기초과학연구원
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Searches for Power-law Warped Extra Dimensions

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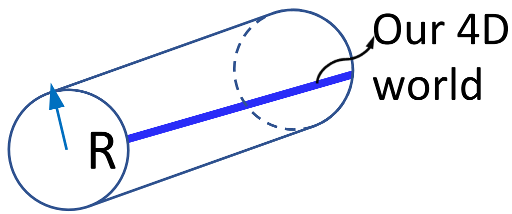
Outline

- Motivations and a model for a power-law warped extra dimension
- New KK graviton phenomenology
- Experimental and astrophysical limits

Extra spatial dimensions beyond 4D spacetime

- String theory (6 extra dimensions for consistency of theory)
- Flat N extra dimensions (ADD model)

N Arkani-Hamed, S Dimopoulos, G Dvali '98



Compact extra dimension

$$V(r) \sim \begin{cases} \frac{1}{(M_{4+N})^{2+N}} \frac{m}{r^{1+N}}, & r \ll R \\ \frac{1}{(M_{4+N})^{2+N} R^N} \frac{m}{r}, & r \gg R \end{cases}$$

M_{4+N} : Planck scale in $4+N$ spacetime

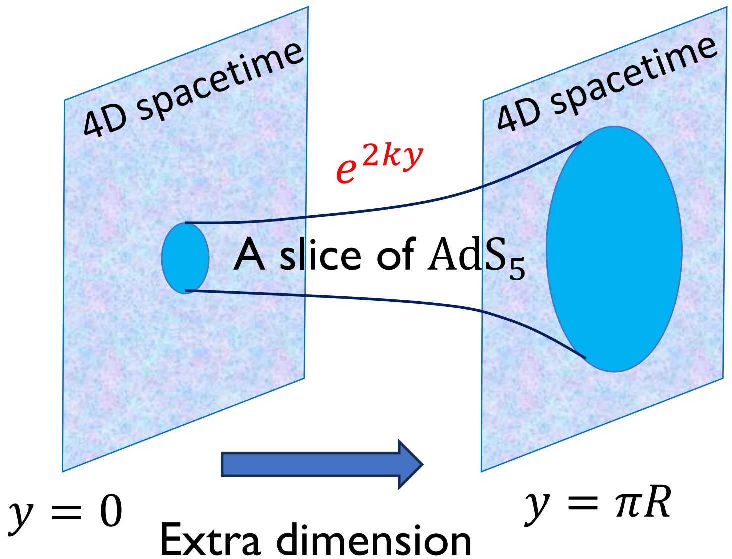
Effective 4D Planck scale $M_P^2 = (M_{4+N})^{2+N} R^N$

$$\Rightarrow \quad M_{4+N} = \left(\frac{M_P^2}{R^N}\right)^{1/(2+N)} \sim \text{TeV} \quad \text{for } R \sim 10^{\frac{30}{N}-17} \text{ cm}$$

The hierarchy problem can be solved by large extra dimensions (e.g. ~ 0.1 mm for $N = 2$).

Extra spatial dimensions beyond 4D spacetime

- A warped extra dimension (RS model)   L Randall, R Sundrum '99



$$ds^2 = e^{2ky} g_{\mu\nu} dx^\mu dx^\nu + dy^2$$

Effective 4D Planck scale

$$M_P^2 \simeq \frac{M_5^3}{k} e^{2k\pi R}$$

M_5 : 5D Planck scale

k ($\sim M_5$) : AdS_5 curvature

$$M_5 \sim M_P e^{-k\pi R} \sim \text{TeV} \quad \text{for} \quad R \sim 10^{-3}\ \text{fm}$$

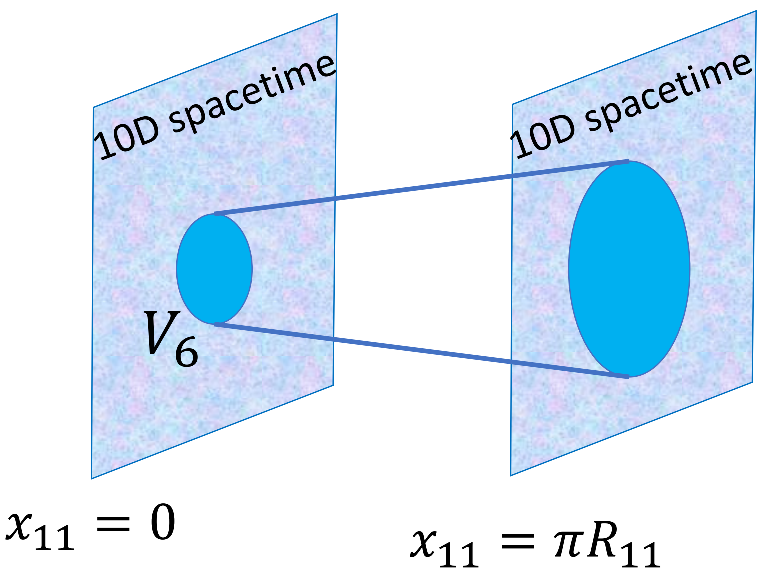
The hierarchy problem can be solved by a small extra dimension.

Extra dimensional models from string theory

- 11D Heterotic M-theory

Dual theory of strongly coupled 10D heterotic string theory

Horava and Witten '96
Lukas, Ovrut, Stelle, Waldram '98
SHI, Nilles, Olechowski '19



- The 11th dimension can be parametrically larger than the other 6D extra dimensional space.
- Consequently, the theory can be described by 5D EFT after integrating out the 6D extra dimensions.
- The effective 5D metric is power-law warped due to the growing 6D extra dimensional space proportional to x_{11}.

$$ds^2_{5\mathrm{D}} = (ky+1)^{2q} g_{\mu\nu} dx^\mu dx^\nu + dy^2 \qquad q = \frac{1}{6}, \frac{1}{7}, \frac{1}{10}, \ldots$$

Extra dimensional models from string theory

- Little string theory

String theory in zero string coupling limit $g_s \to 0$, which may address the hierarchy problem

$$M_P^2 = \frac{1}{g_s^2} M_S^8 V_6$$

M_S : fundamental string scale

V_6 : the volume of 6D extra dimensional space

Its dual theory is given by a 7D theory with a *linear dilaton* background, whose 5D approximate theory is described by a power-law warped extra dimension.

$$ds_{5\mathrm{D}}^2 = (ky+1)^{2q} g_{\mu\nu} dx^\mu dx^\nu + dy^2 \qquad q = 1$$

"Linear dilaton model"

Antoniadis, Arvanitaki, S Dimopoulos, Giveon '11
Giudice, Kats, McCullough, Torre, Urbano '17

Phenomenology of power-law warped extra dimensions?

- Many 5D EFTs from string theory predict a power-law warped extra dimension with $0 \text{ (ADD limit)} < q < \infty \text{ (RS limit)}$

$$ds^2_{5\mathrm{D}} = (ky+1)^{2q} g_{\mu\nu} dx^\mu dx^\nu + dy^2$$

- Nevertheless, its phenomenology has not been studied except $q = 1$ (linear dilaton model; LD).

- In our work, we demonstrate that the KK gravitons from a power-law warped extra dimension with $0 < q < 1$ have quite distinct collider signatures compared with ADD, RS, and LD, while addressing the hierarchy problem.

String-inspired 5D model

$$\boxed{\mathcal{S} = \int d^5x\sqrt{-g}\, e^{S}\left(\frac{1}{2}\mathcal{R} + \frac{Z_S}{2}(\partial_M S)^2 - \Lambda\right)}$$

This form of lagrangian can be generically obtained from string theory with the field *S* identified as the string dilaton or a Kähler modulus.

This is due to *classical scale invariance (CSI)* of EFTs from string theory.

Witten '85

$$S \;\to\; S + \kappa$$

Scale transformation non-linearly realized by dilaton shift symmetry

$$\Rightarrow \quad \mathcal{S} \;\to\; e^{\kappa}\mathcal{S}$$

Overall scaling of the action

$$\frac{\delta\mathcal{S}}{\delta\Phi} = 0$$

Classical e.o.m are invariant (i.e. CSI).

$$\mathcal{A}_{\rm QM} \;\propto\; \exp\left(\frac{i}{\hbar}\mathcal{S}\right)$$

Quantum amplitudes are NOT invariant.

5D CSI action

$$\boxed{\mathcal{S} = \int d^5x\sqrt{-g}\,e^{S}\left(\frac{1}{2}\mathcal{R} + \frac{Z_S}{2}(\partial_M S)^2 - \Lambda\right)}$$

CSI : $S \;\to\; S + \kappa$

$$\mathcal{S} \;\to\; e^{\kappa}\mathcal{S} \qquad \mathcal{A}_{\rm QM} \;\propto\; \exp\left(\frac{i}{\hbar}\mathcal{S}\right)$$

Not respected by quantum effects

Yet invariant under spurious transformation of Planck constant

$$\hbar \;\to\; e^{\kappa}\hbar$$

Selection rule for radiative corrections

$$\mathcal{S}_{\rm 1PI} = \int d^5x\sqrt{-g}\,e^{S}\left[\frac{1}{2}\mathcal{R} + \frac{Z_S}{2}(\partial_M S)^2 - \Lambda + \underbrace{\sum_{n=1}^{\infty} C_n\left(\frac{\hbar}{16\pi^2}e^{-S}\right)^n}\right]$$

Perturbative radiative corrections invariant under

$$\begin{cases} S \rightarrow S + \kappa \\ \hbar \rightarrow e^{\kappa}\hbar \end{cases}$$

Barring possible non-perturbative corrections and assuming

$e^{-S} < 1$ (∵ normally true, since this serves as coupling.)

the leading action would be taken to be stable against quantum effects.

Green, Schwarz, Witten

Giudice, Kats, McCullough, Torre, Urbano 17'

Examples of UV origins

$$\mathcal{S} = \int d^5x\sqrt{-g}\, e^S \left(\frac{1}{2}\mathcal{R} + \frac{Z_S}{2}(\partial_M S)^2 - \Lambda\right)$$

$$Z_S = \begin{cases} 1, & \text{String dilaton} \\ \frac{\mathcal{D}-1}{\mathcal{D}} < 1, & \mathcal{D}\text{-dim internal volume}\, V_{\mathcal{D}} \\ \geq \frac{23}{18}, & \text{Kähler modulus in Heterotic M-theory} \end{cases}$$

$$\Lambda = \begin{cases} \frac{D-10}{3\alpha'}, & \text{Non-critical string} \\ \text{stack of NS5-branes}, & \text{Little String Theory (LST)} \\ \text{c.c. in higher dimension}, & \mathcal{D}+5 \text{ dim theory} \\ \text{4-form flux}, & \text{Heterotic M-theory} \end{cases}$$

Background solution

$$\mathcal{S} = \int d^5x \sqrt{-g}\, e^S \left(\frac{1}{2}\mathcal{R} + \frac{Z_S}{2}(\partial_M S)^2 - \Lambda \right)$$

Dilaton field $S = k_s y$ "Linear dilaton" $k_s = \sqrt{\dfrac{-2Z_S\Lambda}{(4-3Z_S)(5-Z_S)}}$

Metric in the Einstein frame

$$ds_E^2 = e^{\frac{2}{3}S} ds_J^2 = e^{\frac{2}{3}k_s y}\left(e^{2py}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2\right)$$
$$\equiv e^{2k_1 y}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2k_2 y}dy^2$$

AdS_5 in the Jordan frame

$$\frac{p}{k_s} = 1 - Z_S \quad \left(\Leftrightarrow \frac{k_1}{k_2} = 4 - 3Z_S \right)$$

$Z_S = 1$: Linear Dilaton model (LD)

$Z_S \neq 1$: "General" Linear Dilaton model (GLD)

Choi, SHI, Shin '17

Background solution

Metric in the Einstein frame

$$ds_E^2 = e^{2k_1 y}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2k_2 y}dy^2$$
$$= (k_2 z + 1)^{2q}\eta_{\mu\nu}dx^\mu dx^\nu + dz^2$$

$$dz \equiv e^{k_2 y}dy$$

where

$$k_2 = \frac{1}{3}\sqrt{\frac{-2\Lambda}{(4-3Z_S)(5-Z_S)}} \qquad q = \frac{k_1}{k_2} = 4 - 3Z_S$$

The General Linear Dilaton model (GLD) gives rise to a power-law warped extra dimension with a power q determined by the dilaton wave function normalization.

Power-law warping solution to the hierarchy problem

$$ds^2 = (kz+1)^{2q}\eta_{\mu\nu}dx^\mu dx^\nu + dz^2 \qquad z \in [0, L]$$

Effective 4D Planck scale $\quad M_P^2 \sim M_5^3 \underbrace{(kL)^{2q}}_{\text{power-law warping}} \underbrace{L}_{\text{size of the extra dimension}}$

For $k \sim M_5$

$$\Rightarrow \quad M_5 \sim \left(\frac{M_P^2}{L^{1+2q}}\right)^{1/(3+2q)} \sim \text{TeV} \quad \text{for} \quad L \sim 10^{\frac{30}{1+2q}-17} \text{ cm}$$

The hierarchy problem can be solved by an intermediate size of the extra dimension (e.g. ~ 1 nm for $q = 1$).

KK graviton spectrum and couplings

$$ds^2 = (kz+1)^{2q}\eta_{\mu\nu}dx^\mu dx^\nu + dz^2 \qquad \Delta\mathcal{L}_{\rm int} = c_n h^{(n)}_{\mu\nu} T^{\mu\nu}_{\rm SM}$$

$q > 1$ $\quad m_n \sim nk, \quad c_n \sim \frac{1}{M_5} \quad n = 1, 2, 3, \cdots$

$q = 1$ $\quad m_n \sim k\sqrt{1+\left(\frac{n\pi}{\ln(M_P/M_5)}\right)^2}, \quad c_n \sim \frac{1}{M_5}$

RS-like

$q < 1$ $\quad m_n \sim nk\left(\frac{M_5}{M_P}\right)^{\frac{2(1-q)}{1+2q}}, \quad c_n \sim \frac{1}{M_P} n^{\frac{3q}{2(1-q)}}$

Similar to ADD with $N = \frac{1+2q}{1-q}$

but having larger couplings

ADD with
N extra dimensions $\quad m_{n_1\ldots n_N} \sim \sqrt{n_1^2+\cdots+n_N^2}\, M_{4+N}\left(\frac{M_{4+N}}{M_P}\right)^{2/N}, \quad c_{n_1\ldots n_N} = \frac{1}{M_P}$

Characteristic features of KK graviton phenomenology

$$ds^2 = (kz+1)^{2q}\eta_{\mu\nu}dx^\mu dx^\nu + dz^2$$

- RS-like scenarios ($q \geq 1$) : heavy KK gravitons with sizable couplings (determined by $M_5 >$ TeV)

  ➡ Visible KK gravitons at colliders (i.e. short-lived)

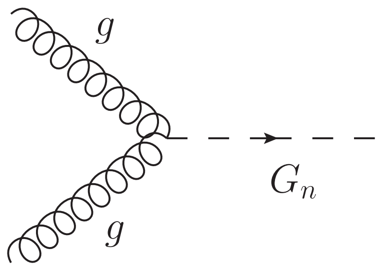
- ADD model : light & heavy KK gravitons with the small coupling $1/M_P$

  ➡ Invisible KK gravitons at colliders (i.e. long-lived), strong astrophysical limits

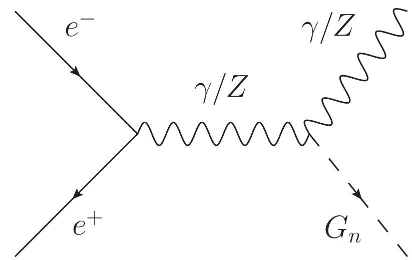
- Scenarios with $0 < q < 1$: light & heavy KK gravitons with couplings growing with KK graviton mass

  ➡ Visible KK gravitons with a small mass gap at colliders, strong or moderate astrophysical limits

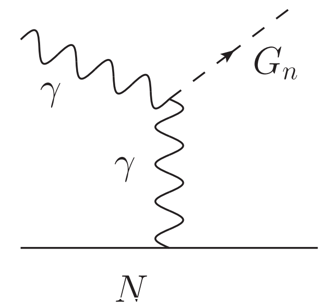
KK gravitons in colliders and astrophysical sources



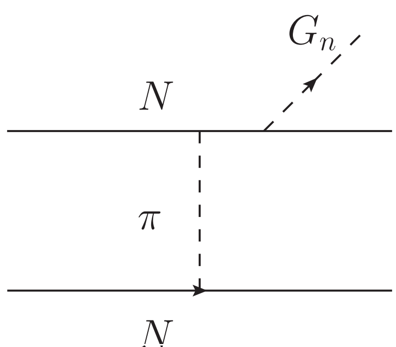
Hadron colliders
(LHC)



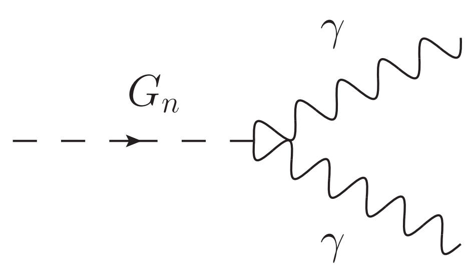
Lepton colliders
(LEP, FCC-ee, CLIC)



Proton beam dumps
(SHiP, NA62, DUNE, …)



Neutron stars and
Supernovae



or long-lived

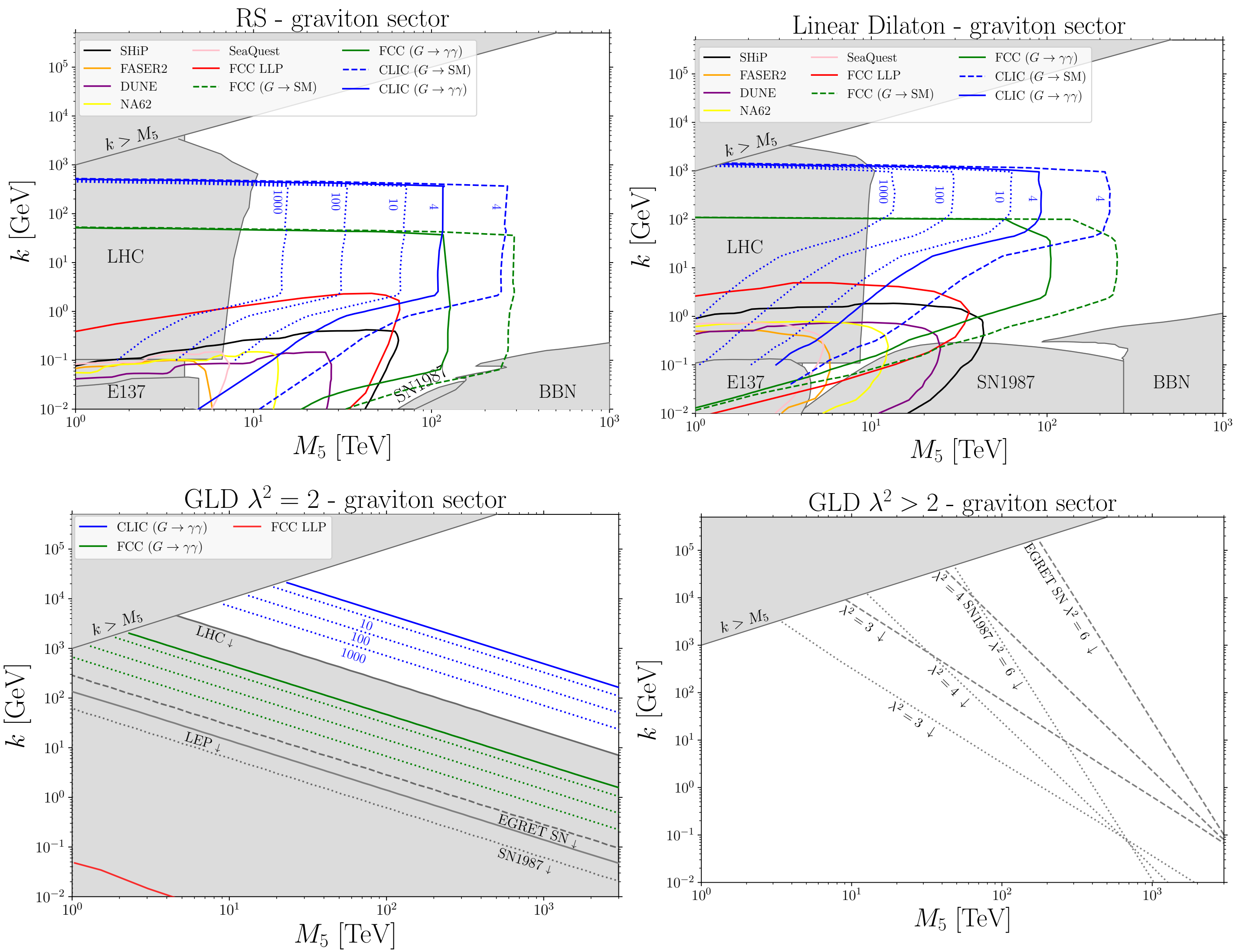
Decay channels

$q \to \infty$

$q = 1$

$q = \frac{1}{2}$

$q < \frac{1}{3}$



Future colliders such as CLIC can search for a power-law warped extra dimension by signatures from densely packed visible KK gravitons.

Conclusions

- Power-law warped extra dimensions are common in string theory, realized by a dilaton field propagating in extra dimensions and the *classical scale invariance*.
- Their phenomenology has never been seriously studied so far except the power $q = 1$ (LD model).
- A power-law warped extra dimension can address the hierarchy problem with an intermediate size of the extra dimension (~ nm).
- The associated KK graviton spectrum and couplings show a distinctive pattern compared with the conventional models such as ADD, RS, and LD.
- Future colliders can test a power-law warped extra dimension by visible KK graviton signatures with a very small mass gap.