Modern Computational Approaches to Post-Inflationary Reheating and Dark Matter Production

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Summer Institute 2025
Utop Marina Hotel & Resort, Yeosu, Korea
21 August 2025

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Inflationary Cosmology

- Inflationary epoch (< 10^-32 s)

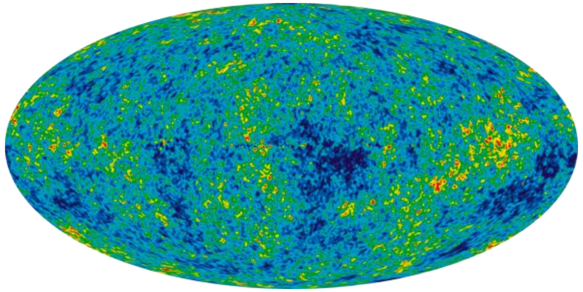
- Solution to Horizon & Flatness problem

- Inflaton?

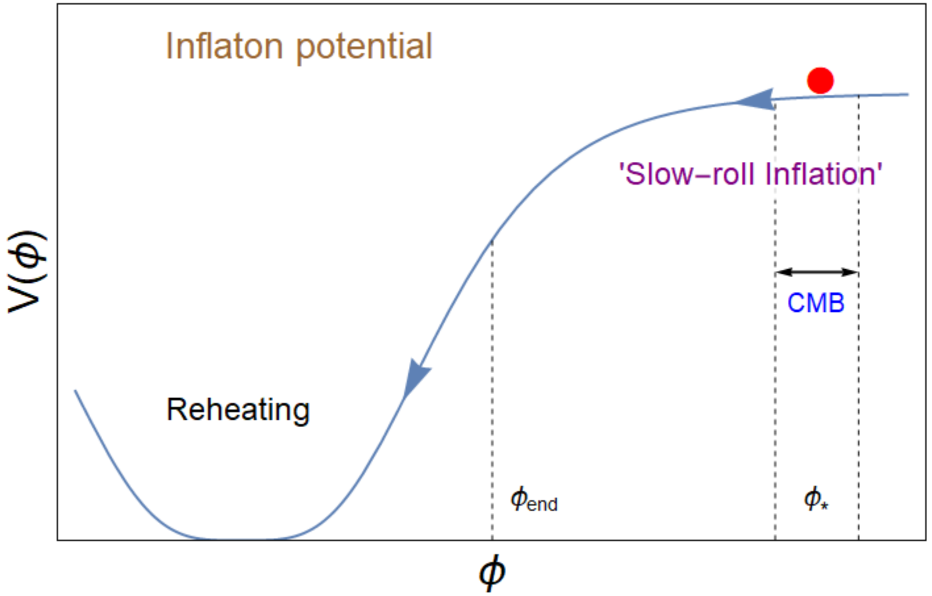
- Real scalar field
- Homogeneity & Inhomogeneity

→ Inflationary Cosmology

(1970~1980s, Alexei Starobinsky, Alan Guth, Paul Steinhardt, and Andrei Linde)



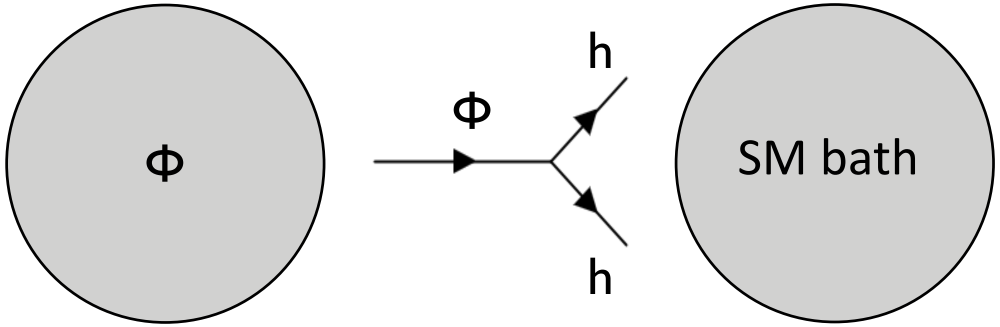
Nine-year Wilkinson Microwave Anisotropy Probe heat map of temperature fluctuations in the CMB



After Inflation

- Reheating (Inflaton → SM bath)

(little known: a few MeV < T_R < 10^13 GeV)



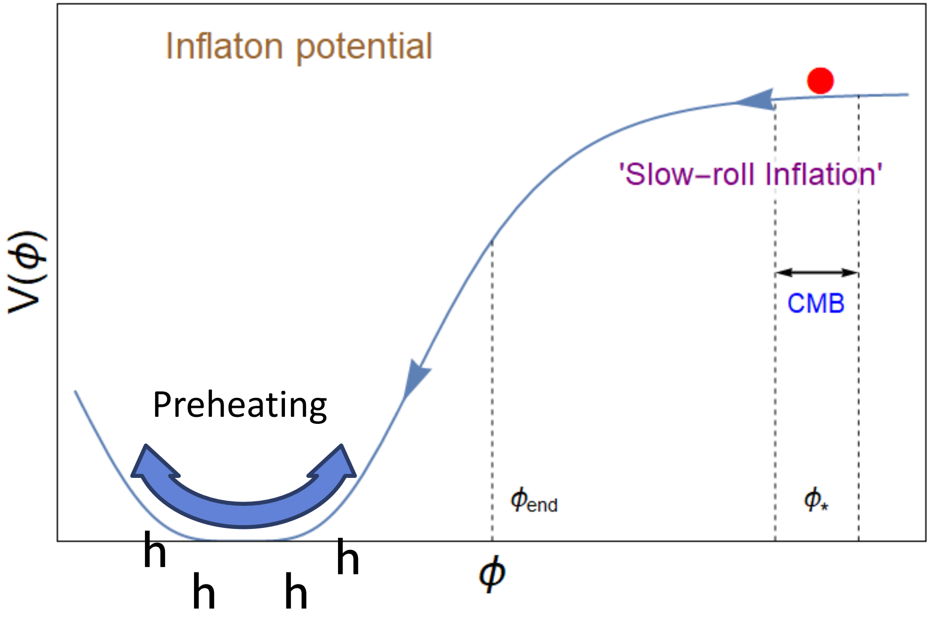
- Simplest reheating model

Inflaton quanta → Higgs

- However, the inflaton field oscillates around the minimum of the potential with large field values

→ Turbulent/non-pert. effects

→ Preheating



Dark Matter in Inflationary Universe

- While Inflaton → SM (reheating the universe) in the long run,
- DM is produced during preheating:

Inflaton=DM
Inflaton-DM scattering
Inflaton freeze-out, decay to DM
Inflaton-DM non-renormalizable couplings
Inflaton-DM via gravity

Dark Matter in Inflationary Universe

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Inflaton-DM via gravity

O. Lebedev, T. Solomko, and **J.-H. Yoon, "Dark matter production via a non-minimal coupling to gravity," JCAP, vol. 02, p. 035, 2023.**

Inflaton-DM via gravity

- Non-minimal coupling to gravity

$$\mathcal{S} = \int d^4x\sqrt{-g}\left(\frac{1}{2}M_{\rm Pl}^2 R - \frac{1}{2}\xi R\, s^2 - \frac{1}{2}g^{\mu\nu}\partial_\mu s\,\partial_\nu s - \frac{1}{2}g^{\mu\nu}\partial_\mu \phi\,\partial_\nu \phi - V\right)$$

ξ: coefficient
R: Ricci scalar
Φ: Inflaton field
s: scalar DM

- R is effectively dominated by Φ, so DM can interact with Φ via

$$R = -\frac{1}{M_{\rm Pl}^2}T^\mu_\mu$$

O. Lebedev, T. Solomko, and **J.-H. Yoon, "Dark matter production via a non-minimal coupling to gravity," JCAP, vol. 02, p. 035, 2023.**

Computing Dark Matter Abundance

- Equation of motion (x, t)

$$\Box s - \xi R s - \frac{\partial V}{\partial s} = 0 \qquad \Delta\mathcal{L}_\xi = -\frac{1}{2}\xi R\, s^2$$

- In momentum space,

$$\ddot{Y}_k + \left(k^2 + \xi R a^2 - \frac{\ddot{a}}{a}\right) Y_k = 0$$

$R = 6\ddot{a}/a^3$

$Y_k \equiv a\, s_k$ a: scale factor

$dt = a\, d\tau$ k: comoving momentum

$\equiv \omega_k^2$ determines the form of the solution

Negative freq. term leads to an exponential solution (gravitational production)

Computing Dark Matter Abundance

- Non-adiabaticity $\frac{\dot{\omega}}{\omega^2} \gg 1$ → Particle production

- Analytic Methods

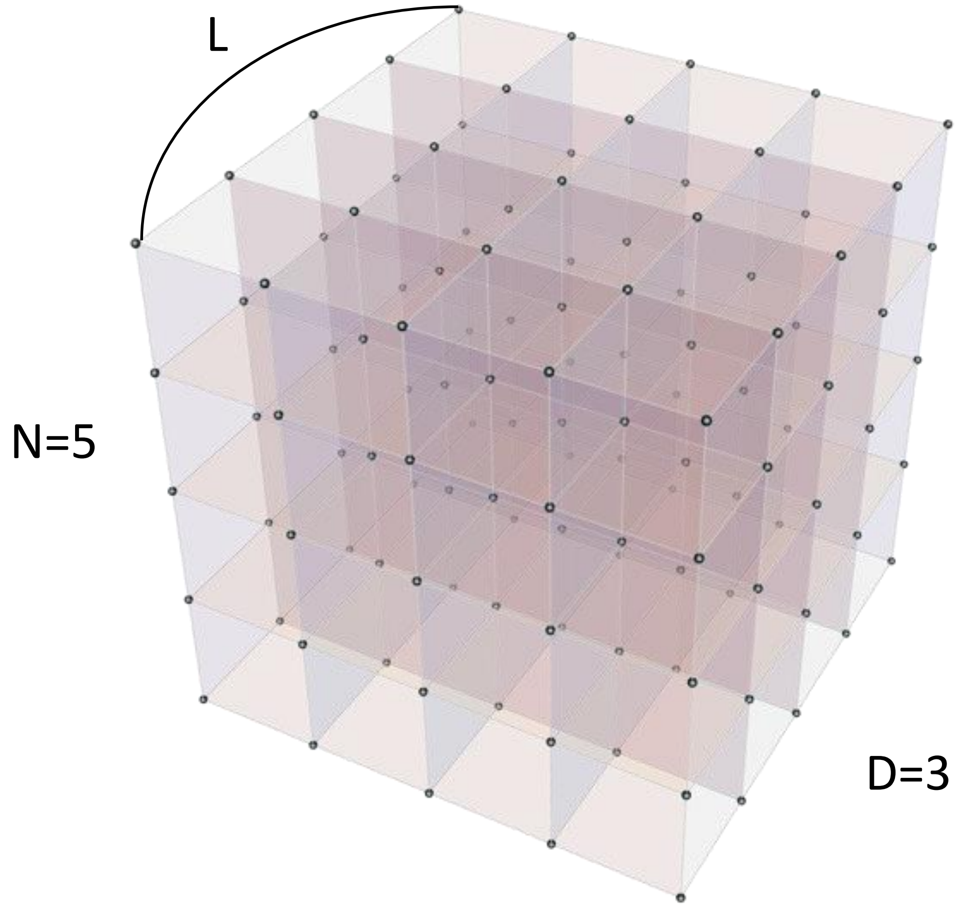
- Bogoliubov approach

- Resonance Structures (Parametric, Tachyonic, etc.)

- When production is copious (large ξ regime), backreaction and rescattering of produced particles become important, requiring a semi-classical numerical treatment

→ we solve the original EOMs defined in spacetime coordinates

Lattice Simulation



$$k_{min} = \frac{2\pi}{L} \qquad k_{max} = k_{min} \times \frac{\sqrt{D}}{2} N$$

- Equations of Motion for Particle Production

$$\ddot{f} + 3\frac{\dot{a}}{a}\dot{f} - \frac{1}{a^2}\nabla^2 f + \frac{\partial V}{\partial f} = 0$$

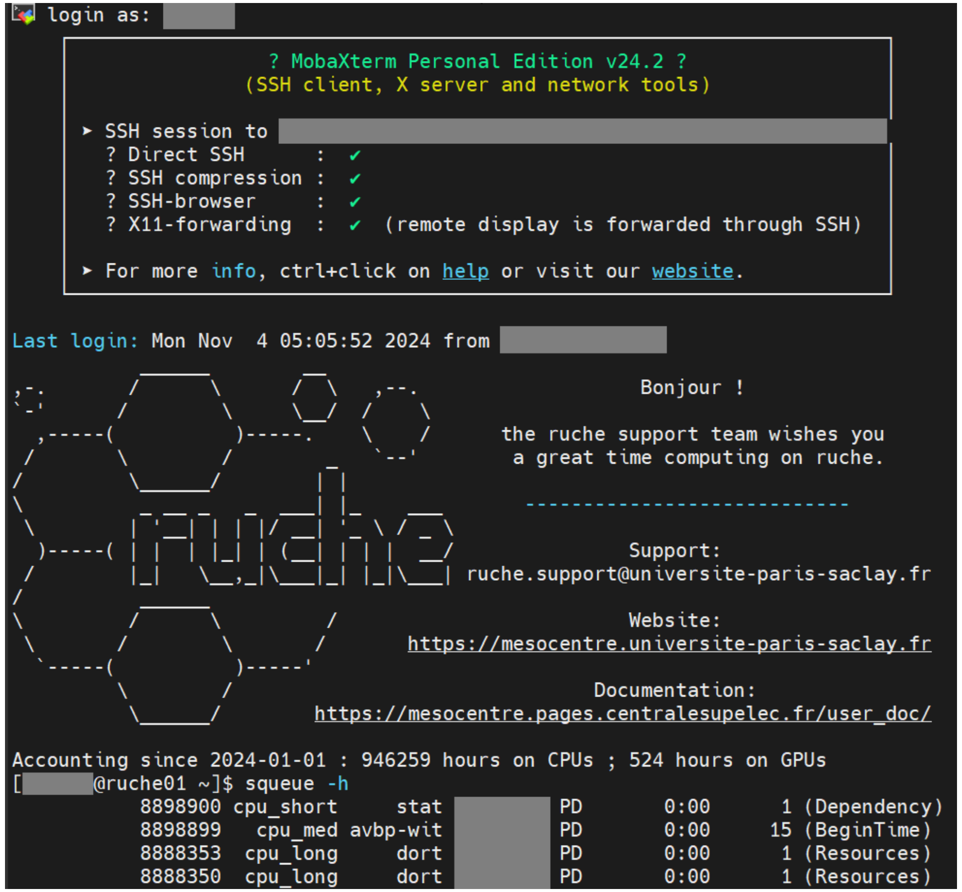
$$\ddot{a} = -\frac{4\pi a}{3}(\rho + 3p)$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}\rho$$

$$\rho = T + G + V \ ; \ p = T - \frac{1}{3}G - V$$

$$T = \frac{1}{2}\dot{f}^2 \ ; \ G = \frac{1}{2a^2}|\nabla f|^2 \ .$$

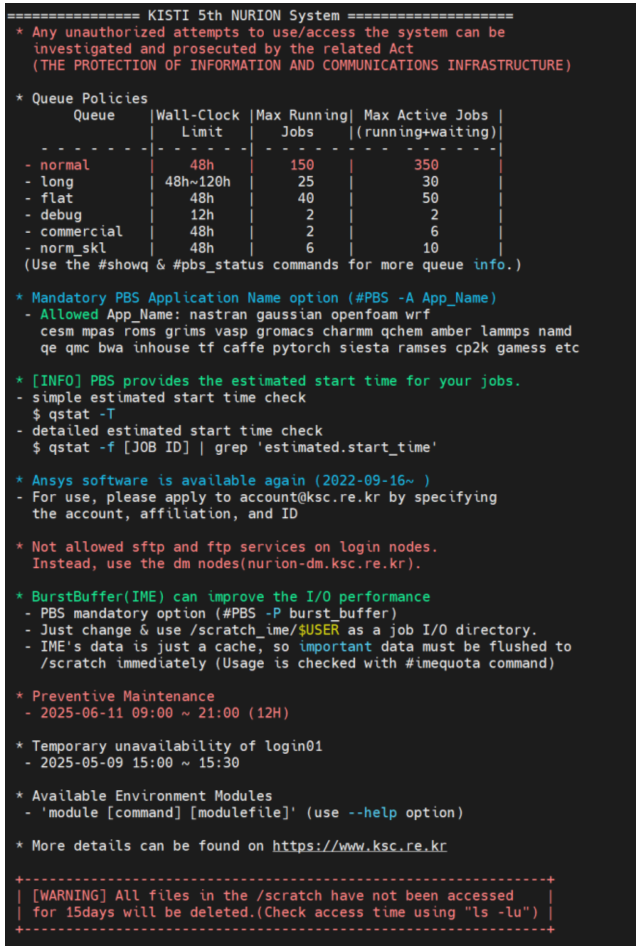
High-Performance Computing



Hardware (ruche cluster at Paris-Saclay University)

Software (CosmoLattice)

High-Performance Computing



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=============== KISTI 5th NURION System ====================
* Any unauthorized attempts to use/access the system can be
  investigated and prosecuted by the related Act
  (THE PROTECTION OF INFORMATION AND COMMUNICATIONS INFRASTRUCTURE)

* Queue Policies
      Queue     |Wall-Clock |Max Running| Max Active Jobs |
                |   Limit   |   Jobs    |(running+waiting)|
   - - - - - - -|- - - - - -|- - - - - - - - - - - - - - -|
  - normal      |    48h    |    150    |      350        |
  - long        | 48h~120h  |     25    |       30        |
  - flat        |    48h    |     40    |       50        |
  - debug       |    12h    |      2    |        2        |
  - commercial  |    48h    |      2    |        6        |
  - norm_skl    |    48h    |      6    |       10        |
  (Use the #showq & #pbs_status commands for more queue info.)

* Mandatory PBS Application Name option (#PBS -A App_Name)
 - Allowed App_Name: nastran gaussian openfoam wrf
   cesm mpas roms grims vasp gromacs charmm qchem amber lammps namd
   qe qmc bwa inhouse tf caffe pytorch siesta ramses cp2k gamess etc

* [INFO] PBS provides the estimated start time for your jobs.
- simple estimated start time check
  $ qstat -T
- detailed estimated start time check
  $ qstat -f [JOB ID] | grep 'estimated.start_time'

* Ansys software is available again (2022-09-16~ )
- For use, please apply to account@ksc.re.kr by specifying
  the account, affiliation, and ID

* Not allowed sftp and ftp services on login nodes.
  Instead, use the dm nodes(nurion-dm.ksc.re.kr).

* BurstBuffer(IME) can improve the I/O performance
 - PBS mandatory option (#PBS -P burst_buffer)
 - Just change & use /scratch_ime/$USER as a job I/O directory.
 - IME's data is just a cache, so important data must be flushed to
   /scratch immediately (Usage is checked with #imequota command)

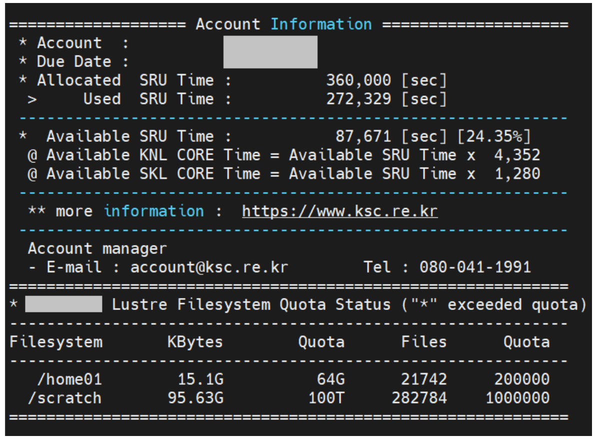
* Preventive Maintenance
 - 2025-06-11 09:00 ~ 21:00 (12H)

* Temporary unavailability of login01
 - 2025-05-09 15:00 ~ 15:30

* Available Environment Modules
 - 'module [command] [modulefile]' (use --help option)

* More details can be found on https://www.ksc.re.kr

+----------------------------------------------------------------+
| [WARNING] All files in the /scratch have not been accessed     |
| for 15days will be deleted.(Check access time using "ls -lu")  |
+----------------------------------------------------------------+
```



```
================== Account Information ===================
 * Account  :  [redacted]
 * Due Date :
 * Allocated  SRU Time :          360,000 [sec]
  >     Used  SRU Time :          272,329 [sec]
------------------------------------------------------------
 *  Available SRU Time :           87,671 [sec] [24.35%]
  @ Available KNL CORE Time = Available SRU Time x  4,352
  @ Available SKL CORE Time = Available SRU Time x  1,280
------------------------------------------------------------
  ** more information :  https://www.ksc.re.kr
------------------------------------------------------------
  Account manager
  - E-mail : account@ksc.re.kr        Tel : 080-041-1991
============================================================
* [redacted] Lustre Filesystem Quota Status ("*" exceeded quota)
------------------------------------------------------------
Filesystem       KBytes        Quota       Files       Quota
------------------------------------------------------------
   /home01        15.1G          64G       21742      200000
  /scratch       95.63G         100T      282784     1000000
============================================================
```

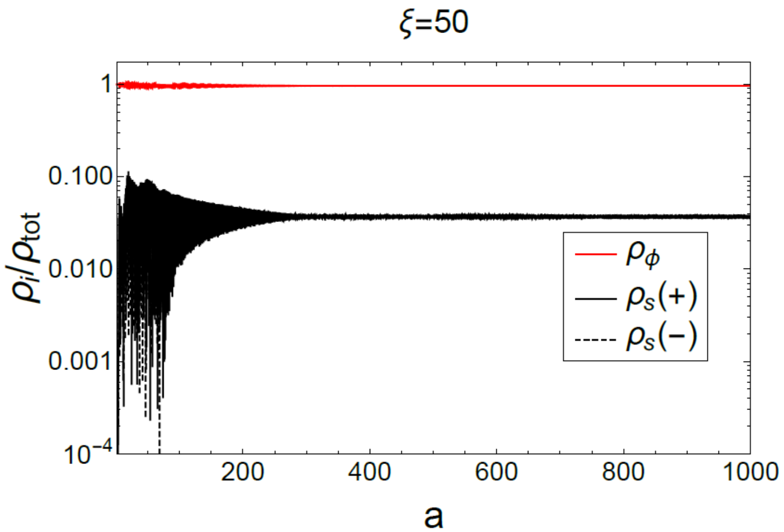
https://www.ksc.re.kr/

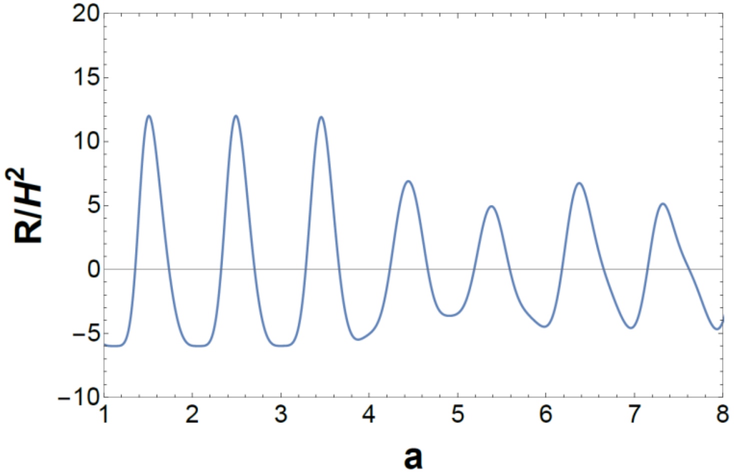
Nurion @ KISTI (Korea Institute of Science and Technology Information)

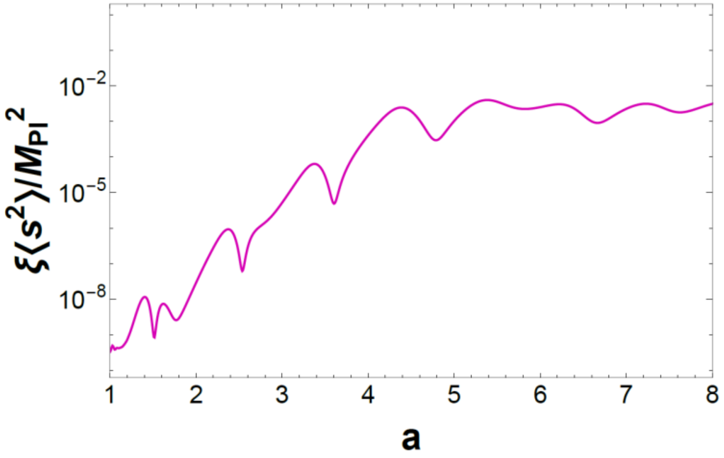
Simulation Outcome

- CosmoLattice customized for NMC

$$\rho(s) = \langle T_{00}(s) \rangle = \frac{1}{2}\langle \dot{s}^2 \rangle + \frac{1}{2a^2}\langle (\nabla s)^2 \rangle + \langle V(s) \rangle + 3\xi H^2 \langle s^2 \rangle + 6\xi H \langle s\dot{s} \rangle$$







- Energy distribution, R breakdown, resonant production, etc.

- Simulations provide intuitive insights into events in the early universe

Dark Matter Relic Abundance

- DM relic abundance (conserved since reheating)

$$Y = \frac{n}{s_{\rm SM}} \quad , \quad s_{\rm SM} = \frac{2\pi^2}{45} g_{*s} T^3$$

$$Y_\infty = 4.4 \times 10^{-10} \left(\frac{\rm GeV}{m_s} \right)$$

- Reheating via inflaton decay into Higgs

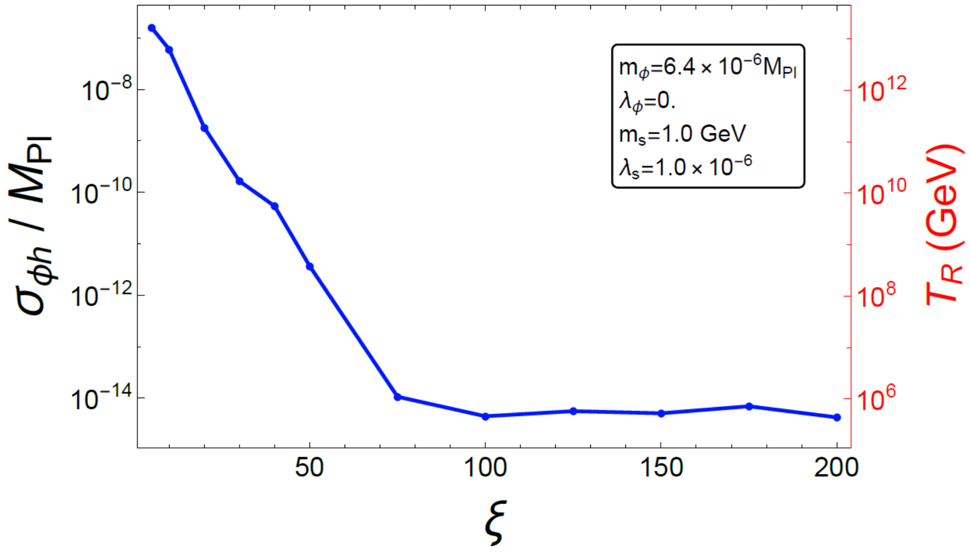
$$H_R \simeq \Gamma_{\phi\to hh} \ , \quad \Gamma_{\phi\to hh} = \frac{\sigma_{\phi h}^2}{8\pi m_\phi} \qquad H_R = \sqrt{\frac{\pi^2 g_*}{90}} \frac{T_R^2}{M_{\rm Pl}}$$

T_R: Reheating temperature

$$a_e \xrightarrow{\rm rel} a_* \xrightarrow{\rm nrel} a_R$$

Nonrelativistic expansion phase due to massive inflaton

Final Results



$$\sigma_{\phi h} \simeq 5 \times 10^{-9} \frac{m_\phi^{3/2}}{M_{\rm Pl}^{1/2}} \frac{n_e(\phi)}{n_e(s)} \left(\frac{\rm GeV}{m_s}\right)$$

- DM production ∝ ξ

→ More DM requires longer dilution

→ Lower T_R (Reheating temperature)

- Early DM production explains observed relic abundance

O. Lebedev, T. Solomko, and **J.-H. Yoon, "Dark matter production via a non-minimal coupling to gravity," JCAP, vol. 02, p. 035, 2023.**

Applications of Simulation Results

- Intense Particle Production → GWB production
- BSM in the early universe: Phase Transition, Axion Inflation, Higgs-like inflation, etc.
- Thermalization

Simulating Thermal Universe

- Simulating the hot universe is very challenging

- Timescale: Preheating << Reheating

- Limited momentum window $k_{min} = \frac{2\pi}{L}$ $k_{max} = k_{min} \times \frac{\sqrt{D}}{2} N$

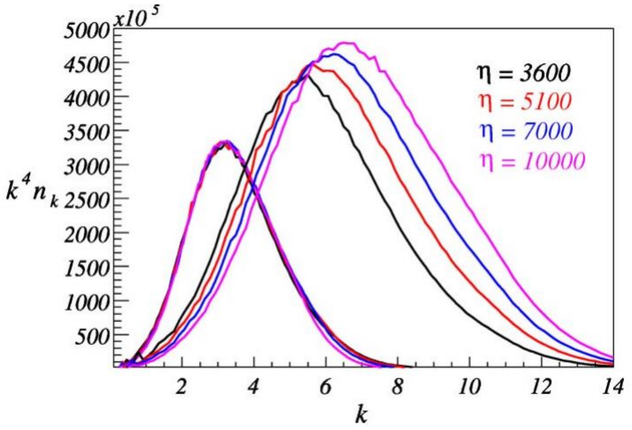
- Classical approximation: valid only when f >> 1

Turbulent Thermalization

- Late-time preheating dynamics exhibits a universal form:

Self-similar evolution of self- or gauge interacting field

→ Implies patterns and trends



Distribution of **Φ field in Φ^4 model**

R. Micha and I. Tkachev, “Turbulent thermalization,” arXiv:hep-ph/0403101

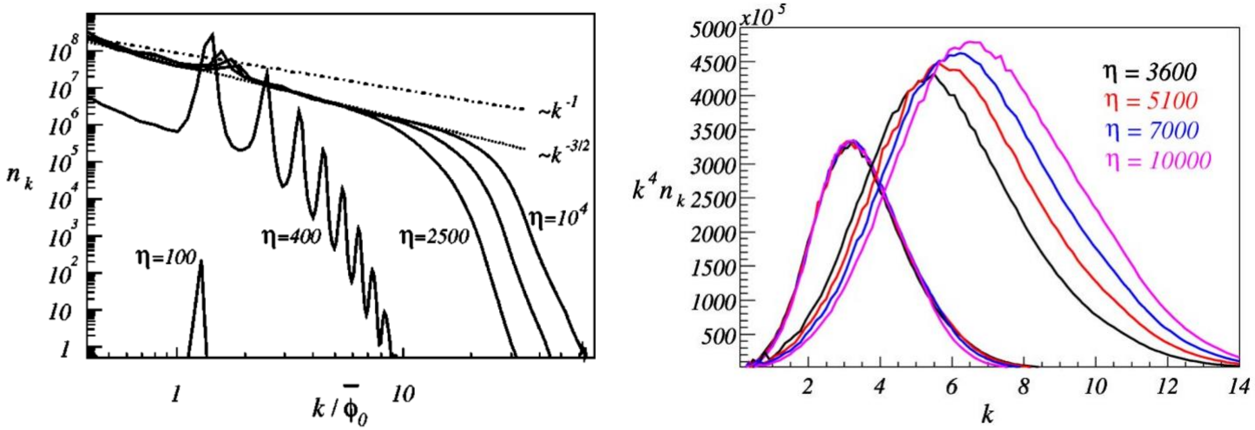
Turbulent Thermalization

- 3-Stage Process

Parametric Resonance

→ Driven Turbulence

→ Free Turbulence



Distribution of Φ **field in Φ^4 model**

- Self-similar evolution from Wave Kinetic Theory

k: comoving momentum
nk: occupation number
dη: dt/a

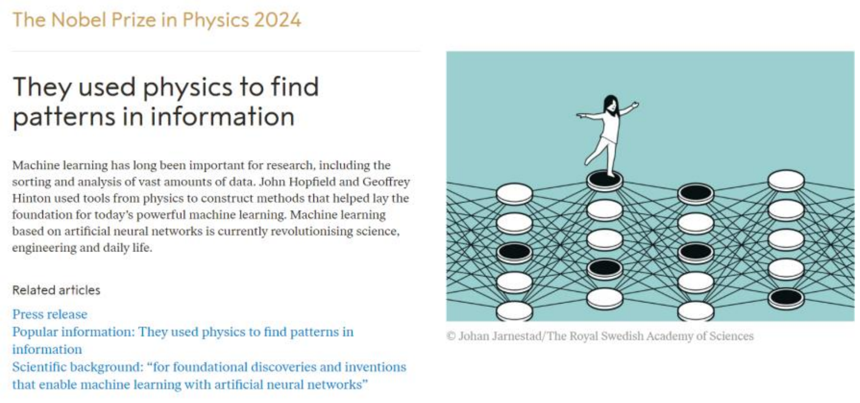
$$n(k,\tau) = \tau^{-q} n_0(k\tau^{-p}) \qquad \tau \equiv \eta/\eta_c$$

m-particle interactions determine scaling: e.g. p = 1/(2m-1)

Observed: p ≈ 1/5 → 3-particle dominance

R. Micha and I. Tkachev, “Turbulent thermalization,” arXiv:hep-ph/0403101

Integrating Deep Learning



The Nobel Prize in Physics 2024

They used physics to find patterns in information

Machine learning has long been important for research, including the sorting and analysis of vast amounts of data. John Hopfield and Geoffrey Hinton used tools from physics to construct methods that helped lay the foundation for today's powerful machine learning. Machine learning based on artificial neural networks is currently revolutionising science, engineering and daily life.

Related articles

Press release

Popular information: They used physics to find patterns in information

Scientific background: "for foundational discoveries and inventions that enable machine learning with artificial neural networks"

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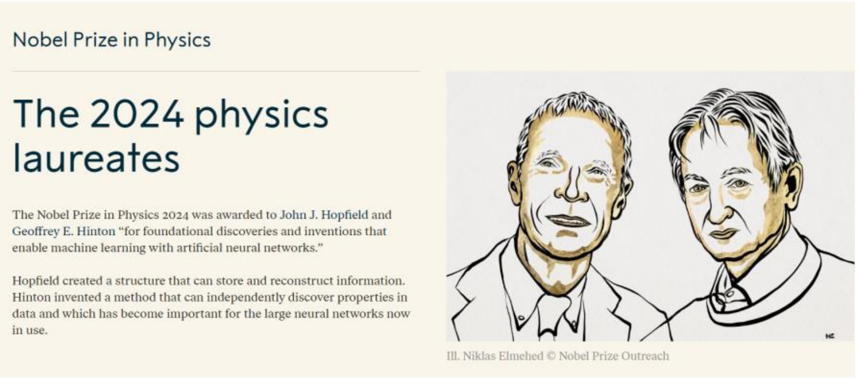


Nobel Prize in Physics

The 2024 physics laureates

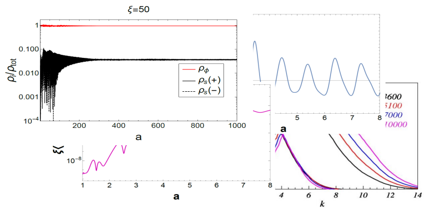
The Nobel Prize in Physics 2024 was awarded to John J. Hopfield and Geoffrey E. Hinton "for foundational discoveries and inventions that enable machine learning with artificial neural networks."

Hopfield created a structure that can store and reconstruct information. Hinton invented a method that can independently discover properties in data and which has become important for the large neural networks now in use.

Ill. Niklas Elmehed © Nobel Prize Outreach

https://www.nobelprize.org/prizes/physics/

- Simulations generate data that can be analyzed by Deep Learning



→ ?

J.-H. Yoon, S.Clery, M.Gross, Y.Mambrini, **"Preheating with deep learning,"** *JCAP*, vol. 08, p. 031, 2024. [arXiv:hep-ph/2405.08901]

- LatticeQCD, CMB, LHC, DM Exp., etc. wherever we have data

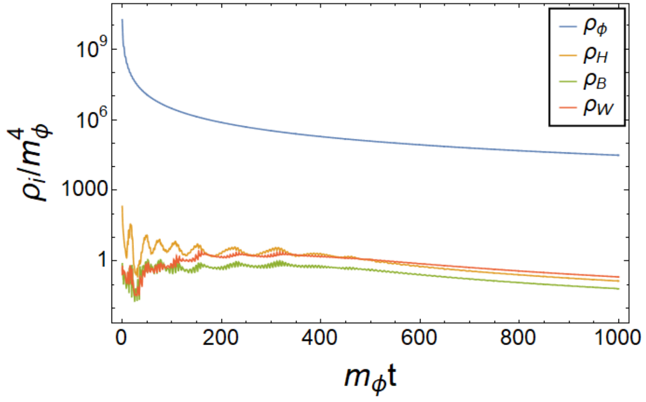
Generating Data for training

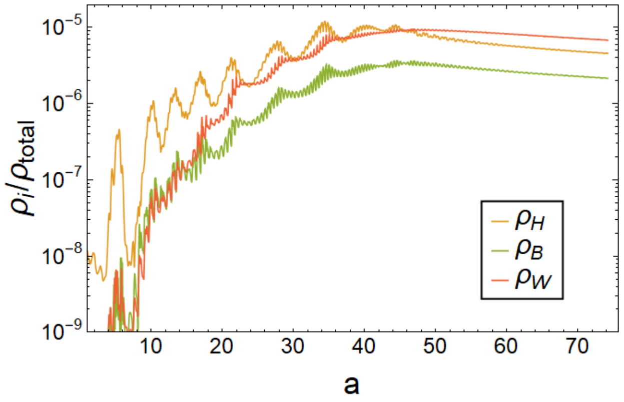
- Preheating model involving Higgs

→ Minimal reheating scenario + self- and gauge interaction

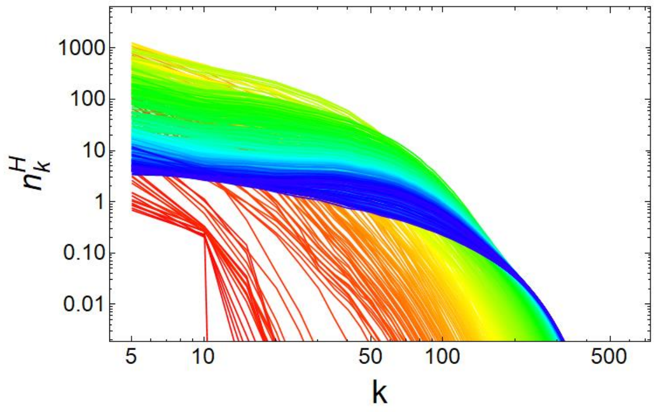
$$\Delta V = \frac{1}{2} m_\phi^2 \, \phi^2 + \frac{1}{4}\lambda_\phi \, \phi^4 + \frac{1}{2}\lambda_{\phi h} \, \phi^2 \, H^\dagger H + \sigma_{\phi h} \, \phi \, H^\dagger H - m_h^2 \, H^\dagger H + \lambda_h \, (H^\dagger H)^2$$

- Input/output=Higgs occupation number, n(k,t)





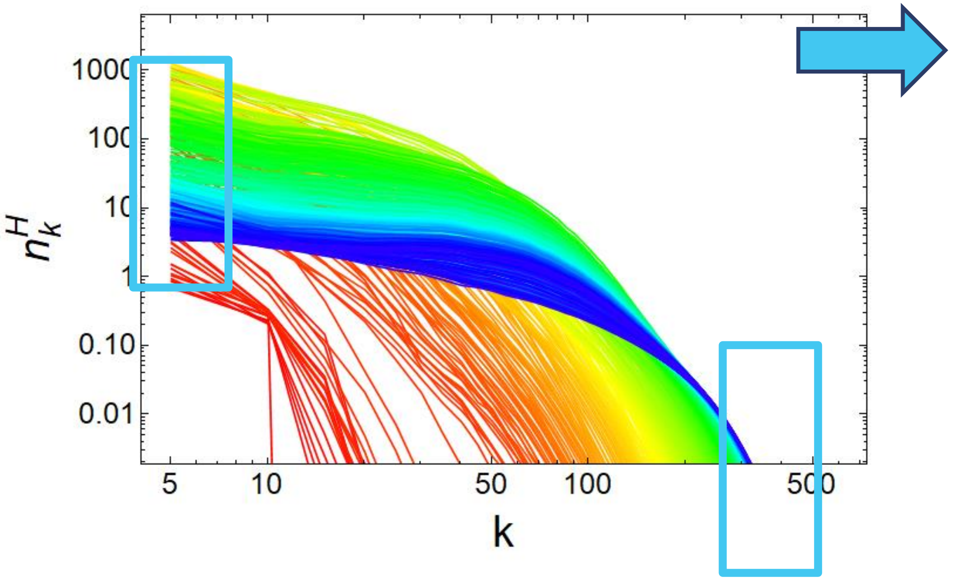
Energy distributions over time/scale factor



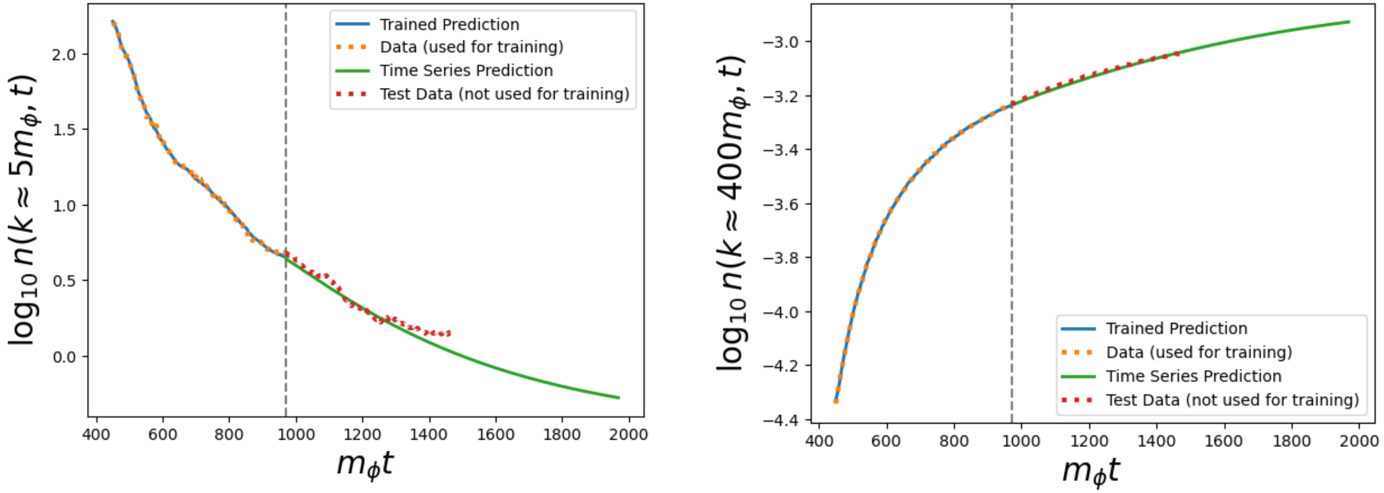
Occupation number of Higgs ~ distribution function (red to blue over time)

Train, Validation & Predictions

- Extending Simulation Results



15 k-modes used for training the DL model
(only two of them are represented here)

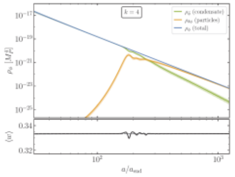
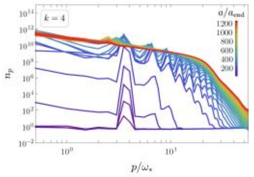
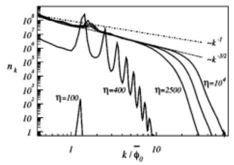
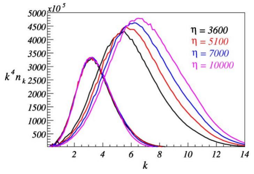


CNN-LSTM time series analysis

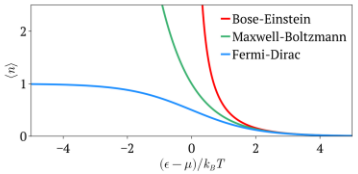
Managed to extend simulation outcomes with DL
→ Once trained, the DL model's predictions are almost instantaneous
→ Successfully laid the groundwork for future developments

Towards Complete Thermal Universe Simulation

- Lattice simulations (f>>1) → ML modeling → Boltzmann eq. (f~1)

$$\frac{df(k,t)}{dt} = \sum \mathcal{C}_{\text{coll}}[f]$$



- Connecting inflation to thermal equilibrium

- End-to-end simulation of reheating

Boltzmann Equation Solver for Thermalization

$$\frac{df(k,t)}{dt} = \sum \mathcal{C}_{\mathrm{coll}}[f]$$

$$C[f](p_1) = \int d^3p_2\, d^3p_3\, d^3p_4\, (2\pi)^4 \delta^4(p_1+p_2-p_3-p_4)\, |M|^2$$
$$\times\ [f_3 f_4(1 \pm f_1)(1 \pm f_2) - f_1 f_2(1 \pm f_3)(1 \pm f_4)]$$

- Vegas adaptive Monte Carlo integration for multidimensional collision integrals
- Flexible deployment: automatic MPI scaling from PC to cluster
- Any process, any species: bosons, fermions, N→M collisions

G. P. Lepage, J. Comput. Phys. 27(1978) 192

For each momentum magnitude |p|:

→ Compute C[f](|p|) via (9-12)D Vegas integration

→ Update f(|p|,t) using explicit time stepping

→ Periodically refine |p|-grid & Vegas maps

※ Currently in development - public release coming soon

Conclusions

- We learned about (p)reheating and minimal cosmological models (e.g. DM via gravity)

→ Preheating effects are often unavoidable and require numerical approaches

- Simulating the early universe with HPC is interesting and useful

→ Intuitive insights, GW search, DM production, BSM physics, Reheating, etc.

- Deep learning can be applied to late-time self-similar systems in the early universe

→ It represents a step toward simulating the entire history of thermal universe