Cosmological Phase Transition and Gravitational Wave (lecture 2)

Summer institute 2025 (at Yeosu)

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Summary: 1'st lecture

- Order of phase transition can be determined by computing the Landau free energy.

- A thermal field theory is a field theory on $\mathbf{R}^3 \times \mathbf{S}^1$.

- At high-temperature, $\mathbf{S}^1$ direction can be integrated out and obtain three-dimensional effective theory.

- A non-perturbative fluctuation should be taken into account to determine the order of a thermal phase transition.

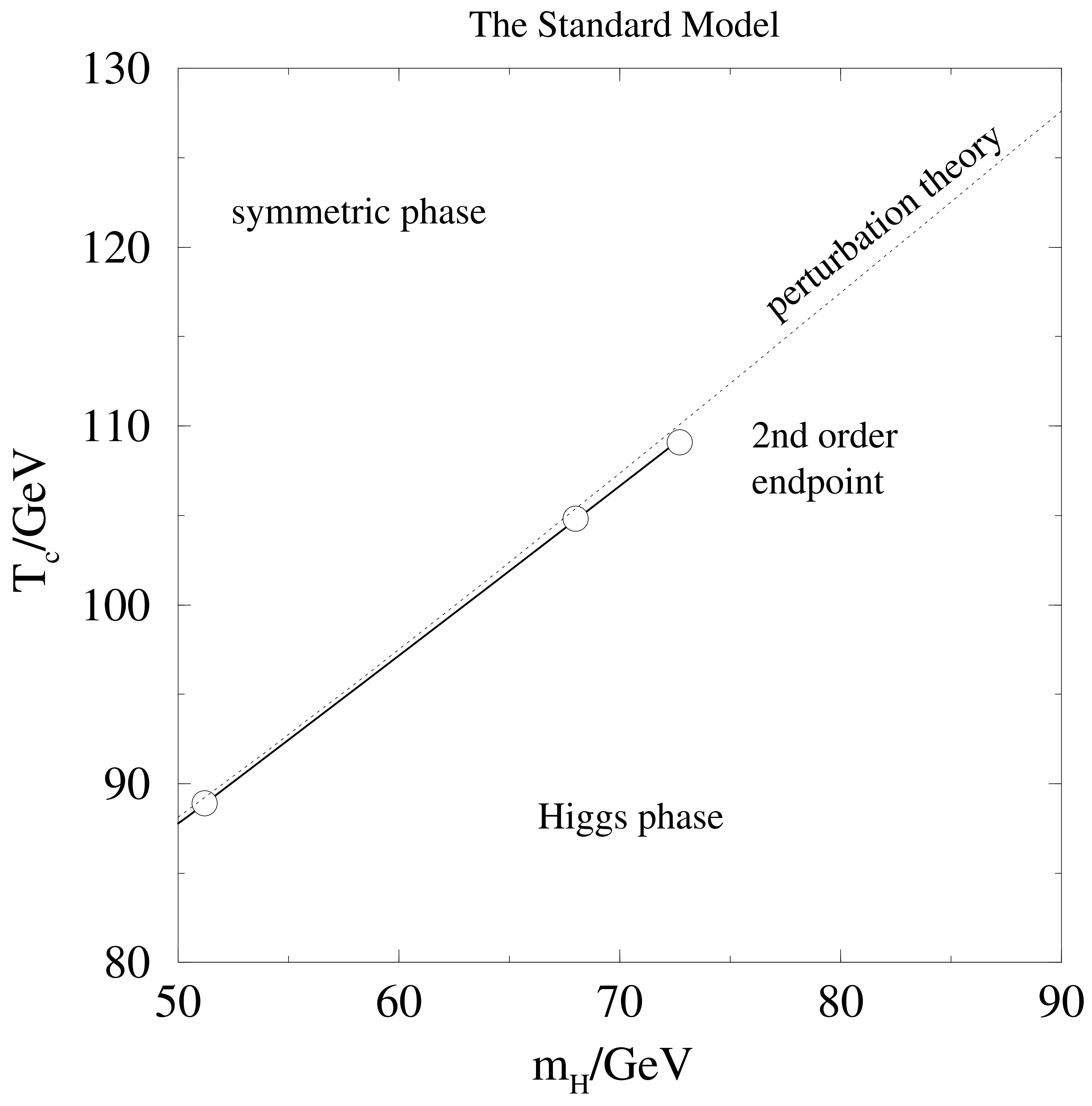
Outline

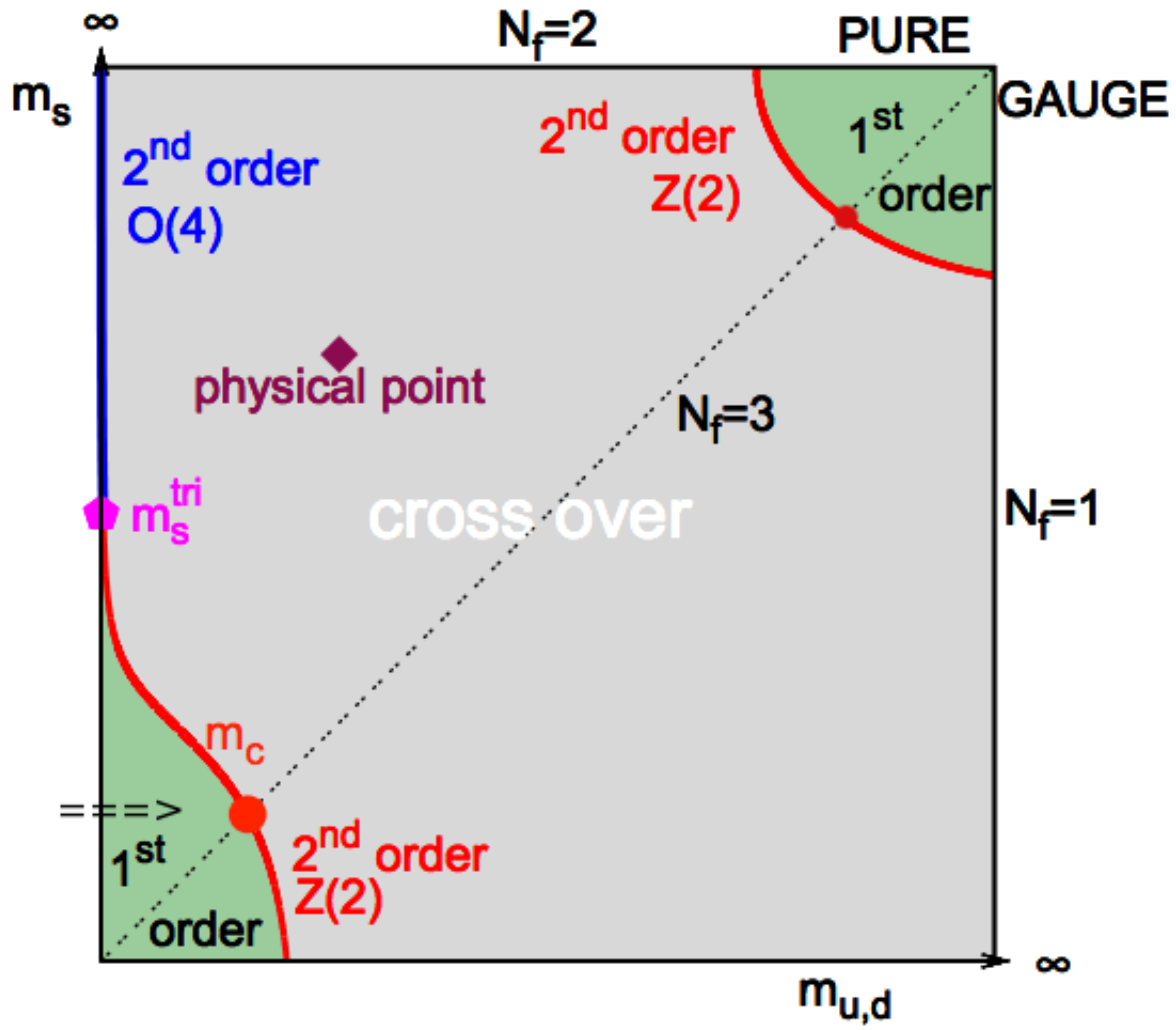
- Review of the hot electroweak and QCD phase diagrams

- Review of gravitational waves from a first-order phase transition

- Particle physics models with ultra-supercooling

I try to explain following phase diagrams.





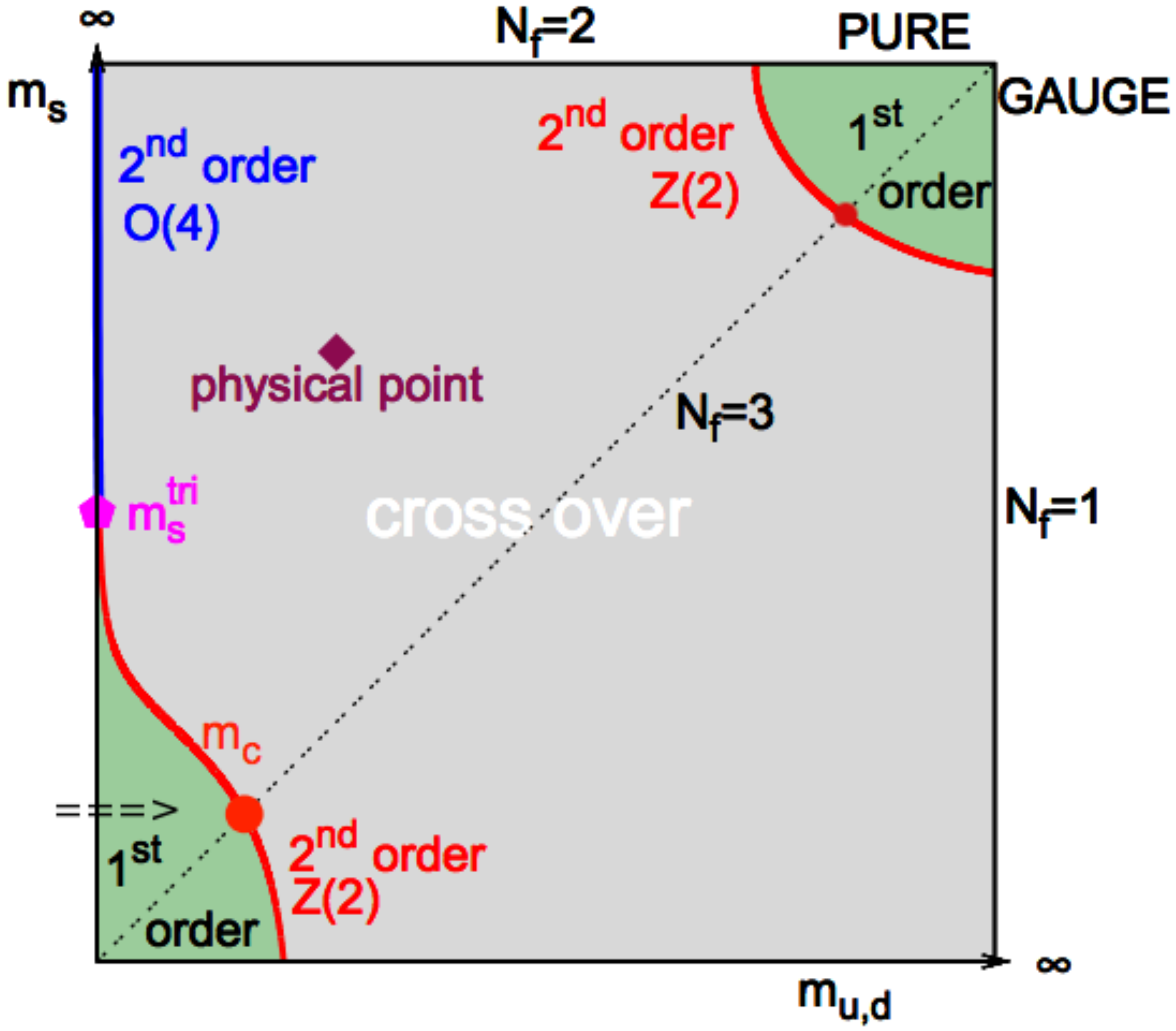
[Forcrand and D'elia (2017)]

[Kajantie, Laine, Rummukainen, Shaposhnikov (1996)]

[Kajantie, Tsyin, Laine, Rummukainen, Shaposhnikov (1998)]

Hot QCD phase diagram $N_c = 3, N_f = 2 + 1$

Colombia Plot *(Expected phase diagram)*



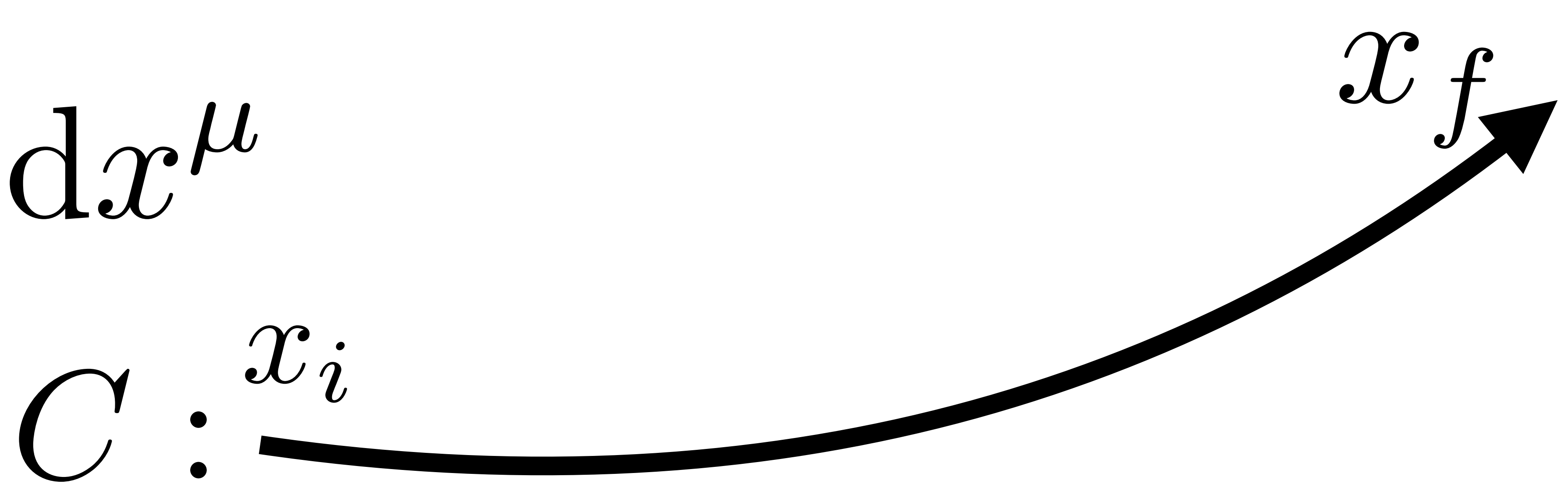
[Forcrand and D'elia (2017)]

A zero baryon chemical potential is assumed.

Thermal Wilson loop

- A Wilson line of an $SU(N_c)$ gauge theory

$$U(x_i, x_f) \sim P \exp\left(\int_C A\right) \qquad A = A_\mu^a T^a \mathrm{d}x^\mu$$

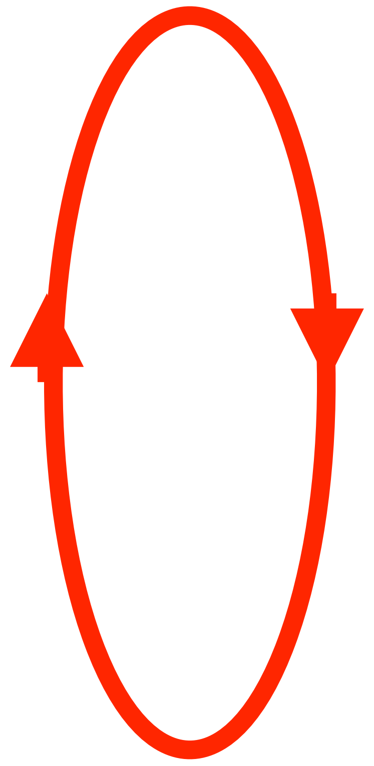


A_μ: gauge field $\qquad T^a$: generator of $SU(N_c)$

Gauge transformation: $W(x_i, x_f) \to U(x_i) W(x_i, x_f) U^\dagger(x_f),\ U(x) \in SU(N_C)$

- At finite-temperature, Polyakov loop (Wilson line wrapping on the $\mathbf{S}^1$) is a gauge-invariant operator.

$$l_P \sim \mathrm{Tr}\, P \exp\left(\int_{S^1} A_4\right)$$



Confinement deconfinement phase?

- For a pure $SU(N_c)$ gauge theory, there is a thermal confinement-deconfinement phase transition. ($N_c > 2$: 1'st order)

Q :probe quark

$$l_P \sim \mathrm{Tr}\, P \exp\left(\int_{S^1} A_0\right)$$

$\langle l_P \rangle \sim e^{-F_q/T}$, F_q: free energy of the probe quark

$\langle l_P \rangle = 0$: confinement phase

$\langle l_P \rangle \neq 0$: deconfinement phase

Confinement deconfinement phase?

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$\langle l_P \rangle = 0$: confinement phase

$\langle l_P \rangle \neq 0$: deconfinement phase

- A (fundamental) matter field makes the thermal confinement-deconfinement phase transition to be smooth crossover.

Q :probe quark

$\bar{q}$:dynamical quark

$\langle l_P \rangle \neq 0$: deconfinement phase

(A bound state formation by dynamical quark)

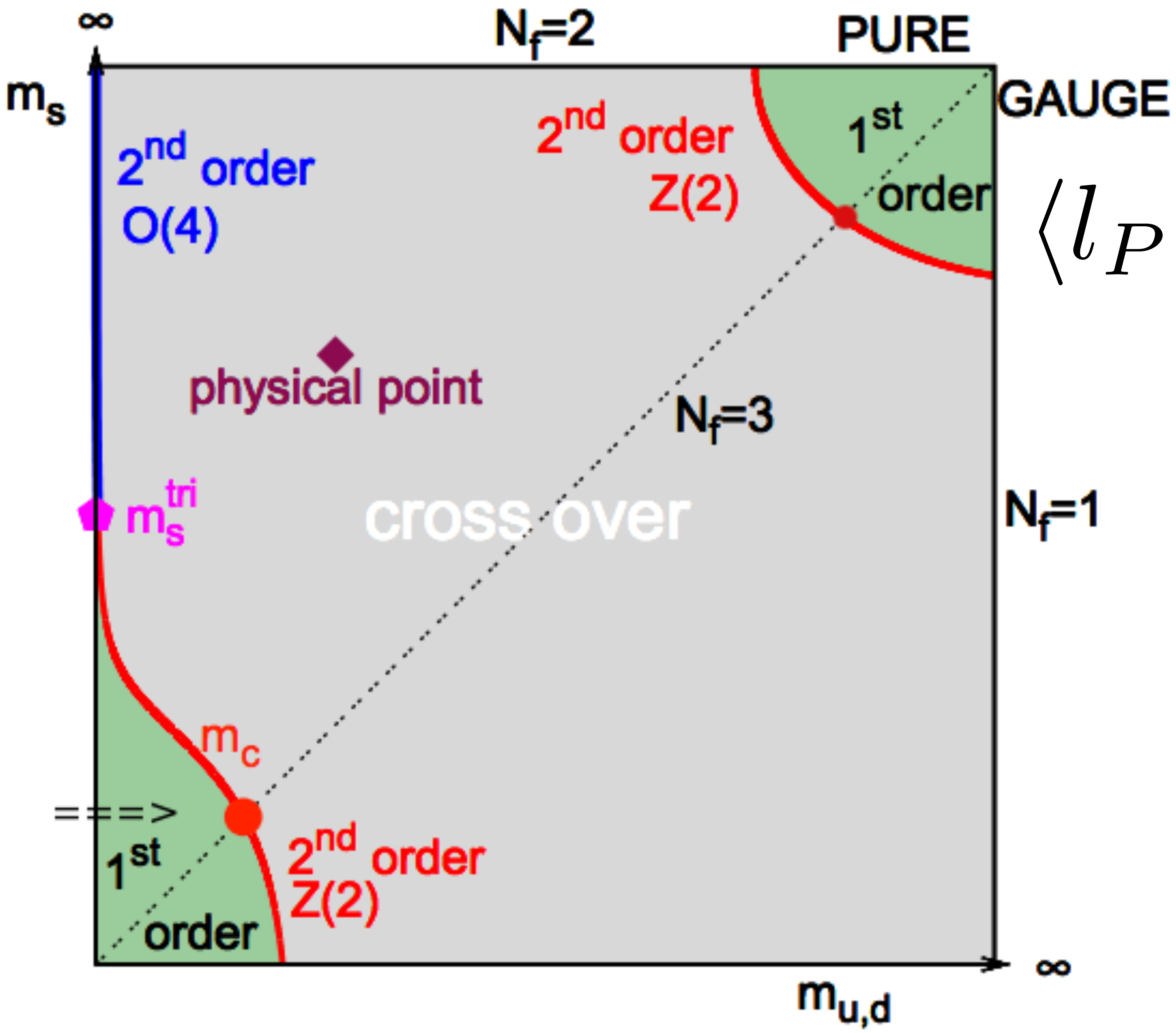
Hot QCD phase diagram

Colombia Plot (*Expected diagram*)

Effective description by Pisarski and Wilczek

A restoration of $U(1)_A$?

$\langle \bar{q}q \rangle$



$\langle l_P \rangle$

Robust
Effective description by the Polyakov loop (Thermal Wilson line)

A smooth crossover is observed at physical points.
(No definite phase transition)

[Aoki, Endrodi, Fodor, Katz, Szyabo (2006)]

Hot electroweak phase diagram

- 1'st order for a light Higgs mass

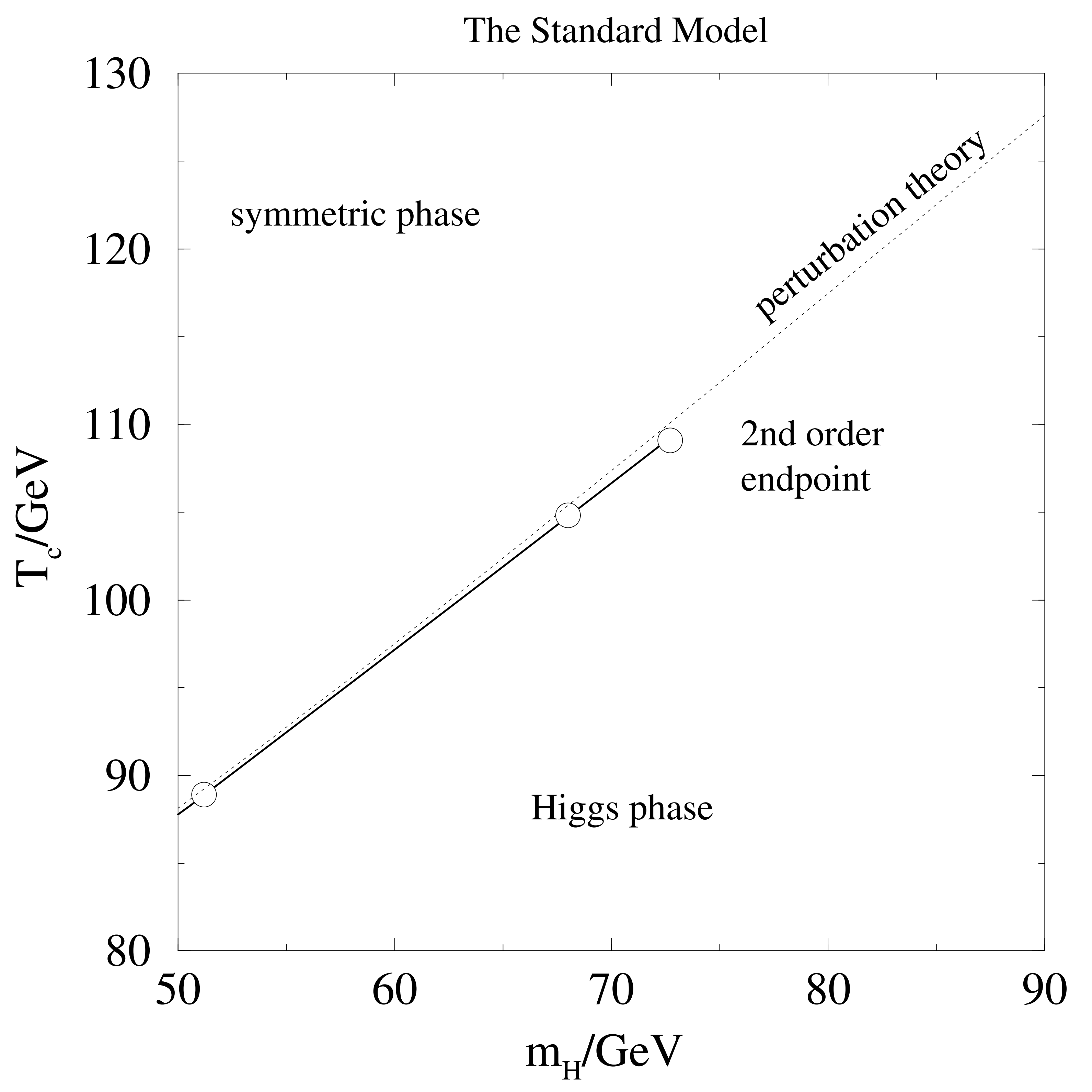
- A smooth crossover for the observed Higgs mass

- There is no local gauge-invariant order parameter.

[Elitzur(1975)]

$\langle H_{\rm SM} \rangle$ is not a gauge invariant.

$\langle H^\dagger_{\rm SM} H_{\rm SM} \rangle$ is a gauge invariant, but there is no obvious global symmetry.



Higgs vs confinement phases

A three-dimensional effective theory:

$SU(2)$ gauge theory with a fundamental Higgs field

$$L = \frac{1}{4}F^2 + |DH|^2 + m(T)^2|H|^2 + \lambda|H|^4$$

$m^2(T) > 0$: Higgs can be integrated out (would-be confinement phase)

$m^2(T) < 0$: A gauge field can be integrated out (would-be Higgs phase)

A phase transition between confinement and Higgs phases?

However, the (deep) Higgs phase and the confinement phase can be smoothly connected… (Fradkin-Shenker's theorem)

[Fradkin, Shenker (1976), Banks, Rabinovici (1979)]

Lattice Simulation

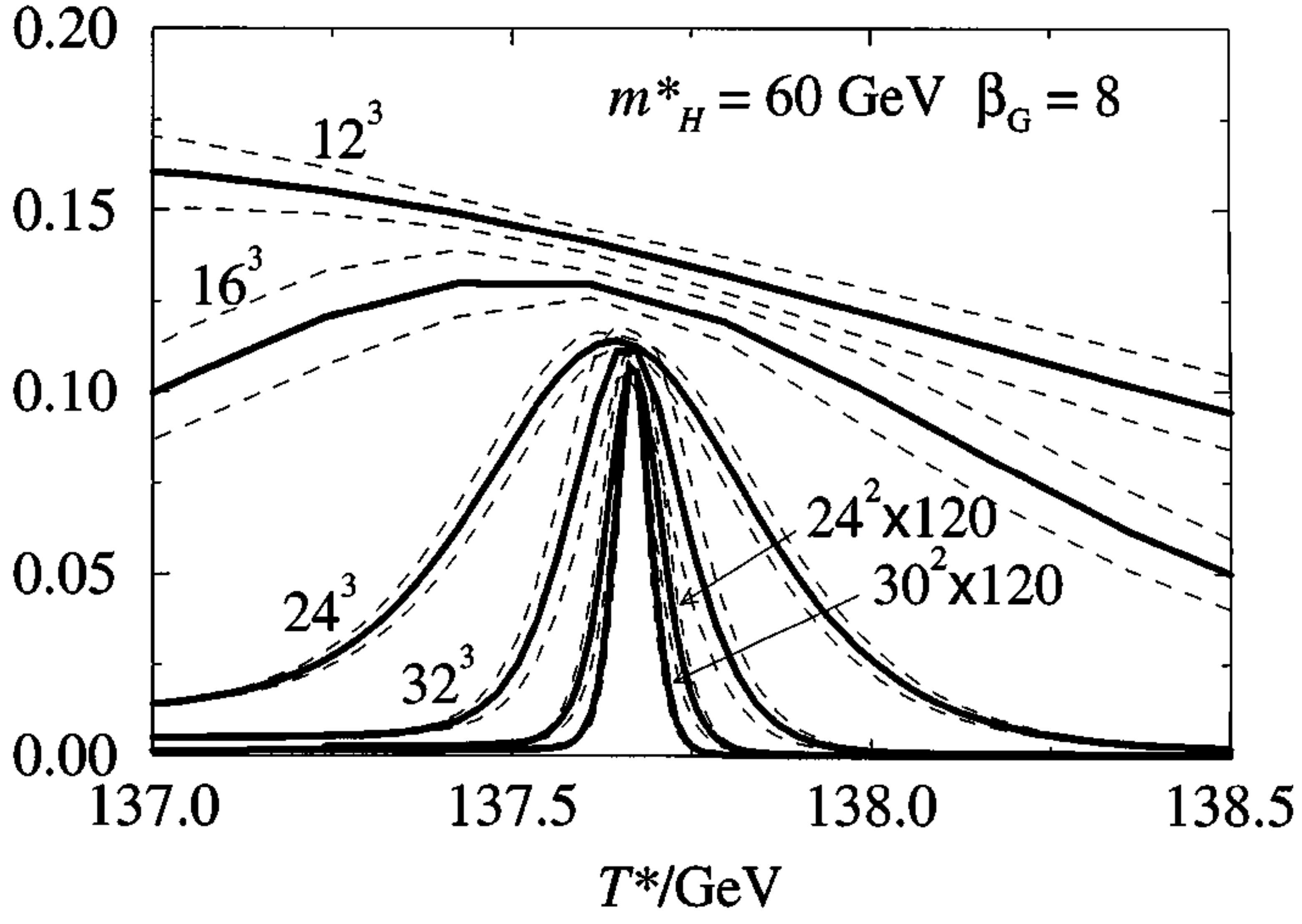
Only bosonic degrees of freedoms appear (suitable for lattice simulation).

On a finite box, path-integral is numerically computed.

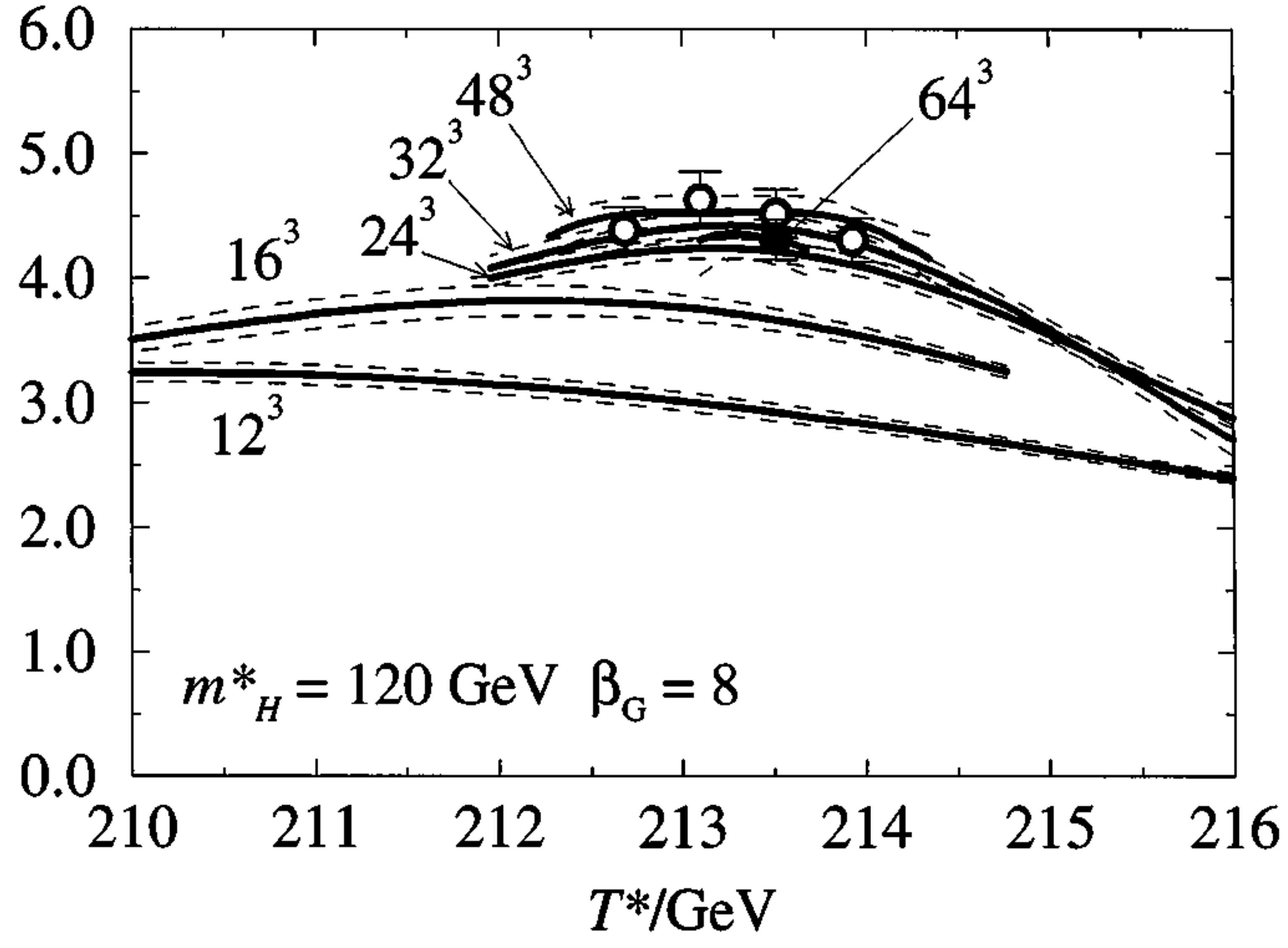
[K. Kajantie, M. Laine, K. Rummukainen and M.E. Shaposhnikov (1996)]

Susceptibility: $\chi \sim V\langle (H^\dagger H - \langle H^\dagger H\rangle)^2\rangle$ $\quad V$: volume

1'st order $\quad \dfrac{\chi}{V}$

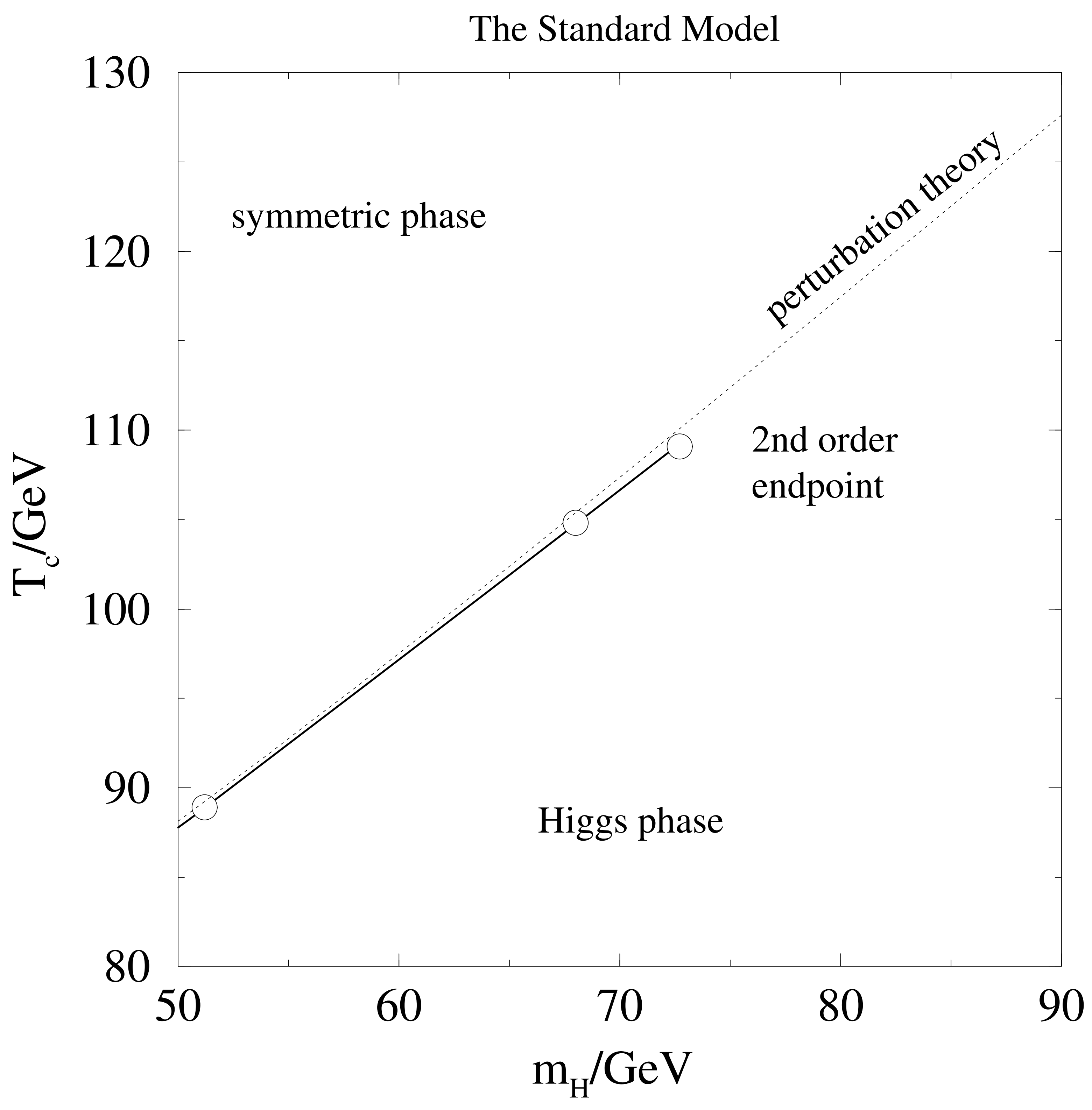


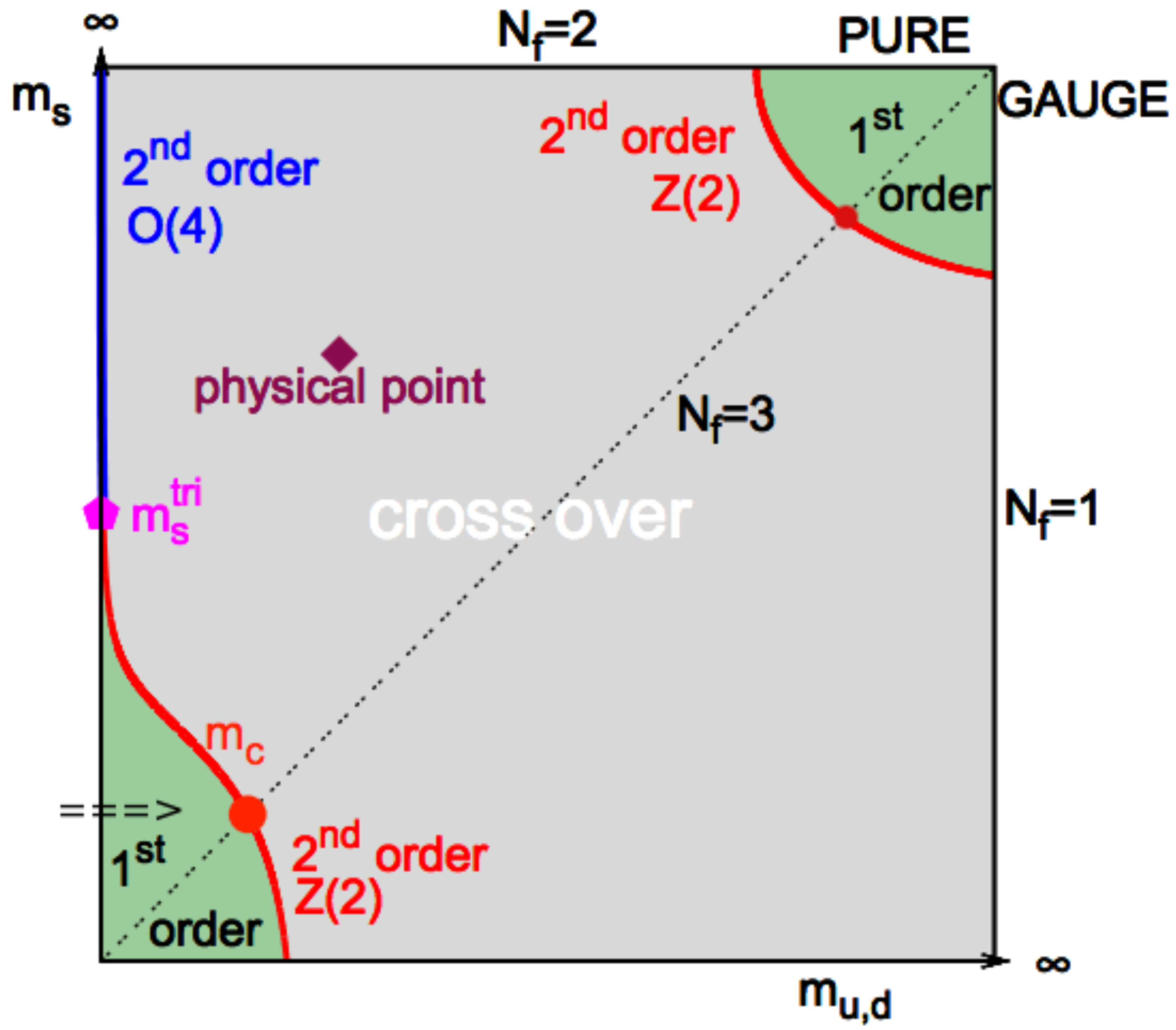
Crossover $\quad \chi$



At a physical point, a smooth crossover is observed (by a finite-size scaling).

Summary: No phase transitions in SM





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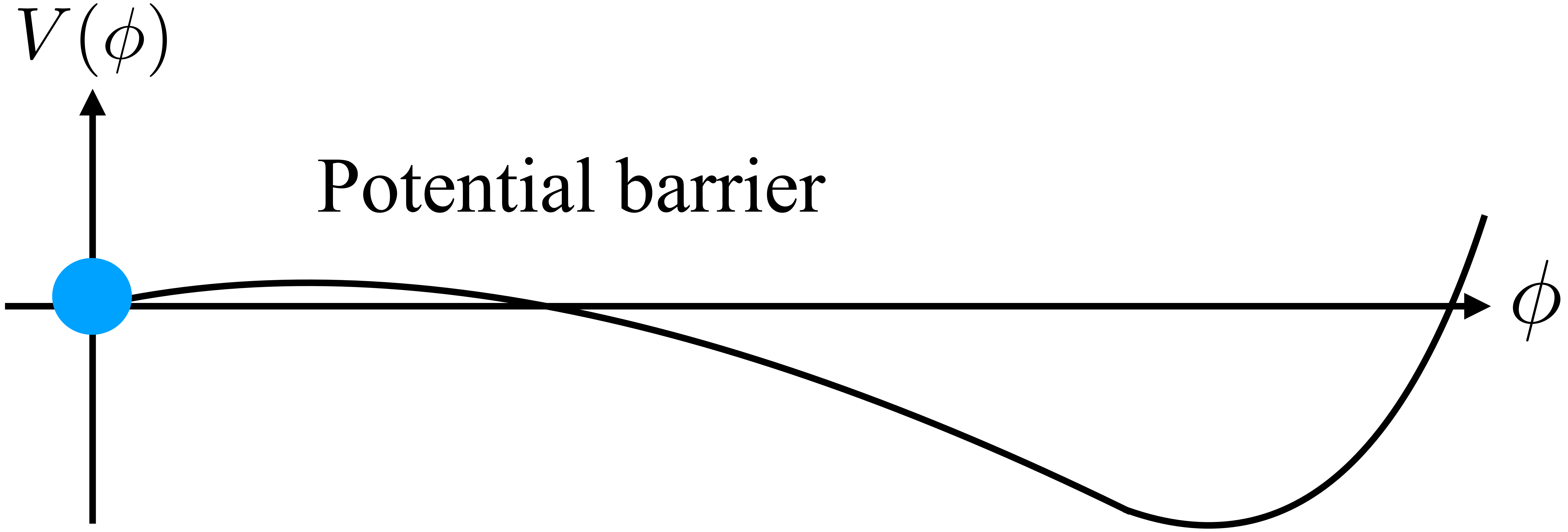
Phase Transitions in BSM

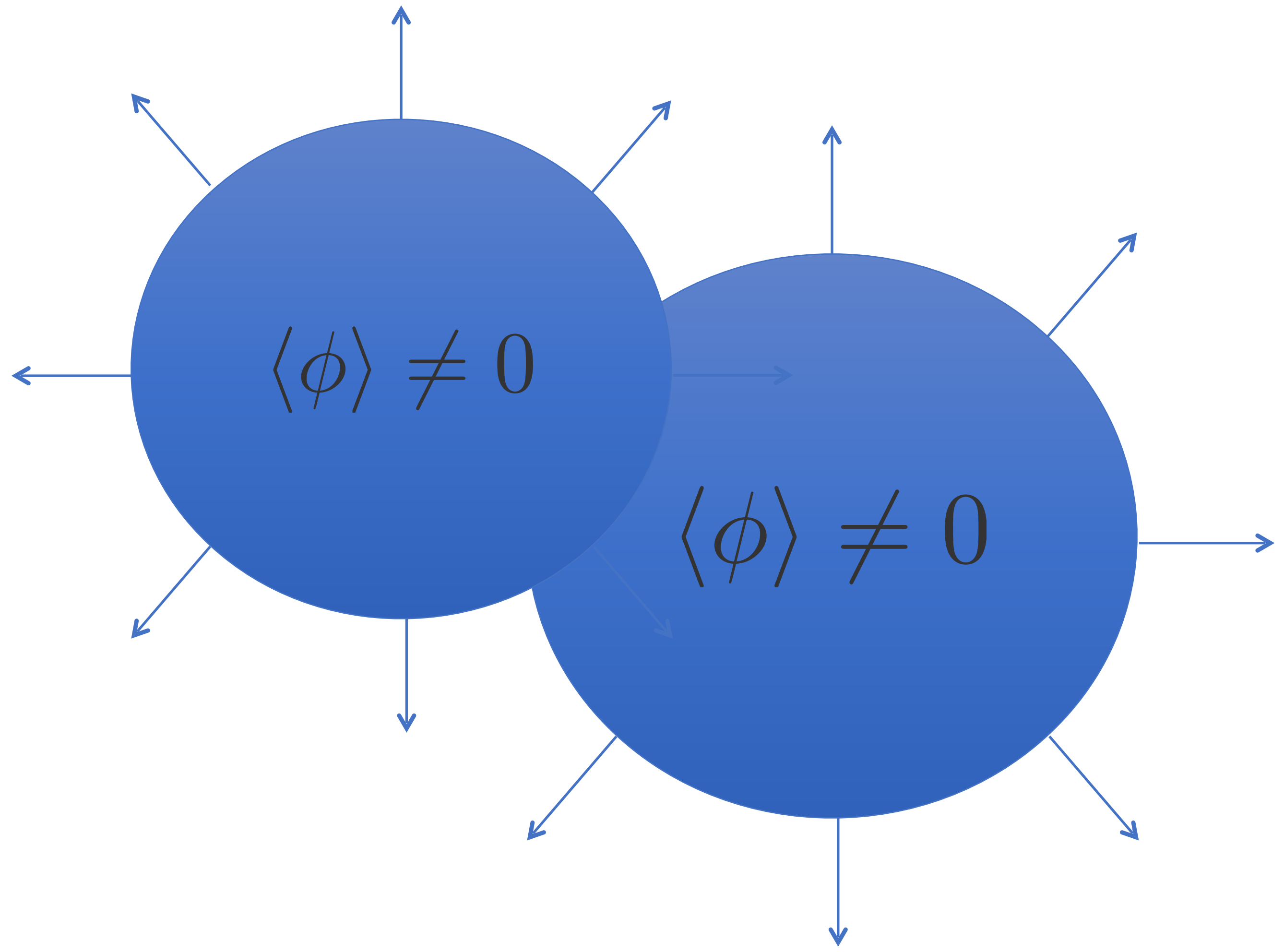
Here, I consider the following situation and assume a mean field analysis is somewhat good.

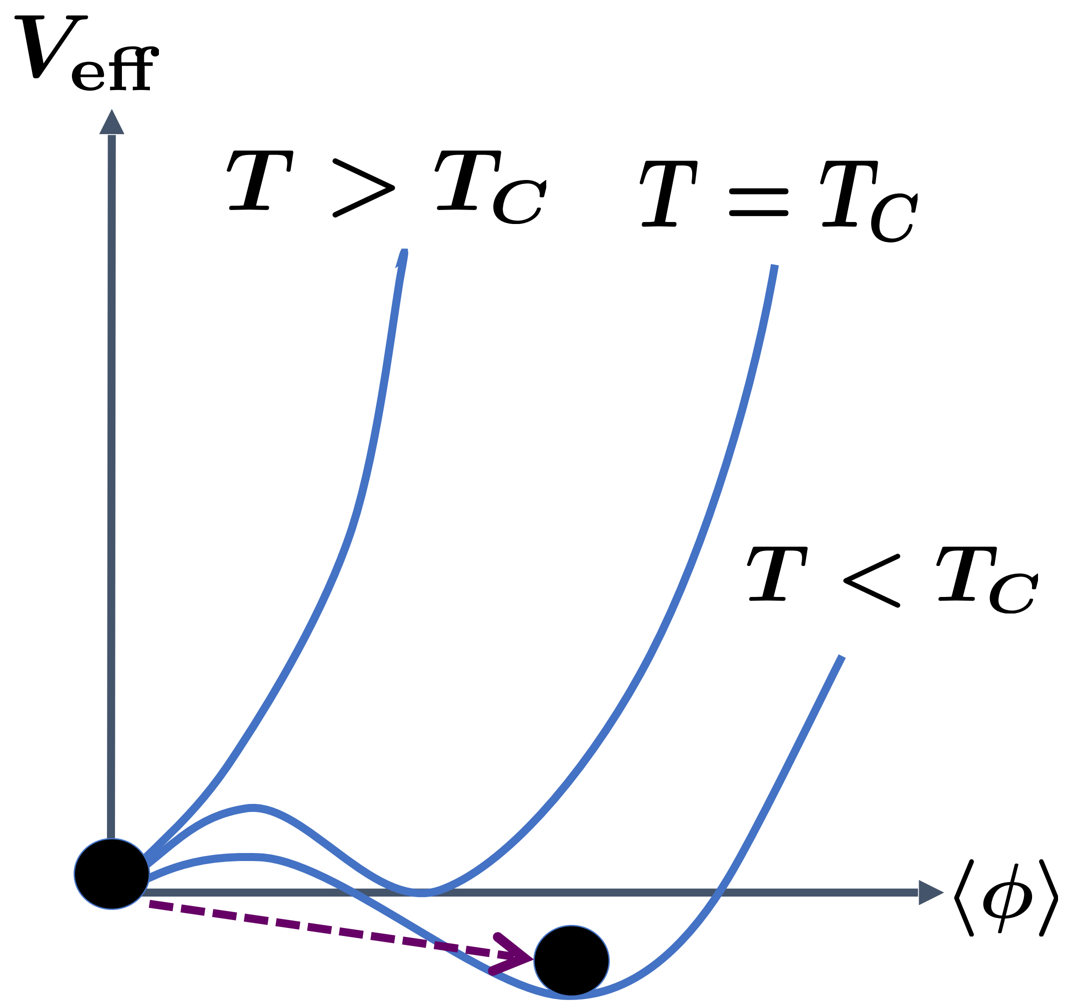


It is known that theories with many bosonic degrees of freedoms or an approximate scale invariance realize the above situation.

Cosmological Phase Transitions

Cosmological first order phase transition ⇒ Formation of bubbles





Very roughly speaking, bubble nucleations take place when the temperature of the Universe $\simeq$ mass scale of symmetry breaking

(There exists exceptions.)

After bubble nucleations, they expand due to a released energy.

A decay rate of metastable state

Nucleation of bubbles ~ decay of the metastable state

A metastable decay rate is estimated by the imaginary part of the energy (or free energy for a thermal system).

[Coleman (1977), Linde (1981)]

$$\langle \mathrm{FV}|e^{-HT}|\mathrm{FV}\rangle = \int D\phi e^{-S[\phi]} \propto e^{-i\Gamma VT/2}$$

T: Euclidean time, $|\mathrm{FV}\rangle$: the false vacuum state, H : Hamiltonian, Γ: decay rate

Γ can be regarded as the decay rate in the real time.

(It may be interesting to see the real-time evolution of the density matrix.)

Bounce Action

A saddle-point approximation:

$$\int D\phi\, e^{-S[\phi]} \simeq e^{-S_{\rm cl}[\phi_{\rm cl}]} \int D(\delta\phi) e^{-\delta\phi \mathcal{M} \delta\phi} \qquad \mathcal{M} = \left.\frac{\delta^2 S}{\delta\phi\delta\phi}\right|_{\phi=\phi_{\rm cl}}$$

Bounce action

A single negative mode (Imaginary part)

Bounce solution (with spherical symmetry):

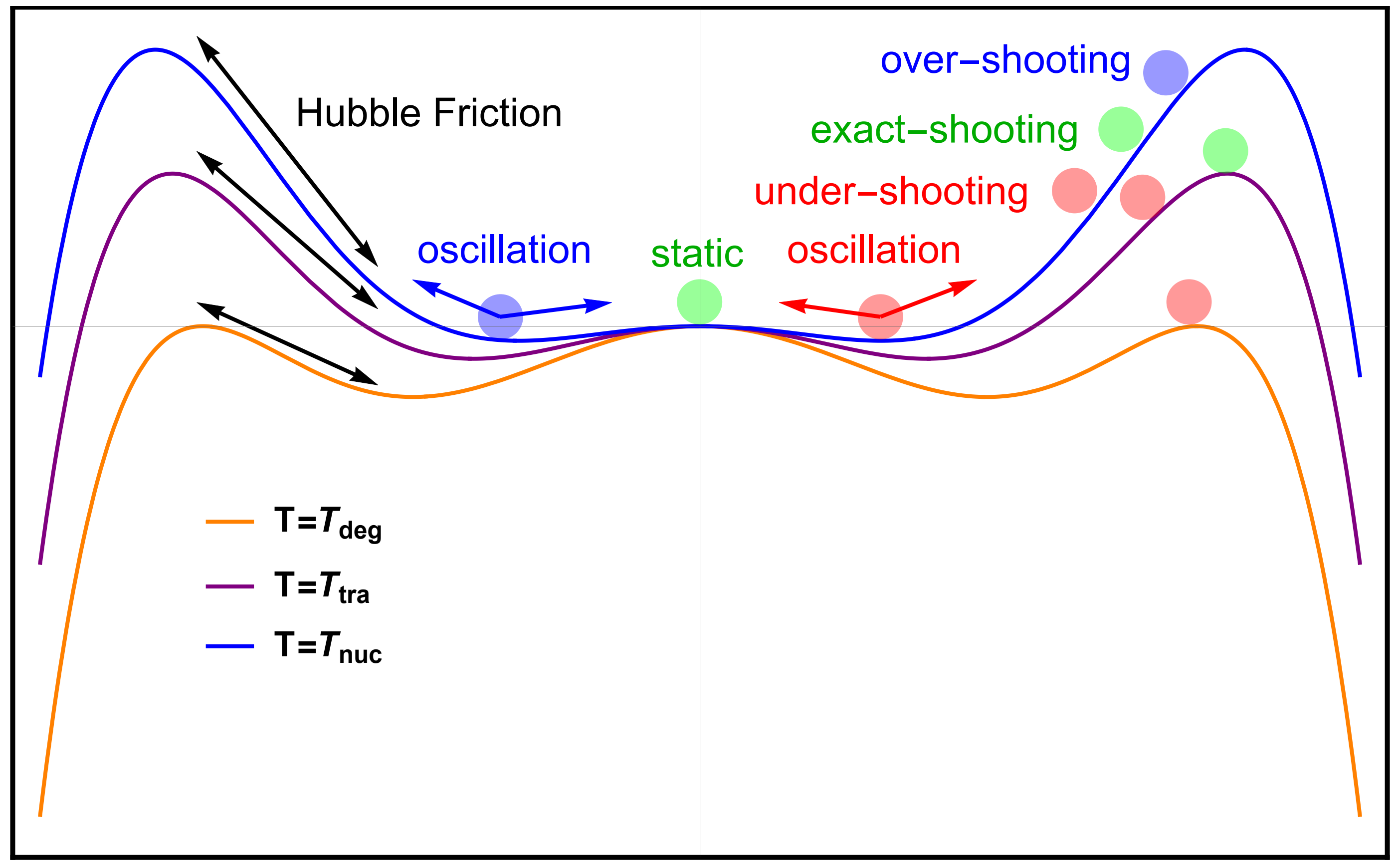
$$\frac{\partial^2\phi}{\partial r^2} + \frac{D-1}{r}\frac{\partial\phi}{\partial r} - \frac{\partial V}{\partial\phi} = 0$$

$r = \sqrt{\tau^2 + \mathbf{x}^2}$

$\phi(r=\infty) = \phi_{\rm FV}$

D: spacetime dimension

$\frac{{\rm d}\phi}{{\rm d}r}(r=0) = 0$



A numerical calculation is typically required.

Bubble Nucleation Rate

A metastable decay rate per unit time and per unit volume:

$$\Gamma(T) \simeq T^4 e^{-S_{\rm cl}(T)}$$

The reaction rate depends on the cosmic time through the temperature:

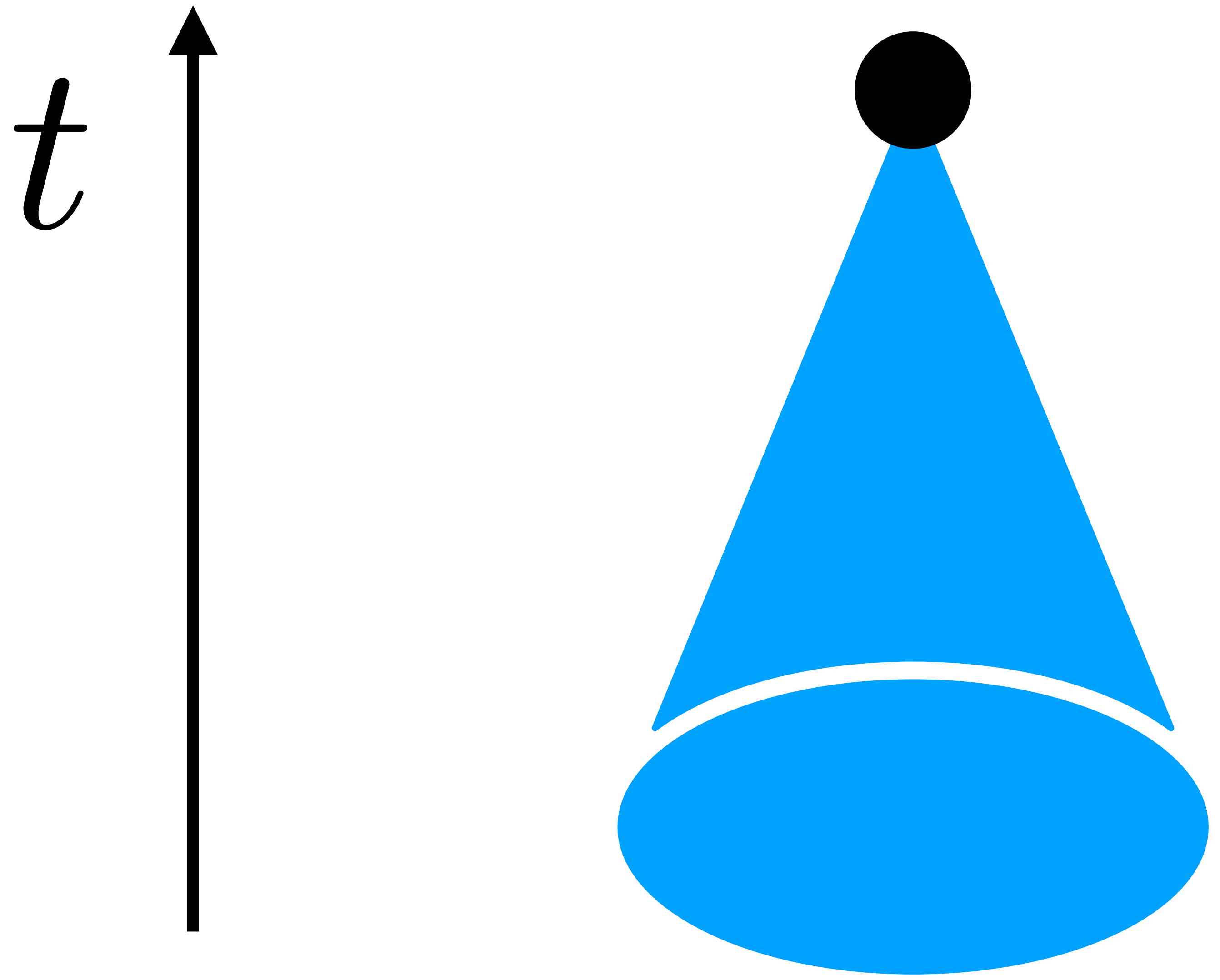
$$\Gamma(T(t)) \simeq T^4(t) e^{-S_{\rm cl}(T(t))}$$

The bounce is typically sensitive to the temperature (the cosmic time).

Bubble Nucleation

Bubble nucleation is treated as the stochastic process.

$P(t)$: A probability that any given point remains in the metastable state:



$$P(t) = e^{-N_b(t)}$$

N_b: Expected number of bubbles

$$N_b \sim \int \mathrm{d}t\, \Gamma H^{-3} \sim \Gamma H^{-4}$$

Condition: No nucleations in past light-cone

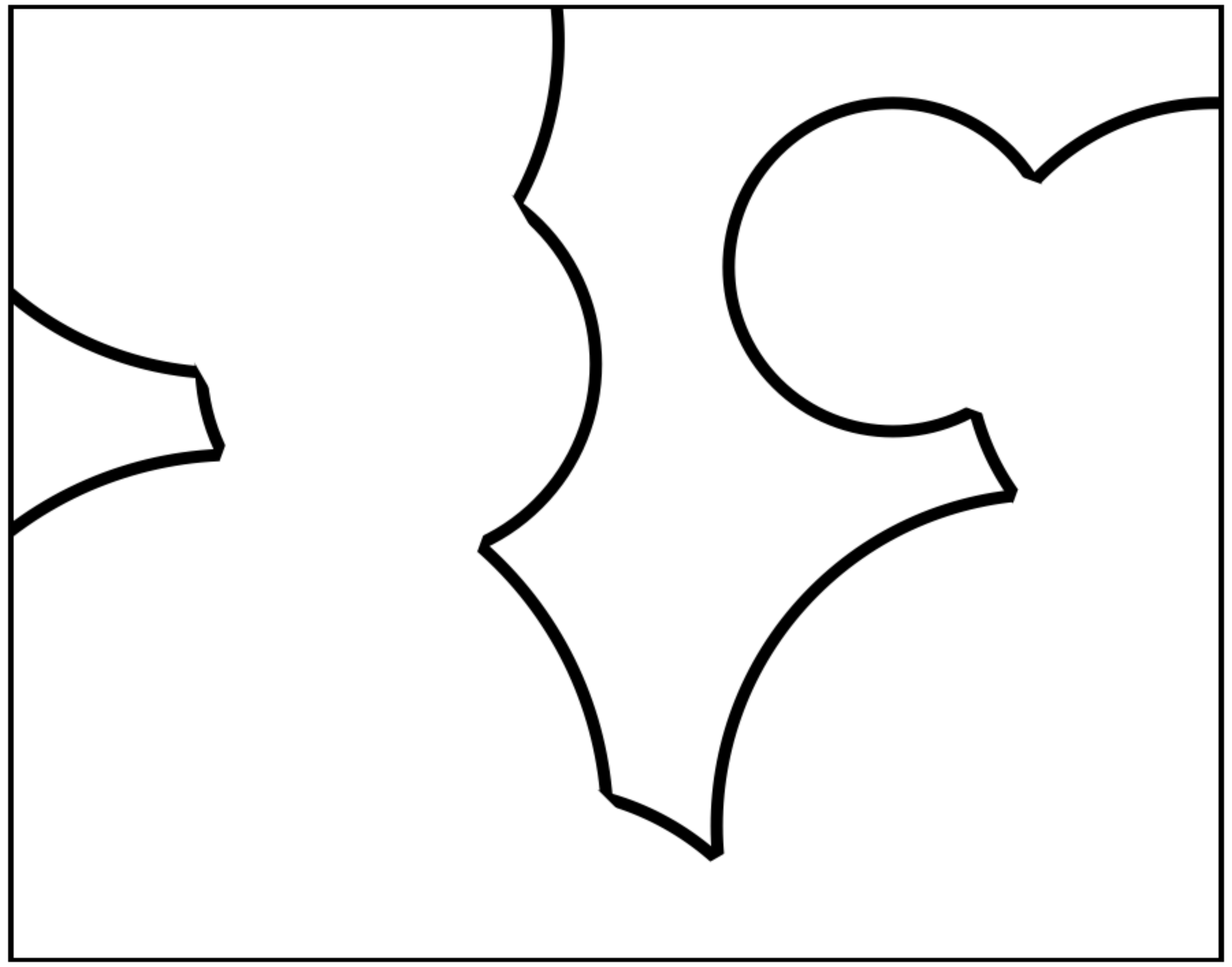
$H \sim 1/t$: The Hubble parameter

A bubble nucleation takes place at $P(t_n) \sim \mathcal{O}(0.1) \rightarrow \Gamma/H^4 \sim 1$.

Bubble Percolation

Bubble nucleation is treated as the stochastic process.

$P(t)$: A probability that any given point remains in the metastable state:



$$P(t) = e^{-N_b(t)}$$

$$N_b \sim \int \mathrm{d}t\, \Gamma H^{-3} \sim \Gamma H^{-4}$$

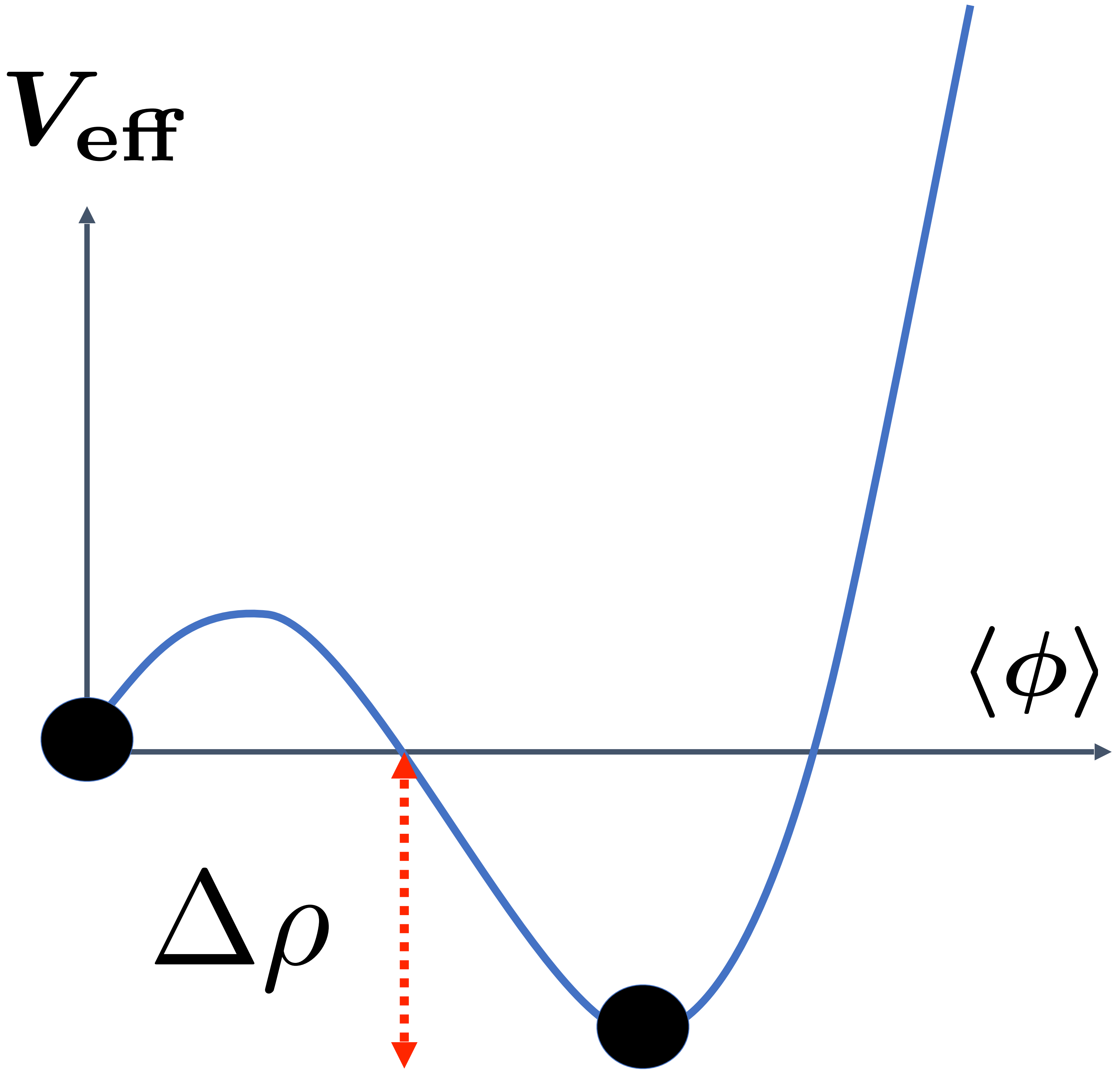
Typically, a reaction rate is quite fast!

$$\Gamma(t) = \Gamma_n e^{\beta(t-t_n)}$$

$$P(t) \sim \exp(-e^{\beta(t-t_n)})$$

The universe soon fills by nucleated bubbles within the time scale $\beta^{-1} \ll H^{-1}$.

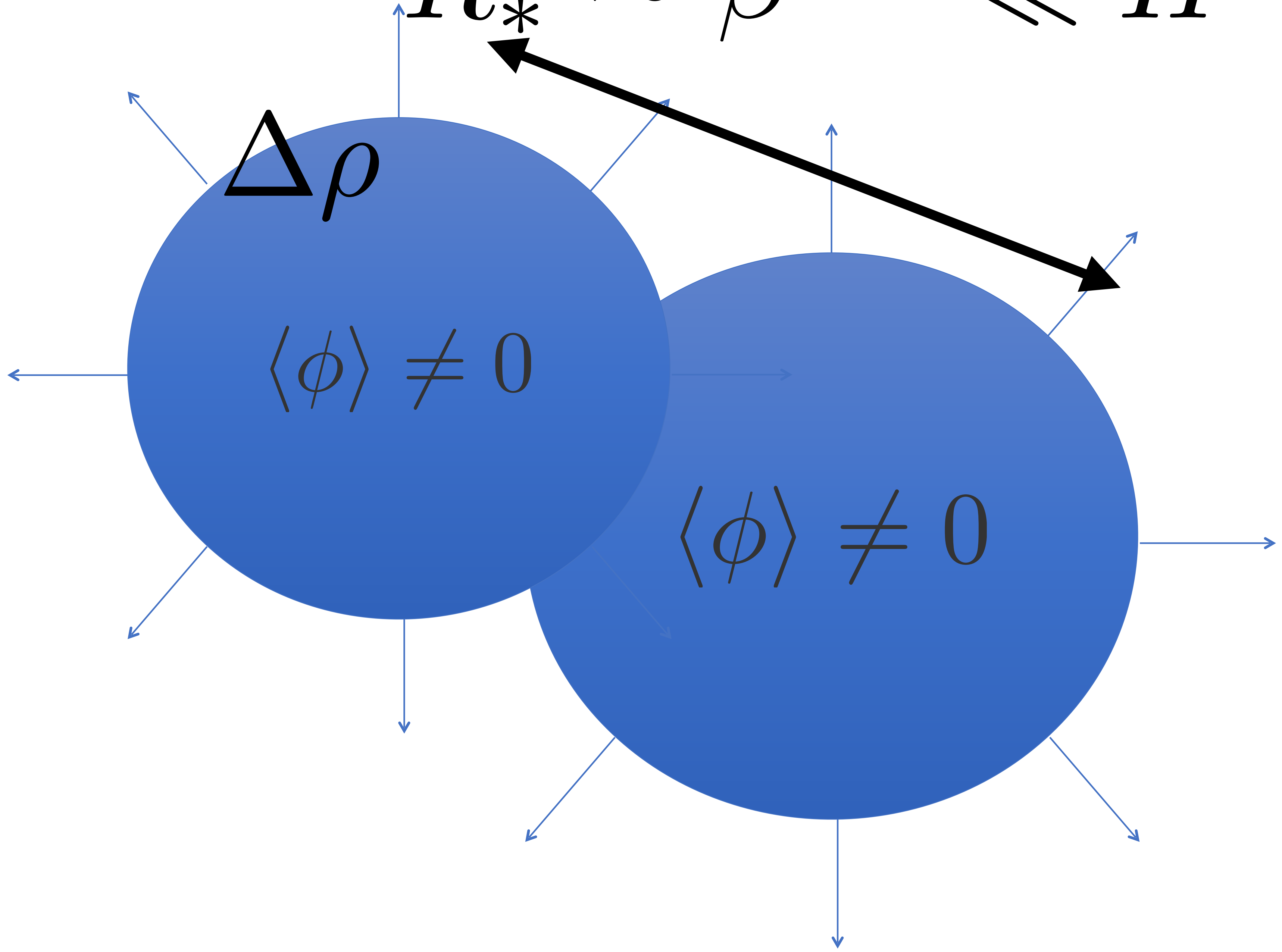
A basic picture



The released energy density

Bubble radius at percolation

$$R_* \sim \beta^{-1} \ll H^{-1}$$



Gravitational wave spectra

Sources of GW spectra:

- Bubble collisions (a vacuum contribution)

[Kosowsky, Turner, Watkins (1992); Kosowsky, Turner (1993)]

[Kamionkowski, Kosowsky, Turner (1993)]

[Caprini, Durrer, Servant (2007)]

[Huber, Konstandin (2008)] *[Jinno, Takimoto (2016,2017)]*

- thermal environment (energy injected by the moving bubble wall)

[Hindmarsh, Huber, Rummukainen, Weir (2013, 2015, 2017)]

[Jinno, Konstandin, Rubira (2021)]

[Auclair, Caprini, Cutting, Hindmarsh, Rummukainen, Steer et al. (2022)]

[Jinno, Konstandin, Rubira, Stomberg (2022)]

And so many works…

Order estimate of peak amplitude

[Kamionkowski, Kosowsky, Turner (1993)]

The released energy: $E_{\rm kin} \sim \Delta\rho \times V$

$\Delta\rho$: released energy density, $V \sim R_*^3 \sim \Delta t^3$: volume

Quadrupole formula: $P_{\rm GW} \sim G_N \dot{E}_{\rm kin}^2 \sim G_N E_{\rm kin}^2 \Delta t^{-2}$

G_N: Newton constant, $P_{\rm GW}$:power of GW

Energy of GW: $E_{\rm GW} \sim P_{\rm GW}\Delta\tau$ ($\Delta\tau$: lifetime of the source of GW)

Energy density of GW: $\rho_{\rm GW} = E_{\rm GW}/V \sim G_N \Delta\rho^2 \Delta\tau \Delta t$

Normalized peak amplitude: $\Omega_{\rm GW} \sim \rho_{\rm GW}/\rho_{\rm rad} \sim \alpha^2 \times (H/\beta) \times H\Delta\tau$

$\Delta t \sim \beta^{-1}, \alpha = \Delta\rho/\rho_{\rm rad}$

Peak amplitude and frequency

A large released energy and slow transition rate amplify the GW spectra.

$$\Omega_{\rm GW} \sim 10^{-5} \times \alpha^2 \times (H/\beta) \times H\Delta\tau$$

(with possible suppression factors.)

Peak frequency today:

$$f_{\rm peak} \sim 10^{-5} \times \frac{T_*}{100\,{\rm GeV}} \times \beta/H \quad {\rm Hz}$$

Electroweak phase transition with $\beta/H \sim 10^2$ → typical peak frequency mHz

LISA, DECIGO and BBO are good.

GW spectra: Tensor perturbation

GW spectra ~ tensor perturbation on the fixed background:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + (\delta_{ij} + 2h_{ij})\mathrm{d}\mathbf{x}^2$$

An energy density of gravitational wave : $\rho_{\mathrm{GW}} \sim \langle \dot{h}_{ij}\dot{h}_{ij} \rangle \times M_{\mathrm{Pl}}^2$

(Ensemble average)

h_{ij} is sourced by the energy momentum tensor via Einstein eq.:

$$h_{ij}(t, \mathbf{k}) \sim \int_{t_i}^{t} \mathrm{d}t\, \underbrace{G(t - t', \mathbf{k})}_{\text{Green function}} T_{ij}^{TT}(t', \mathbf{k}) / M_{\mathrm{Pl}}^2$$

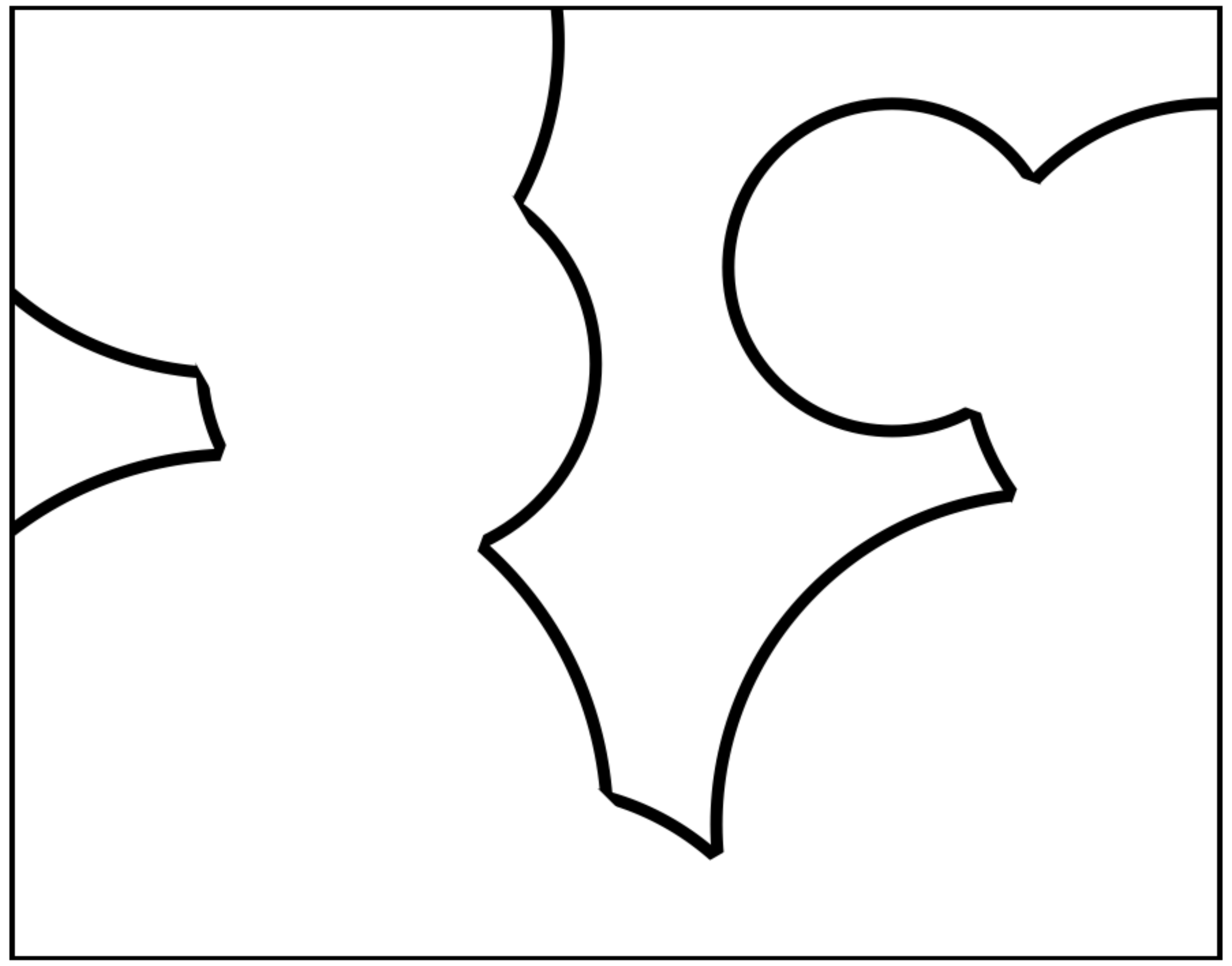
A main claim: GW signals are computed by a two-point function $\langle T_{ij}^{TT} T_{ij}^{TT} \rangle$.

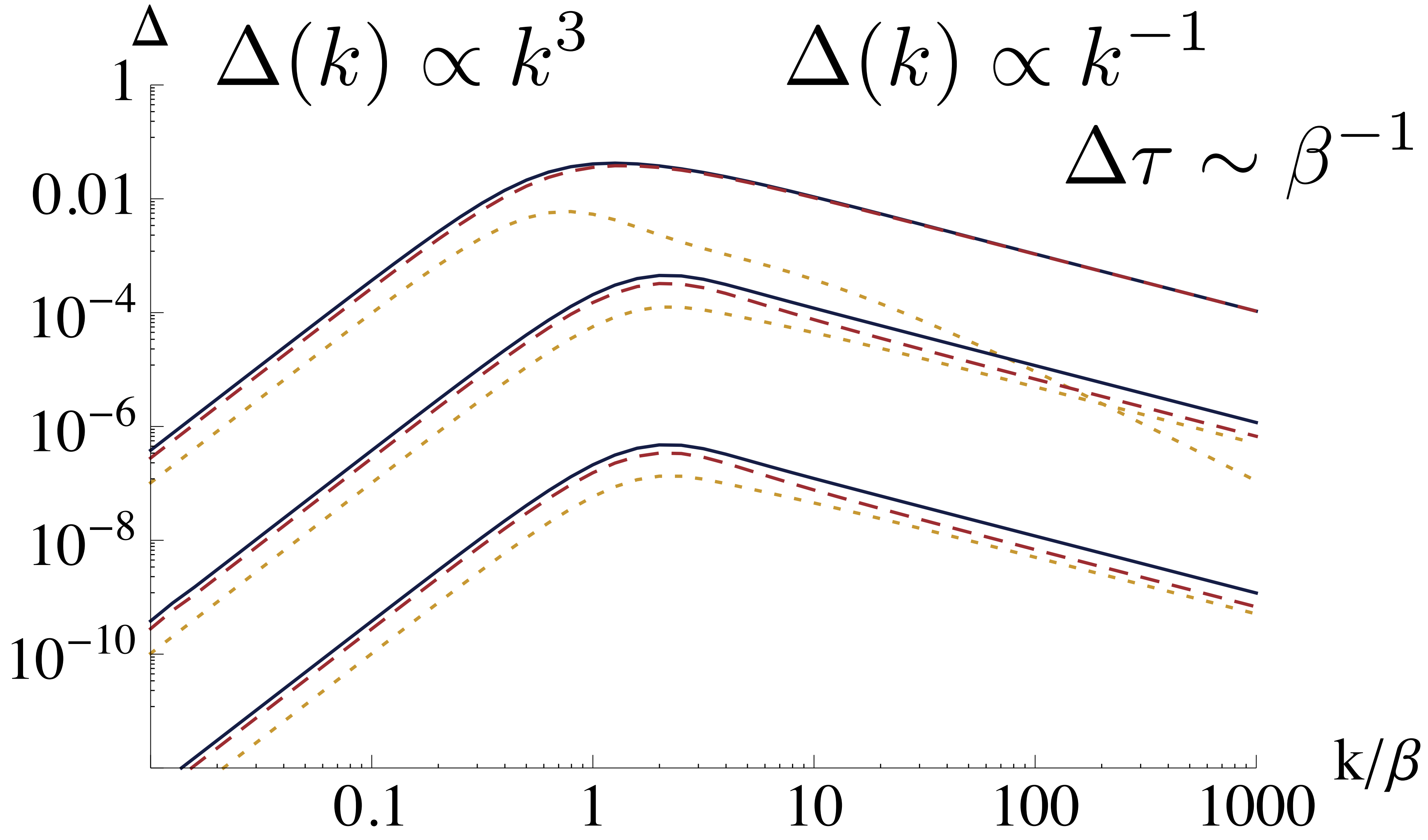
Bubble collisions (A vacuum contribution)

A bubble kinetic energy is a source of the gravitational wave spectra.

Surprisingly, an analytic estimate of $\langle T_{ij}^{TT} T_{ij}^{TT} \rangle$ is found (under thin and envelope approximations).

[Jinno, Takimoto (2016)]





An analytic expression beyond the envelope approximation is also derived.

[Jinno, Takimoto (2017)]

Sound Wave (Thermal environment)

A bulk motion of fluid leads to the source of gravitational wave spectra,
$T^{ij}_{\text{fluid}} \sim \Delta\rho \times U^i U^j$ (U^μ: four velocity).

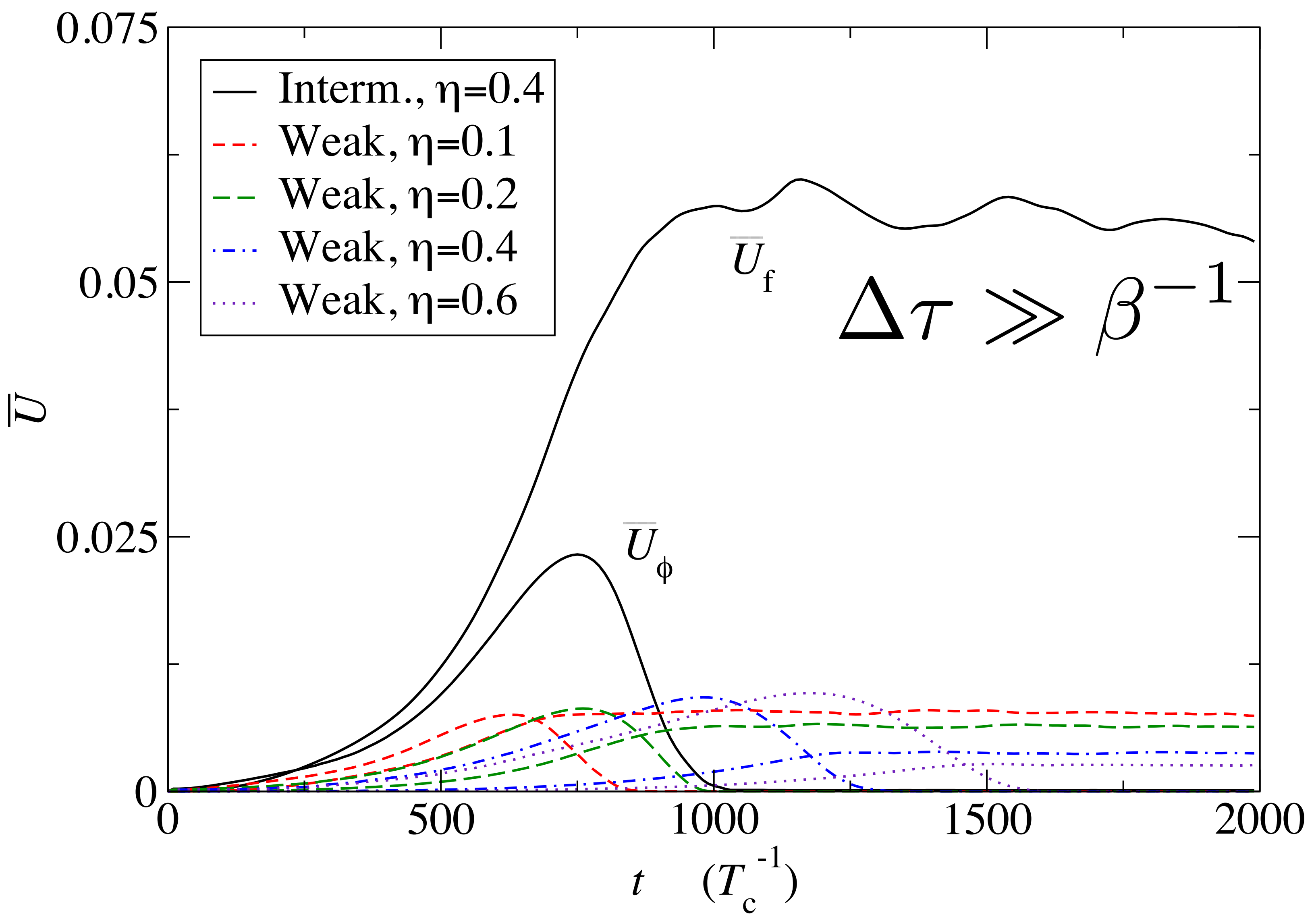
A non-linear fluid dynamics requires the numerical investigation:

setup: Scalar + perfect fluid dynamics

Bubble: $$-\ddot{\phi} + \nabla^2\phi - \frac{\partial V}{\partial \phi} = \eta U^\mu \partial_\mu \phi$$

Fluid: $$\partial_\mu T^{\mu\nu}_{\text{fluid}} = -\eta U^\mu \partial_\mu \phi$$

η: set by hand in the simulation



[Hindmarsh, Huber, Rummukainen, Weir (2013, 2015, 2017)]

It turns out that sound wave contribution is larger than bubble collisions.

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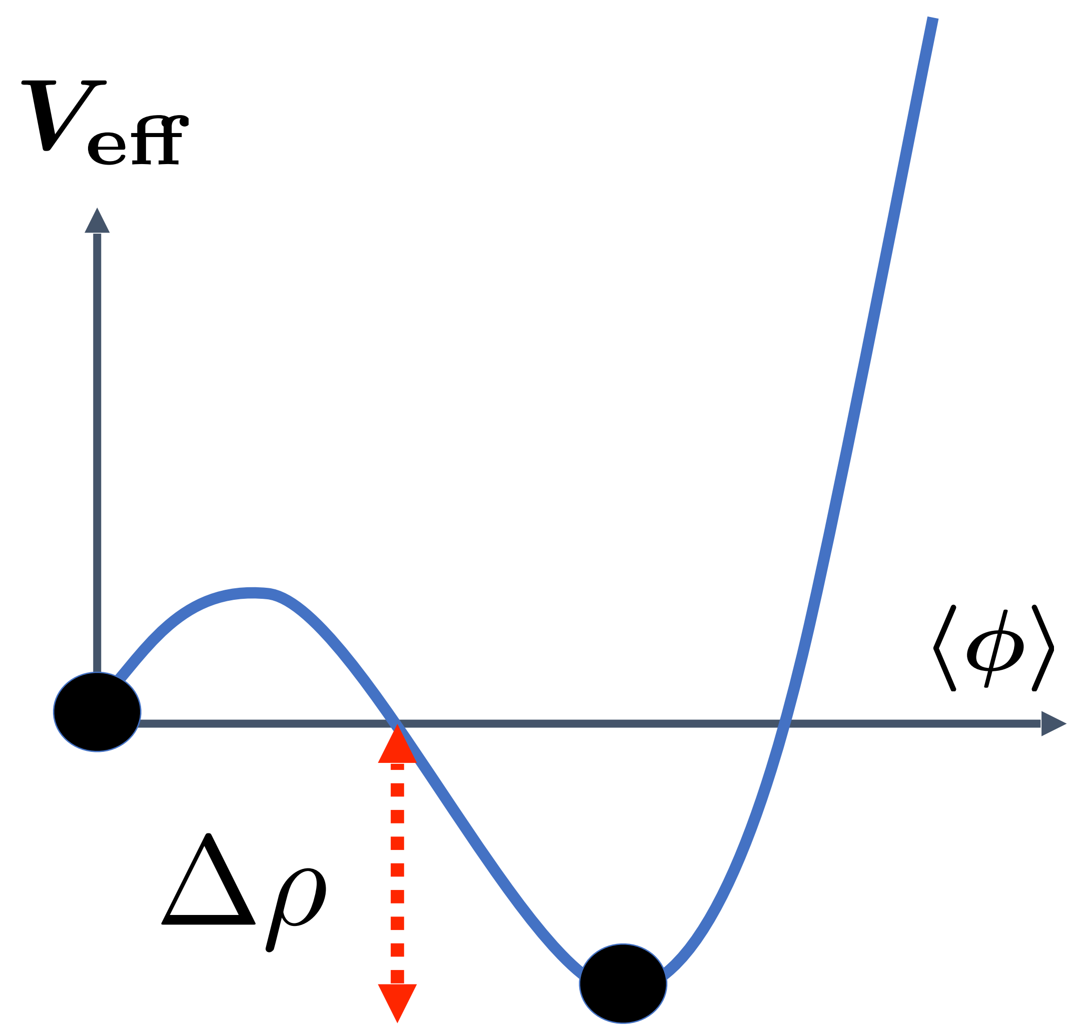
How do we realize the ultra-supercooling?

In any cases, GW spectra from a phase transition with a large released energy density and a slow transition rate are good.

$$\Omega_{\rm GW} \sim \alpha^2 \times (H/\beta) \times H\Delta\tau$$



$$\alpha \sim \Delta\rho/\rho_{\rm rad} \sim \mathcal{O}(1)$$

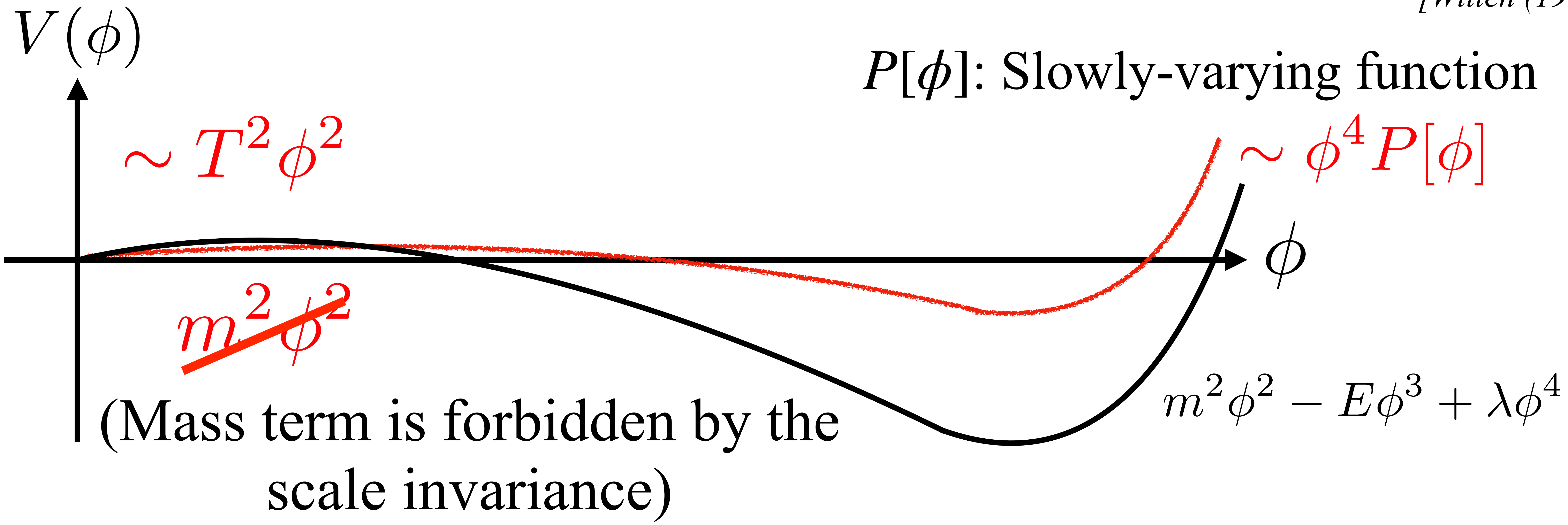
$$\beta/H \sim \left| T\frac{{\rm d}S_{\rm cl}}{{\rm d}T} \right|_{t=t_*} \sim \mathcal{O}(1)$$

What kind of theories lead to this situation (ultra-supercooling)?

Ultra-supercooled phase transition

An approximate scale invariance (Ex. Coleman-Weinberg)

[Witten (1980)]



In the exact scale (or conformal) invariance, there is no transition.

A smallness of the explicit breaking of scale invariance leads to significant supercooling.

Ultra-supercooled Universe in BSM

Ultra-supercooled phase transitions realize various BSM physics.

- QCD with many flavors (AdS dual: Randall-Sundrum model)

[Creminelli, Nicolis, Rattazzi (2001), Randall, Servant (2006), Baratella, Pomarol, Rompineve (2018), KF, Nakai, Yamada (2019) … and so many works]

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- A radical consequence: the graceful exit problem? QCD-induced electroweak phase transition?

The energy density can be dominated by the metastable state : $\alpha \equiv \rho_{PT}/\rho_{\rm rad} \gtrsim 1$

A mini-inflation can takes place.

QCD chiral phase transition may occur before electroweak phase transition…

[Witten (1980), Iso, Serpico, Shimada (2017), Harling Servant(2017)]

- There remains many questions on the wall dynamics (at least for me).

Summary of the second lecture

- No thermal phase transitions in the SM.

- First-order cosmological phase transitions lead to gravitational wave production.

- A nearly scale-invariant theory leads to the ultra-supercooling.