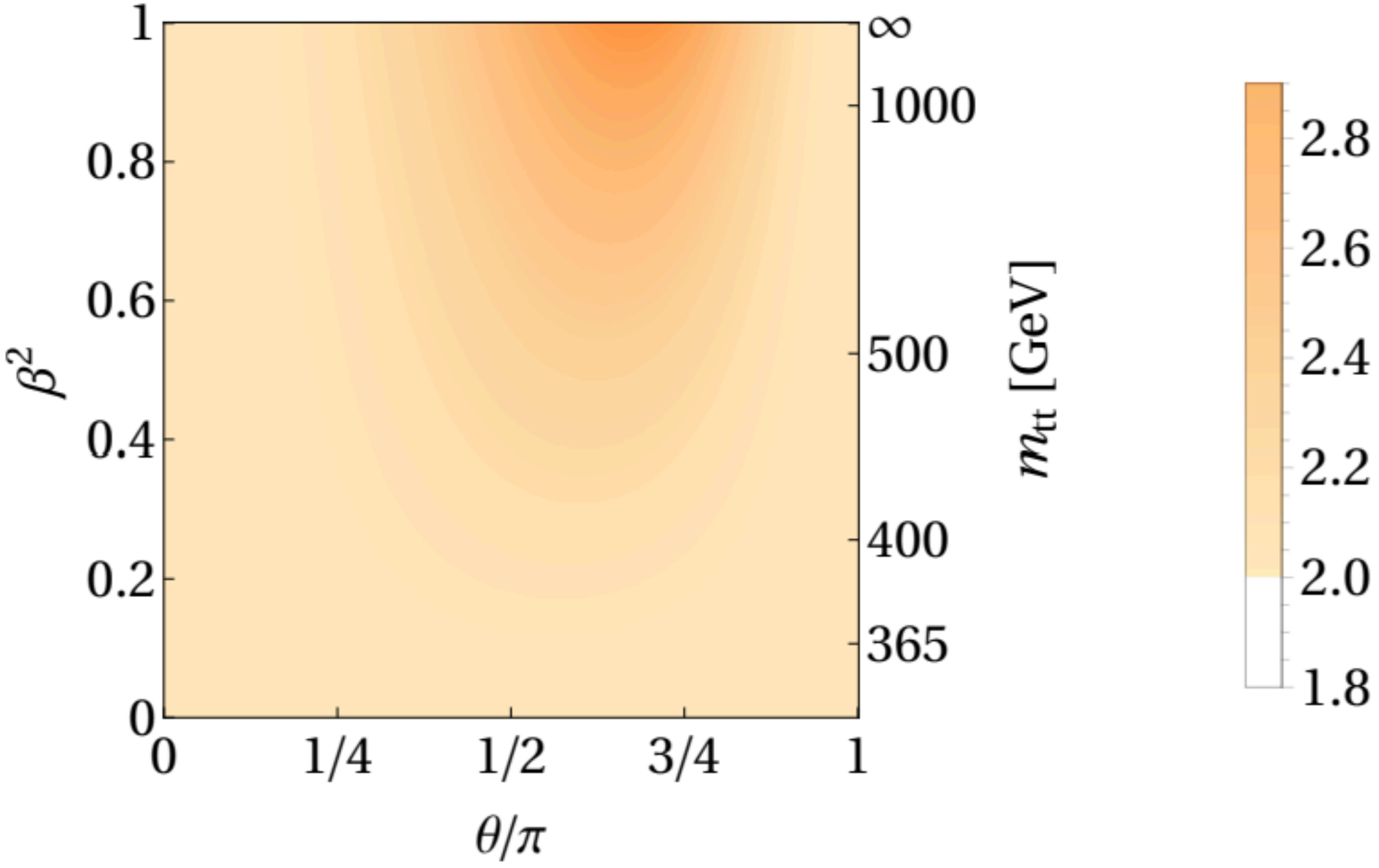
$$D_Z \;=\; c_{\mathrm{W}}^2 s_{\mathrm{W}}^2 (4 m_{\mathrm{t}}^2 - (1 - \beta^2)\, m_{\mathrm{Z}}^2)$$

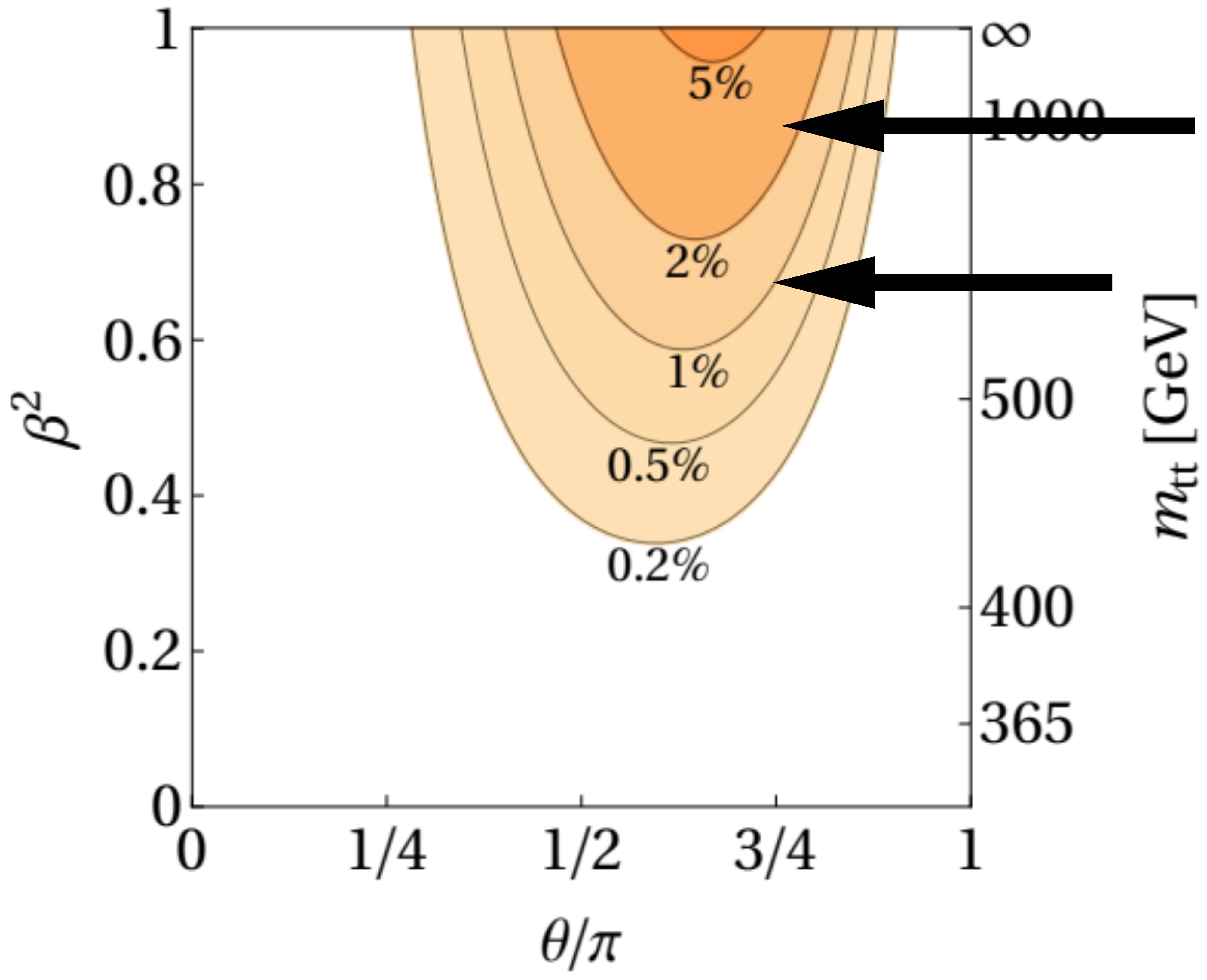
Top quark pairs @ Lepton Collider

[Maltoni, Severi, Tentori, Vryonidoud '24]



Bell's inequality violation

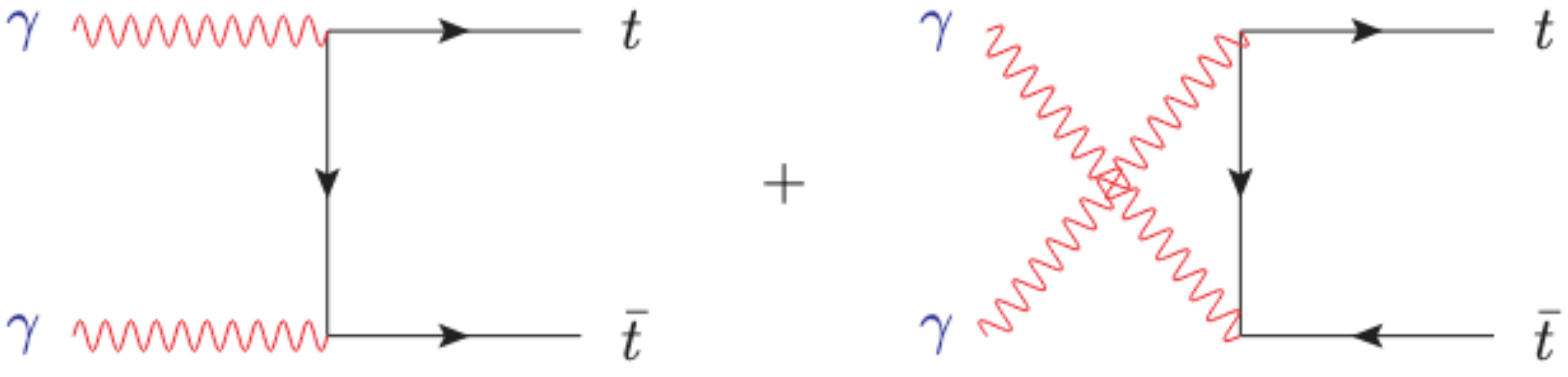




Top quark pairs @ Photon Collider

[Han, Low, Su '25]
[DWK and S.Y Choi work in progress]

- We can also consider the process $\gamma\gamma \to t\bar{t}$.
  The process is very similar to the process $gg \to t\bar{t}$.

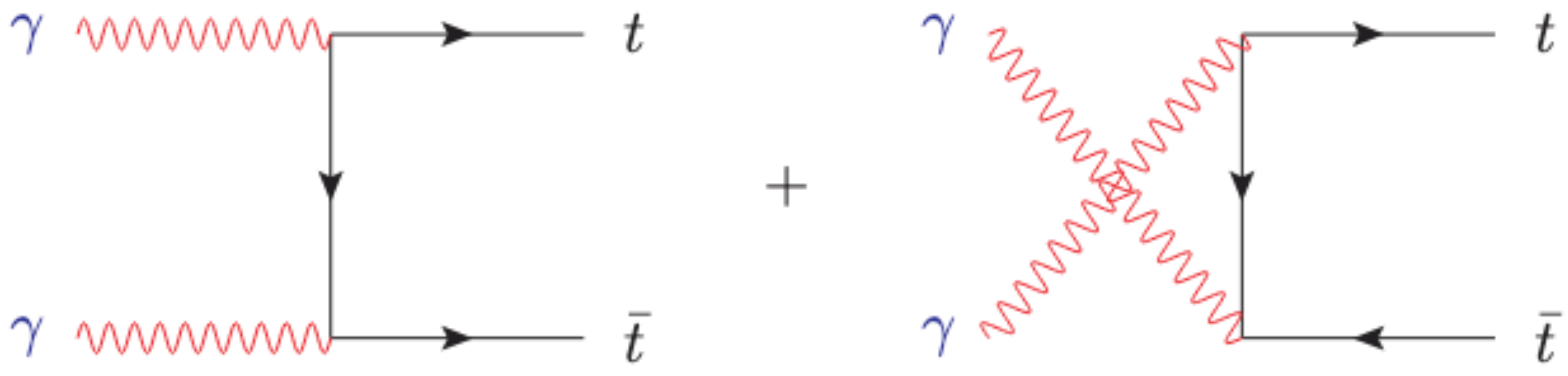


- Also independent with the process $l^+l^- \to t\bar{t}$.

$$F_\gamma = \frac{64\pi^2\alpha^2Q_t^4}{(1-\beta^2z^2)^2}, \qquad \tilde{B}_i^{\pm} = 0,$$
$$\tilde{A}^{\gamma\gamma} = F_\gamma(1 + 2\beta^2(1-z^2) - \beta^4(2 - 2z^2 + z^4)),$$
$$\tilde{C}_{nn}^{\gamma\gamma} = F_\gamma(1 + 2\beta^2(1-z^2) + \beta^4(2 - 2z^2 + z^4)),$$
$$\tilde{C}_{kk}^{\gamma\gamma} = -F_\gamma(1 - 2z^2(1-z^2)\beta^2 - \beta^4(2 - 2z^2 + z^4)),$$
$$\tilde{C}_{rr}^{\gamma\gamma} = -F_\gamma(1 - \beta^2(2-\beta^2)(2 - 2z^2 + z^4)),$$
$$\tilde{C}_{rk}^{\gamma\gamma} = \tilde{C}_{kr}^{\gamma\gamma} = F_\gamma 2z(1-z^2)^{3/2}\beta^2\sqrt{1-\beta^2},$$
$$\tilde{C}_{nr}^{\gamma\gamma} = \tilde{C}_{rn}^{\gamma\gamma} = \tilde{C}_{nk}^{\gamma\gamma} = \tilde{C}_{kn}^{\gamma\gamma} = 0,$$

Top quark pairs @ Photon Collider

[DWK and S.Y Choi work in progress]



- It is well established that at a future linear e^+e^- collider, the Compton backscattering of the laser light with the $e^\pm$ beam can provide very high-energy photons.
- Especially, top-quark pair production is possible with large rates in (un)polarized photon-photon fusion.
- Operation below the threshold of $e^\pm$ pair production in collisions between the laser beam and the Compton-backscattered photon beam requires $x = 4E_e\omega/m_e^2 \leq 2(1+\sqrt{2})$ with the initial $e^\pm$energy $E_e$and laser frequency ω .

Top quark pairs @ Photon Collider

[DWK and S.Y Choi work in progress]

- The normalized photon energy spectrum resulting from Compton backscattering of unpolarized laser light off the unpolarized high-energy $e^{\pm}$ beam is explicitly given by

$$f_{\gamma}(y) = \frac{1}{\mathcal{N}(x)}\left[\frac{1}{1-y} - y + (2r-1)^2\right]$$

[Ginzburg, Kotkin, Serbo, Telnov '83]

with y being the fraction of the $e^{\pm}$ energy transfered to the photon and $r = y/x(1-y)$,

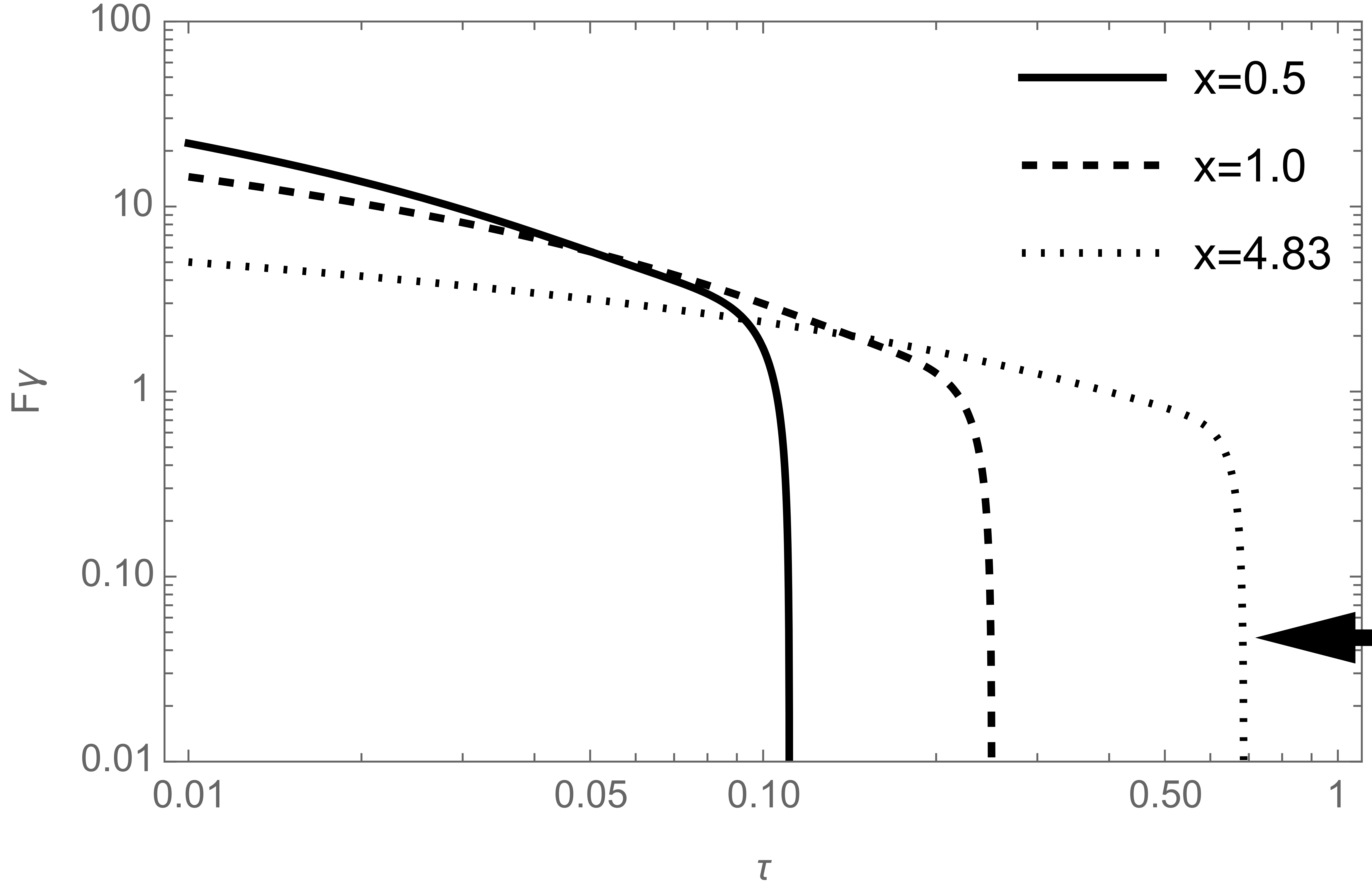
$$\mathcal{N}(x) = \frac{(x^2-4x-8)}{x^2}\ln(1+x) + \frac{1}{2} + \frac{8}{x} - \frac{1}{2(1+x)^2} \quad \text{with} \quad 0 \le y \le \frac{x}{x+1} = y_{\max}$$

Top quark pairs @ Photon Collider

[DWK and S.Y Choi work in progress]

- For the unpolarized laser beams the number of the $t\bar{t}$ is given by laser light off the unpolarized high-energy $e^{\pm}$ beam is explicitly given by

$$\frac{dN}{d\tau} = \kappa^2 L_{ee} F_{\gamma}(\tau)\ \hat{\sigma}(\tau s) \quad \text{with} \quad F_{\gamma}(\tau) = \int_{\tau/y_m}^{y_m} \frac{dy}{y} f_{\gamma}(y) f_{\gamma}(\tau/y)$$



Top quark pairs @ Photon Collider

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with $s = 4E_e^2$, $y_m = x/(1+x)$, the integrated e^-e^+luminosity L_{ee} and the

- The range of τ is from $4m_t^2/s$ to $\tau_m = y_m^2$.
- The differential event rate in terms of the $t\bar{t}$ invariant mass $m_{t\bar{t}} = \sqrt{\tau s}$ is

$$\frac{dN}{dm_{t\bar{t}}} = \frac{2\sqrt{\tau}}{\sqrt{s}}\frac{dN}{d\tau} \quad \text{with} \quad m_{t\bar{t}} = \sqrt{\tau s}$$

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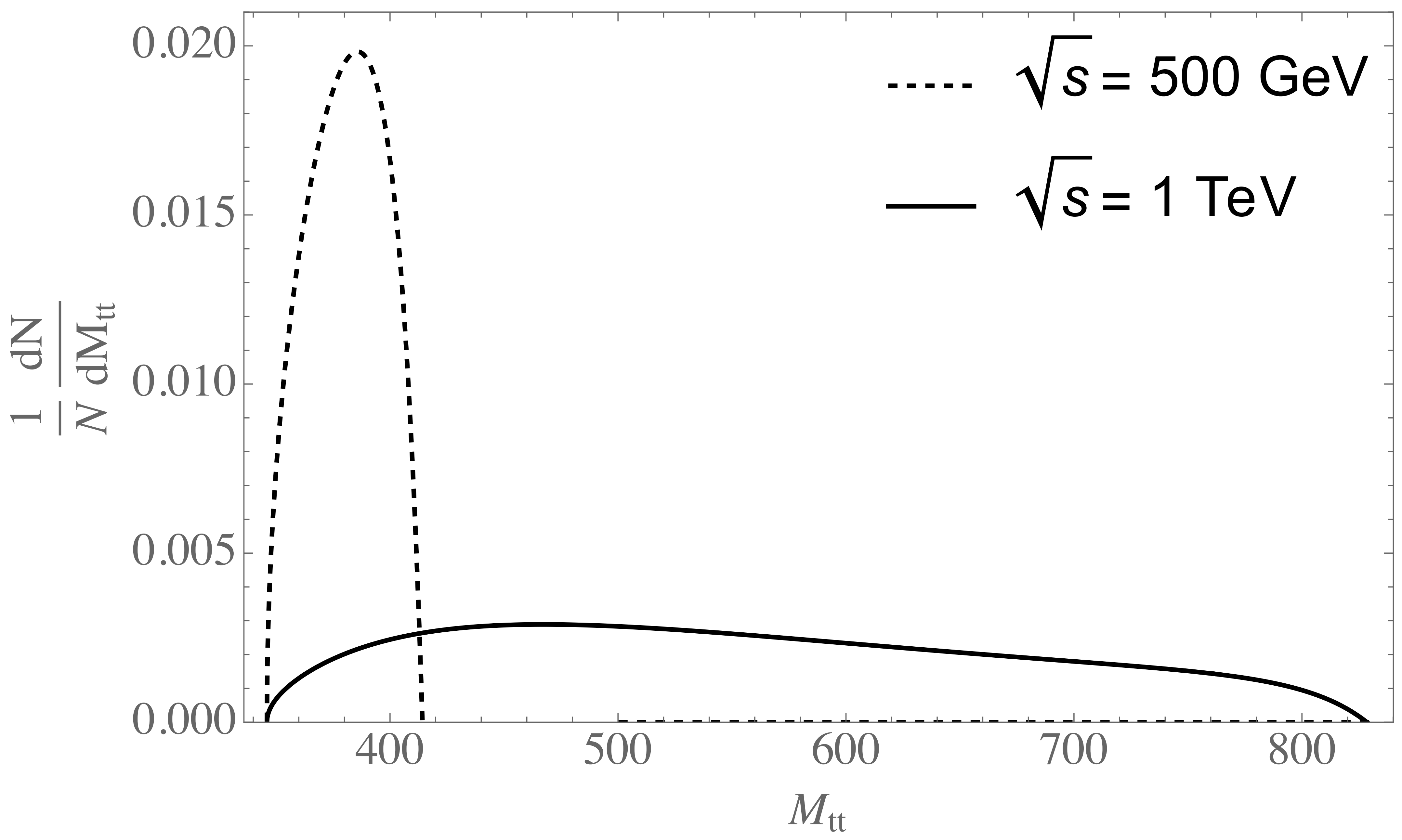
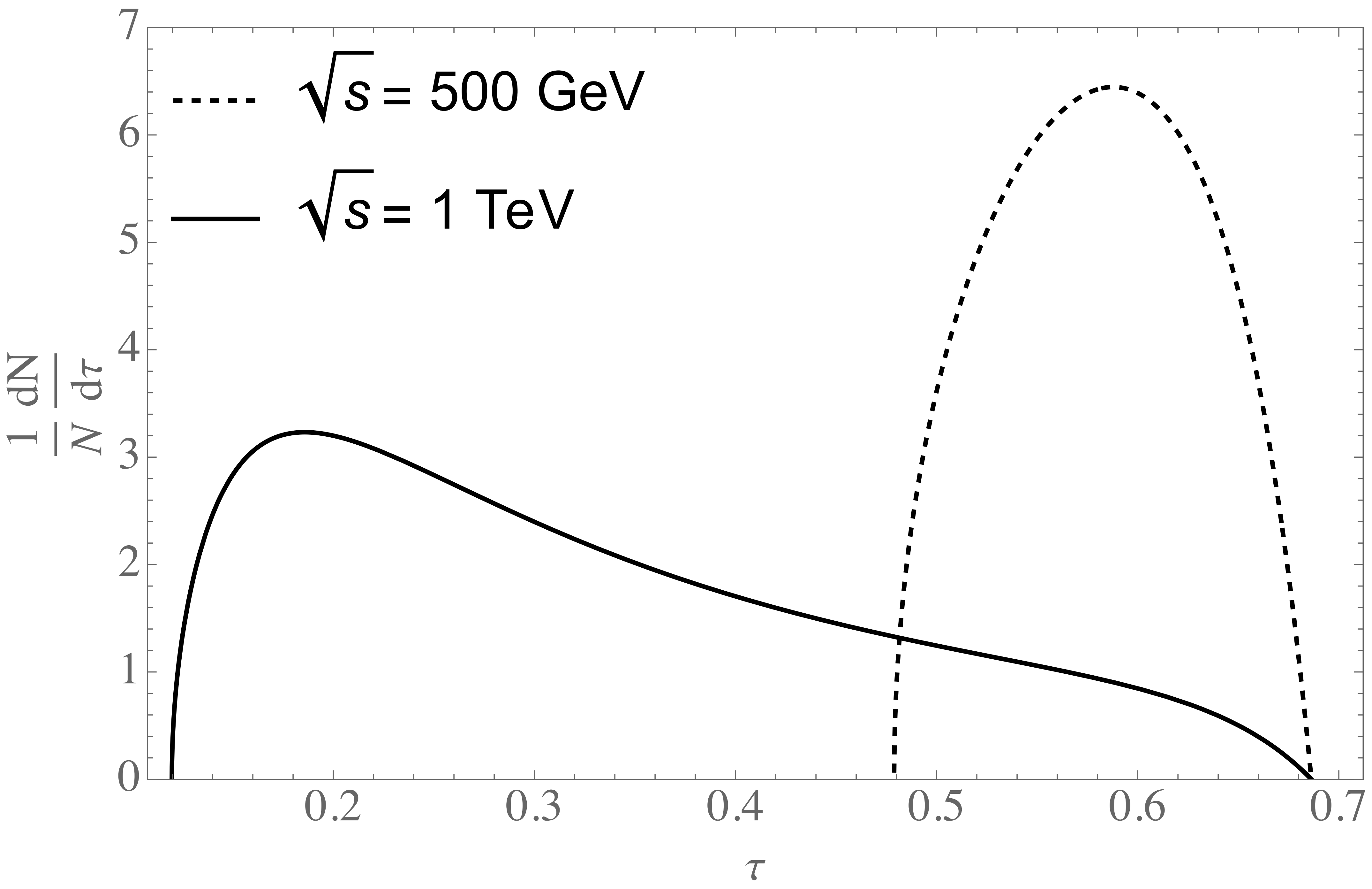
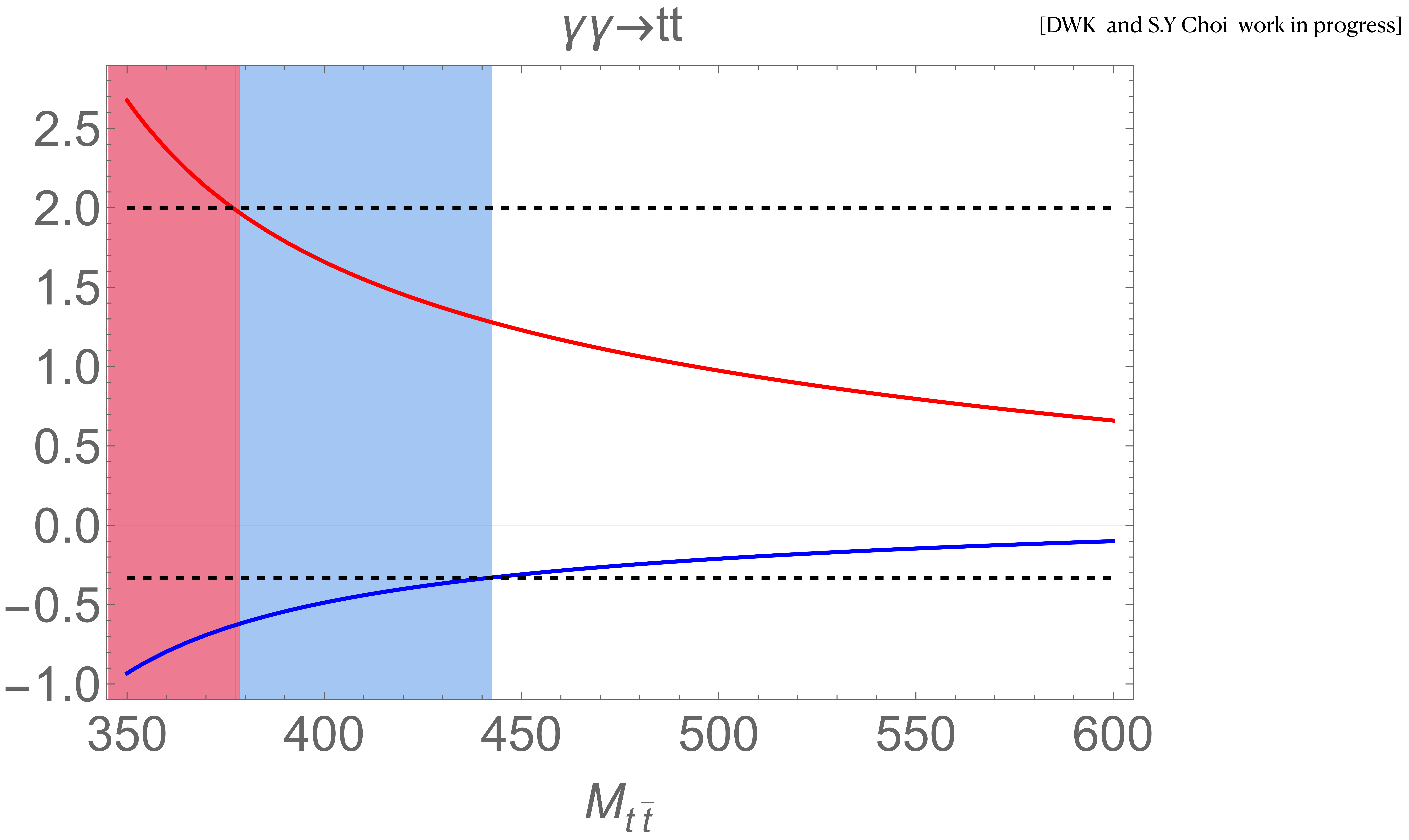
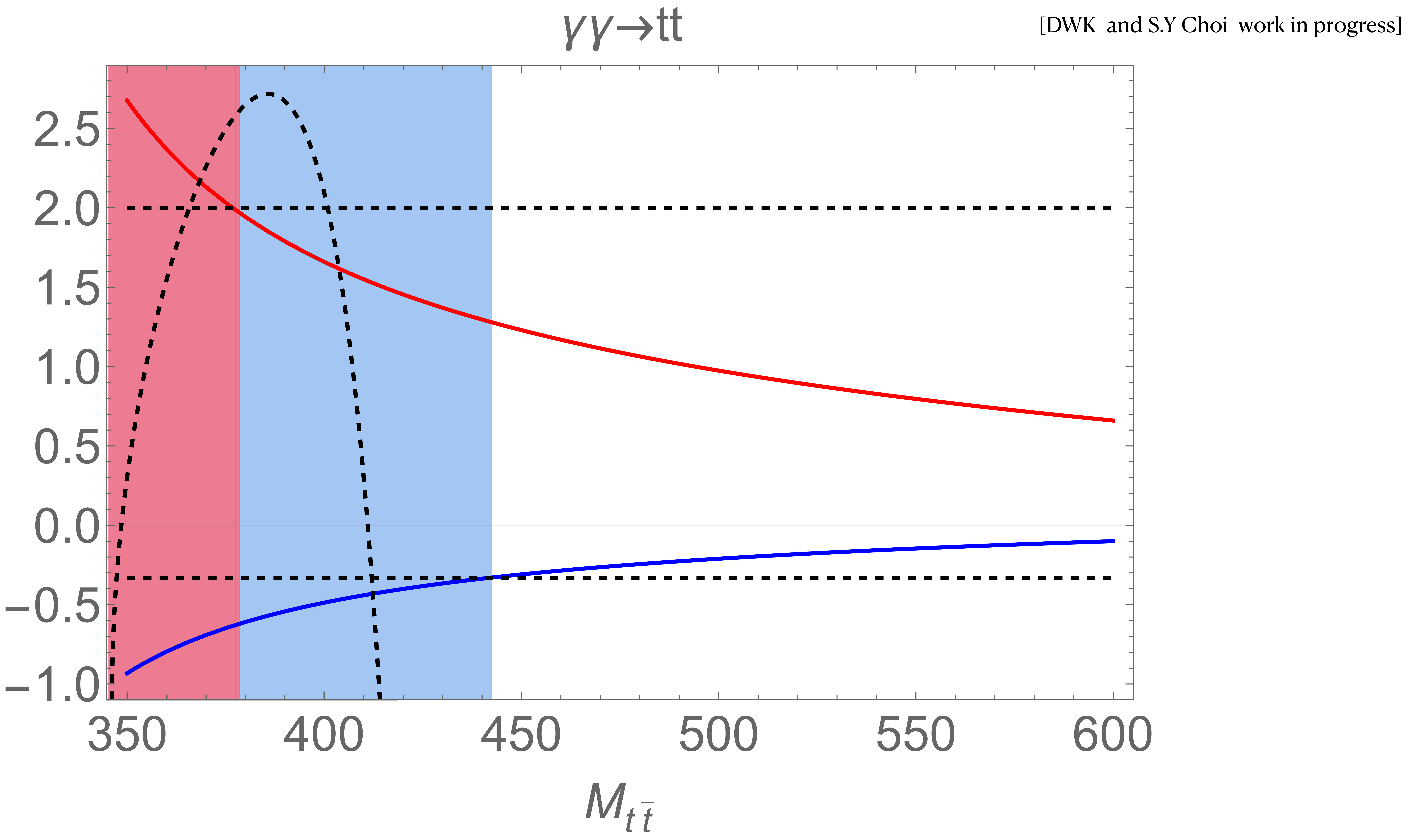





γγ→tt
[DWK and S.Y Choi work in progress]
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
350
400
450
500
550
600
$M_{t\bar{t}}$



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Summary

- Top quark pairs is an ideal testing ground for entanglement at high energy colliders (Hadron, lepton, photon), different degrees of correlations can be observed.
- $\tau^+\tau^-$, ZZ, W^+W^-, ... also very nice to study.
- Bell' inequalities violation at high energy is also could be observed soon.
- Could quantum Information observables help us discover new physics???