“Hairy Black Holes by Spontaneous Symmetry Breaking in ESGB theory"

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7th Cubes Workshop "Chirality in the universe beyond the electroweak scale"

The-K Jirisan Family Hotel, Gurye-gun, S. Korea

April 26-29, 2025

Content

1. Motivation of Modified Gravity
2. EsGB Theory
   - Evasion of No-hair theorem
3. Formation mechanism of Hairy Black Holes
   - Hairy black holes by Spontaneous Scalarization
   - Hairy black holes by Spontaneous Symmetry Breaking (SSB)
4. Scalar Field Perturbation
   - Quasinormal modes (QNM)
5. Future works

In this talk

“Rectifying no-hair theorems in Gauss-Bonnet theory"
Alexandros Papageorgiou, Chan Park, Miok Park, Phys.Rev.D 106 (2022) 8, 084024 (arXiv:2205.00907)
$\longrightarrow$ Evasion of the no-hair theorem

“Hairy Black Holes by Spontaneous Symmetry Breaking"
Boris Latosh, Miok Park, Phys.Rev.D 110 (2024) 2, 024012 (arXiv:2305.19814)
$\longrightarrow$ Formation mechanism of hairy black holes

“Scalar Field Perturbation of Hairy Black Holes in EsGB theory"
Young-Hwan Hyun, Boris Latosh, Miok Park, JHEP 08 (2024) 163 (arXiv:2405.08769)
$\longrightarrow$ (In)stability of hairy black holes

Motivation to study modified gravity

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In this talk, I will consider **Einstein-Scalar-Gauss-Bonnet Theory (ESGB)**.

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{R}{2\kappa^2} - \frac{1}{2}\nabla_\alpha\varphi\nabla^\alpha\varphi + f(\varphi)\mathcal{G}\right],$$

where $\mathcal{G}$ is the GB term

$$\mathcal{G} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2,$$

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- Einstein theory holds the no-hair theorems, which are proved by J. Bekenstein in 1972 and 1995.
- ESGB theory evades **the no-hair theorems** which indicates the existence of hairy black holes.

Evasion of (old/novel) No-Hair theorem

Conflict results in the no-hair theorems

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{R}{2\kappa^2} - \frac{1}{2}\nabla_\alpha\varphi\nabla^\alpha\varphi + f(\varphi)\mathcal{G}\right]$$

Evasion of No-Hair Theorems and Novel Black-Hole Solutions in Gauss-Bonnet Theories #1

G. Antoniou (Ioannina U.), A. Bakopoulos (Ioannina U.), P. Kanti (Ioannina U.) (Nov 9, 2017)

Published in: *Phys.Rev.Lett.* 120 (2018) 13, 131102 • e-Print: 1711.03390 [hep-th]

pdf DOI cite claim reference search 475 citations

Evaded only when $f(\varphi) > 0$

Spontaneous scalarization of black holes and compact stars from a Gauss-Bonnet coupling #1

Hector O. Silva (Mississippi U. and Montana State U.), Jeremy Sakstein (Pennsylvania U.), Leonardo Gualtieri (Rome U. and INFN, Rome), Thomas P. Sotiriou (Nottingham U.), Emanuele Berti (Mississippi U.) (Nov 6, 2017)

Published in: *Phys.Rev.Lett.* 120 (2018) 13, 131104 • e-Print: 1711.02080 [gr-qc]

pdf DOI cite claim reference search 539 citations

Evaded only when $f(\varphi) > 0$

Expanded evasion of the black hole no-hair theorem in dilatonic Einstein-Gauss-Bonnet theory #1

Bum-Hoon Lee (Sogang U. and CQUeST, Seoul), Wonwoo Lee (CQUeST, Seoul), Daeho Ro (APCTP, Pohang) (Sep 15, 2018)

Published in: *Phys.Rev.D* 99 (2019) 2, 024002 • e-Print: 1809.05653 [gr-qc]

pdf DOI cite claim reference search 29 citations

Evaded when $f(\varphi) > 0$ & $f(\varphi) < 0$

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- We found that *there were critical errors in the previous studies.*
- We corrected them and restored the reliability of the no-hair theorems in EsGB theory.

Hairy Black holes

How these hairy black holes are formed?[1]

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- black hole can spontaneously develop a scalar hair via a tachyonic instability.

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- However, it is known that hairy black holes for the case of $f(\varphi) = \alpha\,\varphi^2$ are dynamically unstable under the radial perturbation of scalar fields. [2]
- This implies that this hairy black hole may not be the final stage in the evolution of a non-hairy black hole.

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We suggest the formation of hairy black holes by spontaneous symmetry breaking [3]

[3] Boris Latosh, Miok Park, Phys.Rev.D 110 (2024) 2, 024012, “Hairy black holes by spontaneous symmetry breaking"

Our Lagrangian

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \nabla_\alpha\varphi^*\nabla^\alpha\varphi + f(\varphi)\mathcal{G}\right], \tag{1}$$

$$\mathcal{L}_\varphi = -\nabla_\alpha\varphi^*\nabla^\alpha\varphi + f(\varphi)\mathcal{G} = T - V, \qquad V = -f(\varphi)\mathcal{G} \tag{2}$$

where $\mathcal{G}$ is the Gauss-Bonnet term and the scalar field coupling function is

$$f(\varphi) = \alpha\,\varphi^*(r)\varphi(r) - \lambda\left(\varphi^*(r)\varphi(r)\right)^2, \qquad (\lambda > 0)$$

Our metric ansatz is

$$\mathrm{d}s^2 = -A(r)\mathrm{d}t^2 + \frac{1}{B(r)}\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right). \tag{3}$$

- This Lagrangian respects the global $U(1)$ symmetry

$$\varphi(r) \to e^{i\chi}\varphi(r)$$

- This action allows to have Schwarzschild black holes ($\varphi = const$).

Schwarzschild black holes are stable?

Scalar field perturbation

$$\delta\varphi_1(t,r,\theta,\phi) = \sum_{l,m} \frac{\Phi(r)Y_{lm}(\theta,\phi)}{r}\, e^{-i\omega t} \tag{4}$$

$$\Phi''(r_*) - (V_{\rm eff} - \omega^2)\Phi(r_*) = 0, \qquad \mathrm{d}r_* = \frac{1}{\sqrt{AB}}\mathrm{d}r, \tag{5}$$

$$V_{\rm eff}(r) = \frac{l(l+1)A}{r^2} + \frac{1}{2\,r}\left(A'B + AB'\right) - \frac{1}{2} f_{\varphi_1\varphi_1} A\,\mathcal{G}, \tag{6}$$

where l is the angular momentum.

$$\int_{r_h}^{\infty} dr\, \frac{1}{\sqrt{AB}}\, V_{\rm eff}(r) < 0 \qquad \rightarrow \qquad \alpha > \alpha_{\rm Sch.} = \frac{5}{24} \approx 0.2083, \tag{7}$$

$$A(r) = B(r) = 1 - \frac{2M}{r}, \qquad l = 0 \tag{8}$$

Schwarzschild black holes in EsGB

become unstable beyond the critical value of α $(= \alpha_{\rm Sch.})$

Schwarzschild black holes are stable? Not always

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Hairy Black Holes by SSB

$$V = -f(\varphi)\mathcal{G},$$
$$f(\varphi) = \alpha\,\varphi^*(r)\varphi(r) - \lambda\left(\varphi^*(r)\varphi(r)\right)^2, \qquad (\lambda > 0)$$

- Black holes in the symmetric phase :

$$\langle\varphi\rangle = 0, \qquad \varphi(r) = \frac{1}{\sqrt{2}}\left(\varphi_1(r) + i\,\varphi_2(r)\right) \tag{9}$$

- Black holes in the symmetry-broken phase : the stable minima are determined by

$$\langle\varphi\rangle = v\,e^{i\beta}, \qquad v = \sqrt{\frac{\alpha}{2\,\lambda}} \tag{10}$$

we expand a field around the ground state v by reparameterizing it as follows

$$\varphi(r) = \left(v + \frac{\sigma(r)}{\sqrt{2}}\right)e^{i\theta(r)} \tag{11}$$

Flux near the black hole horizon [4]

The conserved current is given by

$$\partial_\alpha J^\alpha = 0, \qquad J_\alpha = i\, g\, (\varphi^* \partial_\alpha \varphi - \varphi \partial_\alpha \varphi^*).$$

- In the symmetric phase :

$$\int_\Sigma J_\alpha n^\alpha \sqrt{-h}\, \mathrm{d}^3 y = \int_\Sigma \Big[g(\varphi_2 \partial_r \varphi_1 - \varphi_1 \partial_r \varphi_2) \Big] \Big[\sqrt{A(r)B(r)}\, r^2 \sin\theta\, \mathrm{d}\theta\, \mathrm{d}\phi\, \mathrm{d}t \Big] = 0 \quad (12)$$

- In the symmetry-broken phase :

$$\theta'(r) = \frac{c_2}{4r^2 \sqrt{A(r)B(r)}} \left(v + \frac{\sigma(r)}{\sqrt{2}} \right)^{-2}, \quad (13)$$

$$\begin{aligned} \int_\Sigma J_\alpha n^\alpha \sqrt{-h}\, \mathrm{d}^3 y &= \int_\Sigma \left[-2\, g \left(v + \frac{\sigma(r)}{\sqrt{2}} \right)^2 \theta'(r) \right] \left[\sqrt{A(r)B(r)}\, r^2 \sin\theta\, \mathrm{d}\theta\, \mathrm{d}\phi\, \mathrm{d}t \right] \\ &= -8\pi\, g\, c_2 \end{aligned} \quad (14)$$

$c_2 = 0$ is required. The Goldstone boson is trivial.

[4] special thanks to Prof. Seong Chan Park

Hairy Black Holes by SSB in EsGB with global $U(1)$

We are interested in the situation that

“Scalar fields are about to grow from black holes without scalar hair. Finally it evolves to hairy black holes."

ex. Schwarzschild black hole $\longrightarrow$ *Hairy Black holes*

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- "$V = -f(\varphi)\mathcal{G}$" as an "interacting potential" :
  - effective near the black hole horizon
  - not effective at infinity ($V \to 0$ as $r \to \infty$, since $\mathcal{G} \to 0$ as $r \to \infty$)

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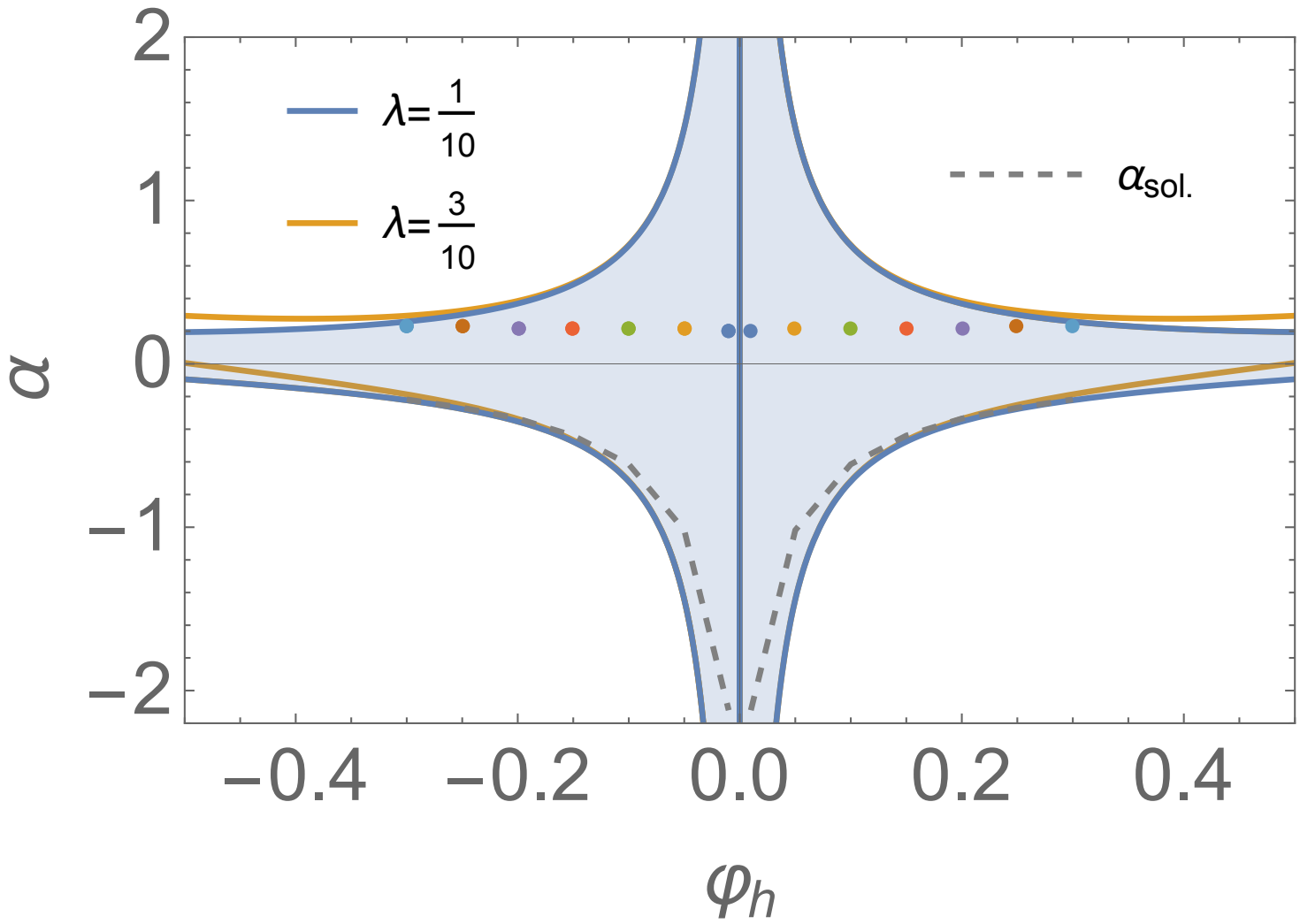
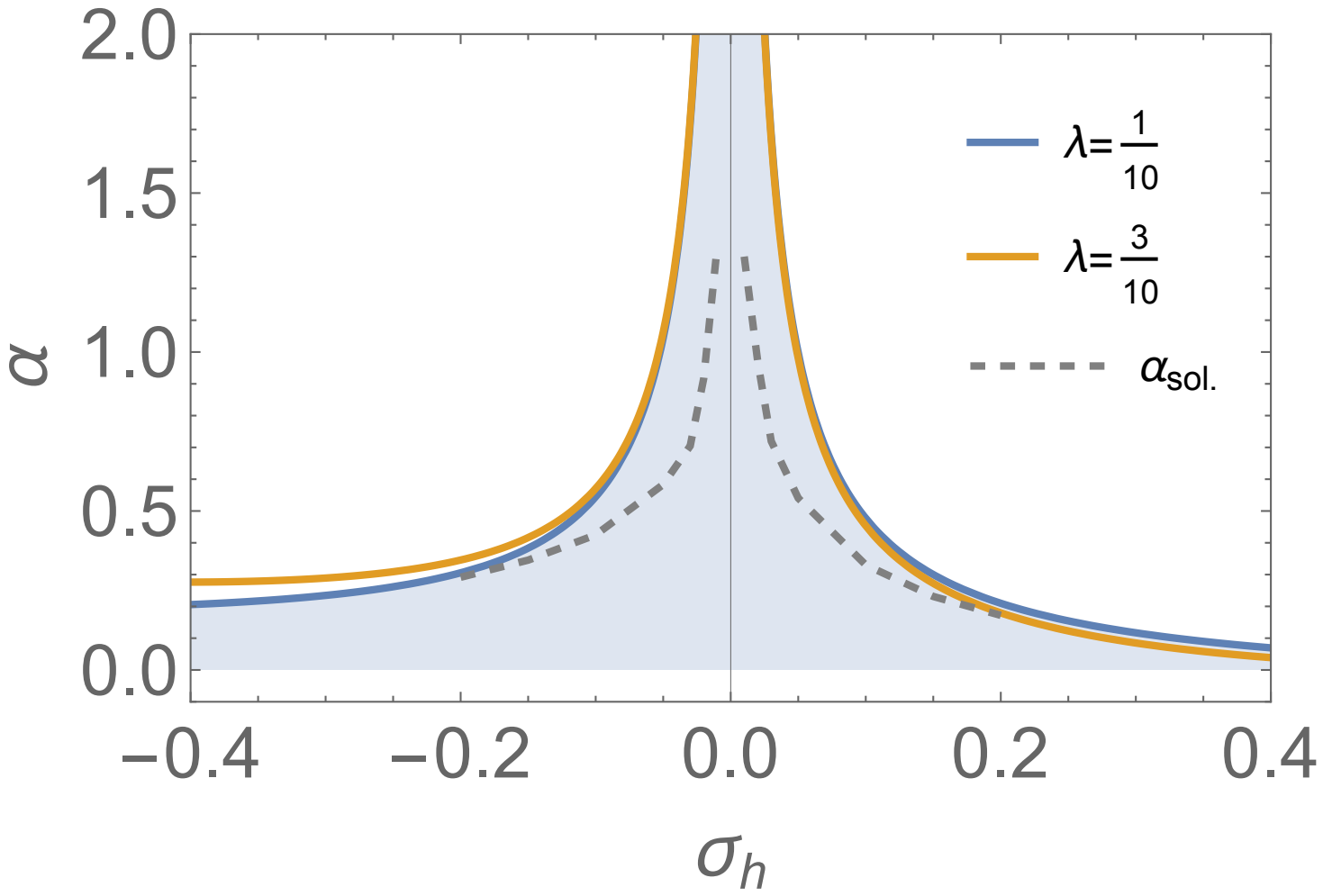
Based on these assumption,

we numerically generated the hairy black holes in symmetric and symmetry-broken phase

Hairy black holes in *symmetric phase*



Hairy black holes in *symmetry broken phase*



Hairy black hole with local $U(1)$

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2\,\kappa^2}R - \frac{1}{4}F^2 - D_\alpha\varphi^* D^\alpha\varphi + f(\varphi^*,\varphi)\,\mathcal{G}\right], \tag{15}$$

where $F = dP$ and $D_\alpha = \nabla_\alpha - iqP_\alpha$. This action is invariant under local $U(1)$ symmetry

$$\varphi \to \varphi\, e^{i\chi(r)} \tag{16}$$

where $\chi(r)$ is an arbitrary function.

$(q = 0)$

- we found that hairy black holes with mass and electric charge but electrically neutral scalar hair.

$(q \neq 0)$

- Asymptotic expansions $(1/r)$ of the gauge and scalar fields give $P_\infty = P_1 = 0$ or $\varphi_\infty = \varphi_1 = 0$.
- In our numerical calculation, we could not find any hairy black hole solutions with electrically charged scalar hair.
- These two findings strongly suggest the absence of charged hairy black holes in our theory.
- Thus, this theory might not realize SSB associated with local U(1) symmetry.

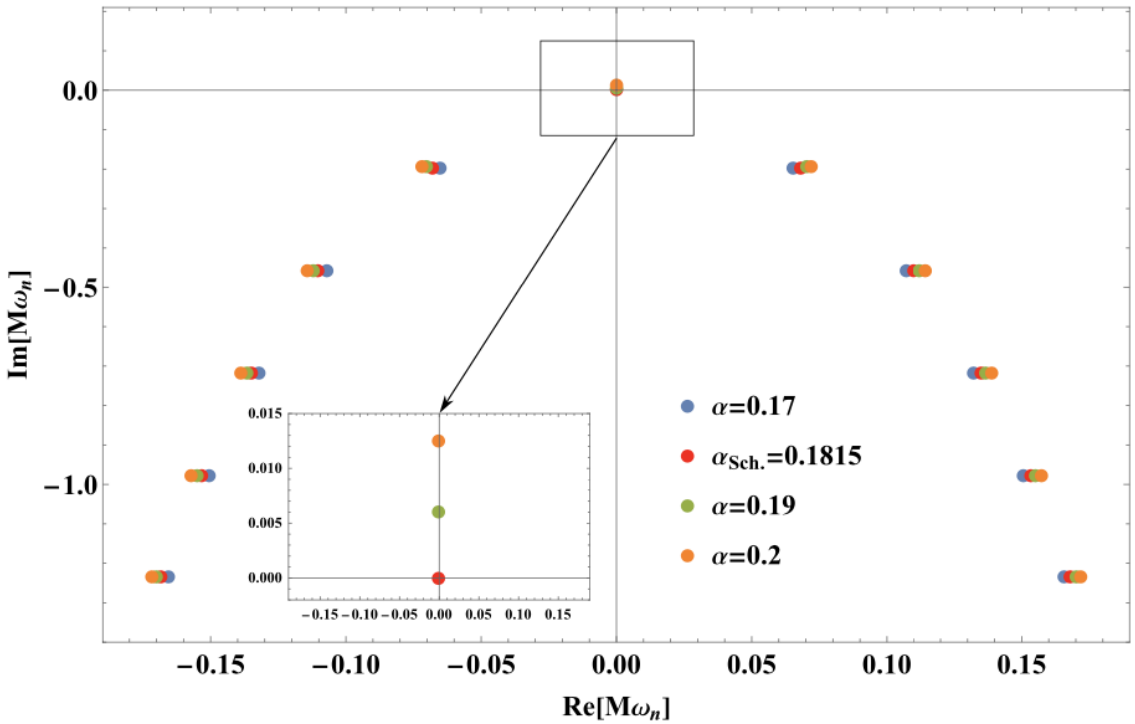
Hairy black holes + Scalar field perturbation

$\rightarrow$ **Quasinormal modes (QNM)** [5]

[5] Young-Hwan Hyun, Boris Latosh, Miok Park, JHEP 08 (2024) 163, “Scalar field perturbation of hairy black holes in EsGB theory"

G_{QNM} : Schwarzschild black holes

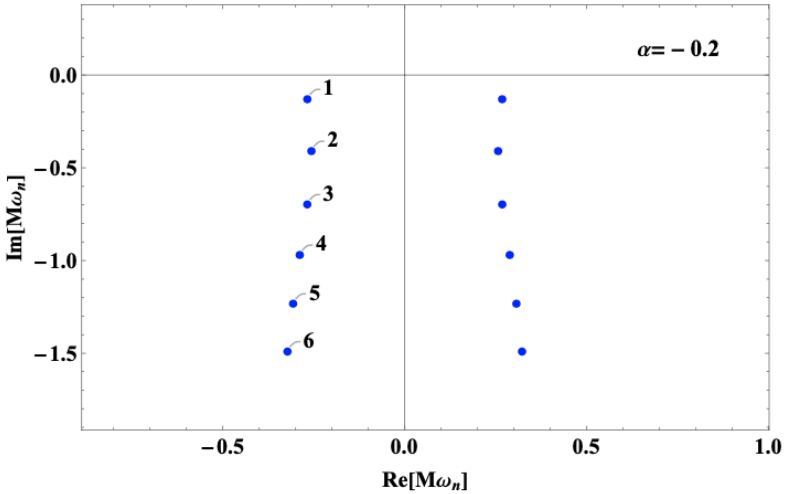
$$\delta\varphi(v,z,\theta,\phi) = \sum_{l,m} \Phi(z) Y_{lm}(\theta,\phi) e^{-i\omega v}, \quad \omega = \omega_{\mathrm{R}} + i\omega_{\mathrm{I}}$$

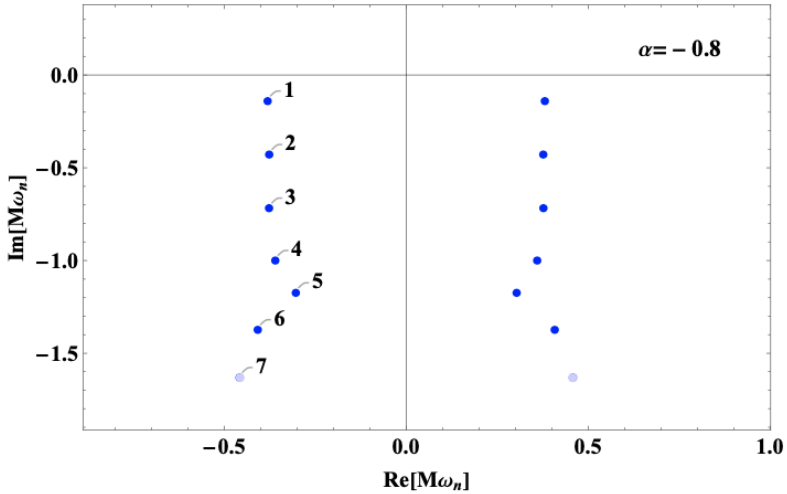


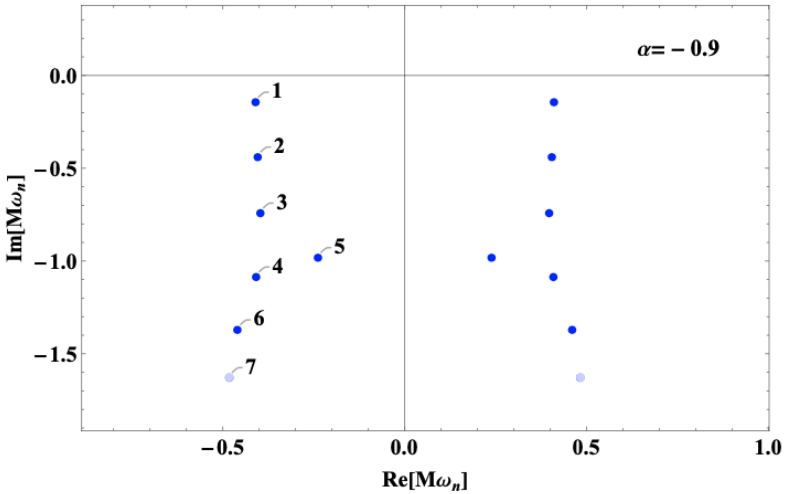
	$\mathrm{Re}[M\omega_n]$	$\mathrm{Im}[M\omega_n]$
α=0.17	± 0.06528	-0.19795
	± 0.10718	-0.45845
	± 0.13196	-0.71851
	± 0.15043	0.97682
$\alpha_{\mathrm{Sch.}} \approx 0.1815$	0	0.00002
	± 0.06826	-0.19618
	± 0.11012	-0.45776
	± 0.13483	-0.71820
	± 0.15324	-0.97669
α=0.19	0	0.00600
	± 0.07013	-0.19489
	± 0.11208	-0.45728
	± 0.13677	-0.71798
	± 0.15513	-0.97662
α=0.20	0	0.01254
	± 0.07198	-0.19340
	± 0.11415	-0.45675
	± 0.13881	-0.71775
	± 0.15712	-0.97654

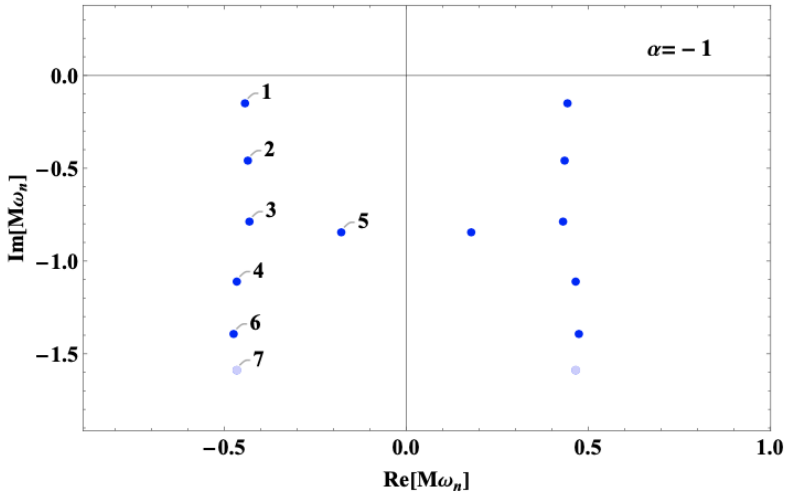
Figure 4 & Table 1: QNMs of Schwarzschild black holes for serval values α

G_{QNM} : Symmetric Phase with $\alpha < 0$

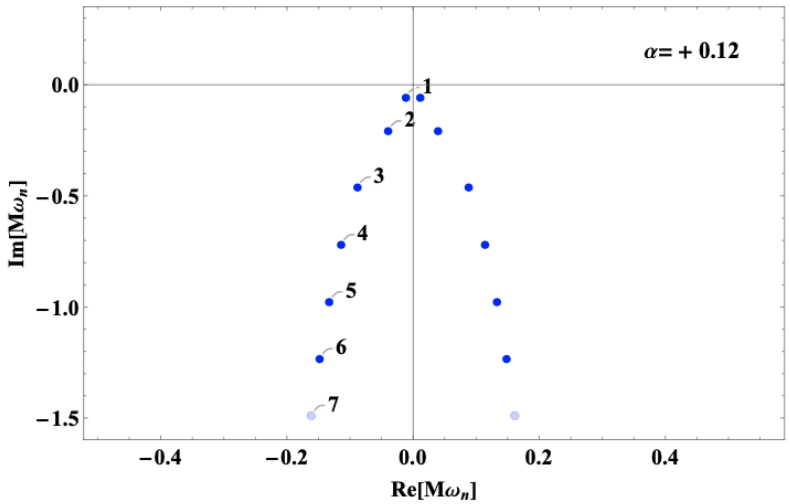


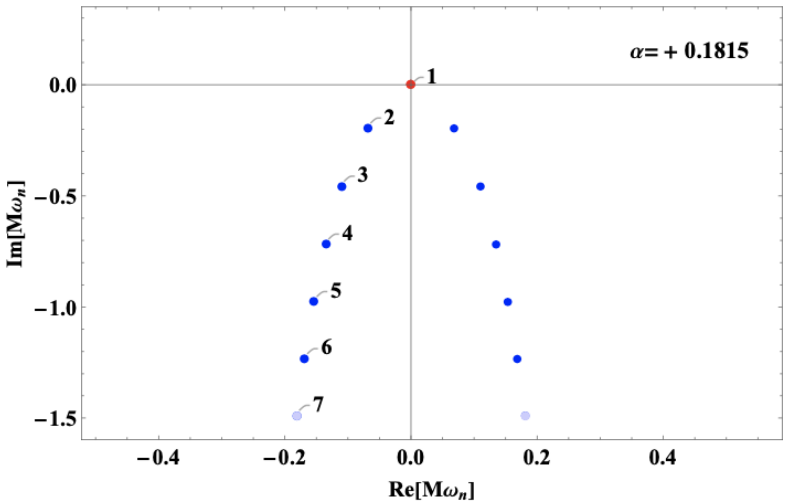


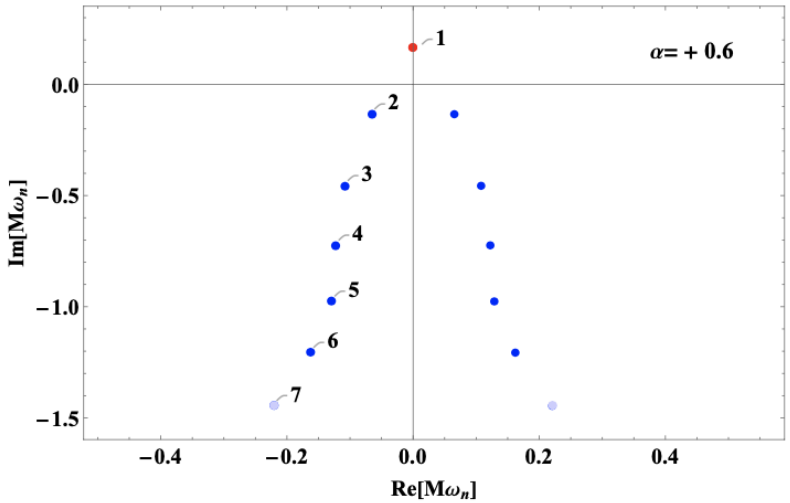


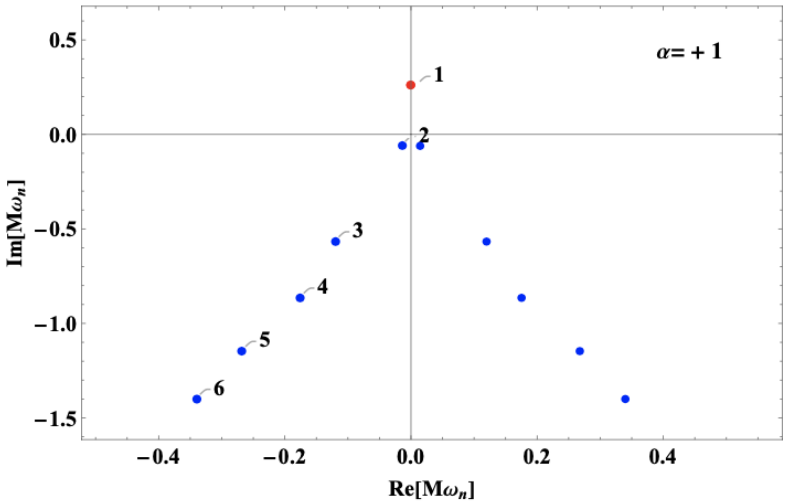


G_{QNM} : Symmetric Phase with $\alpha > 0$









G_{QNM} : Symmetric Phase with $\alpha > 0$

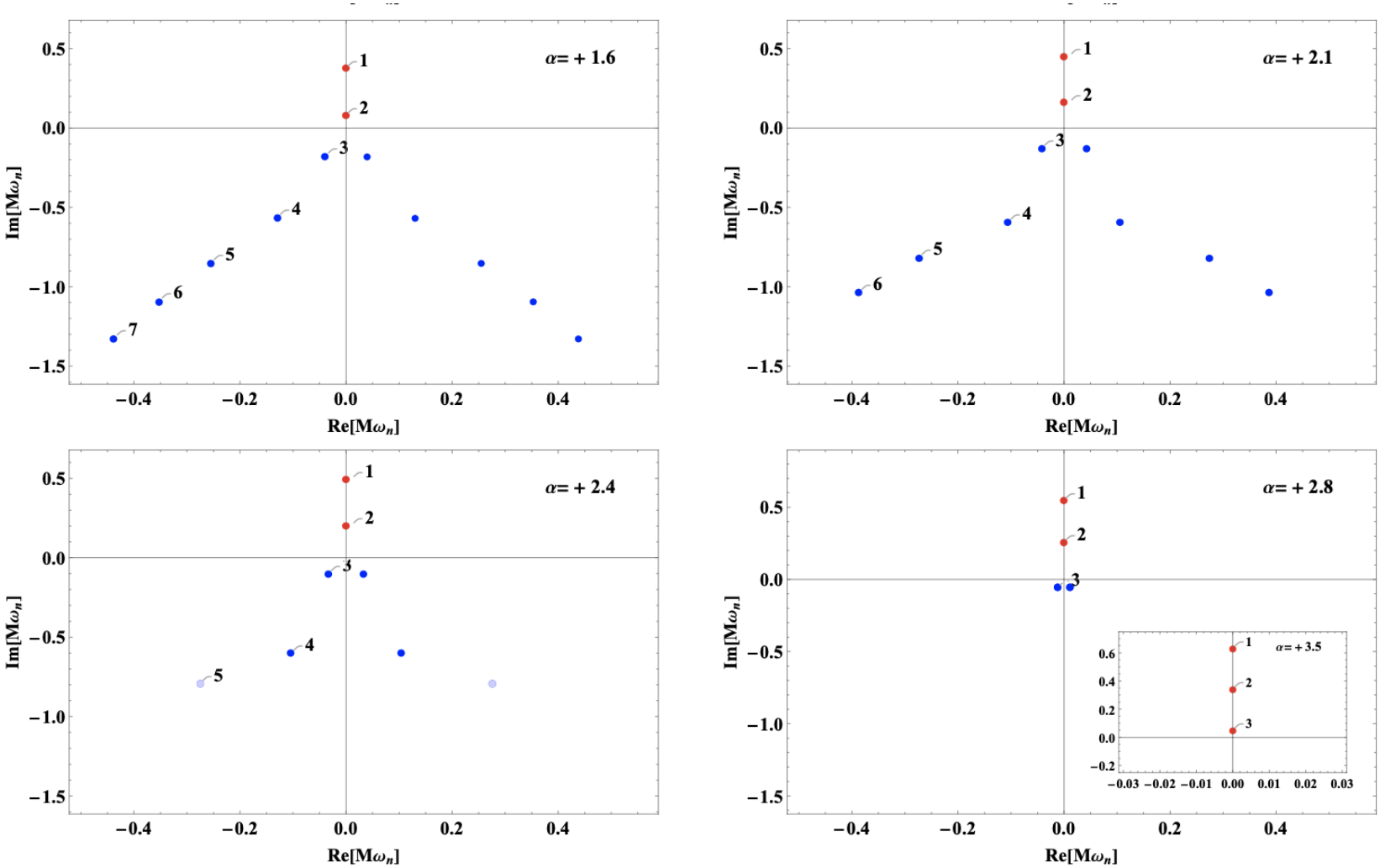
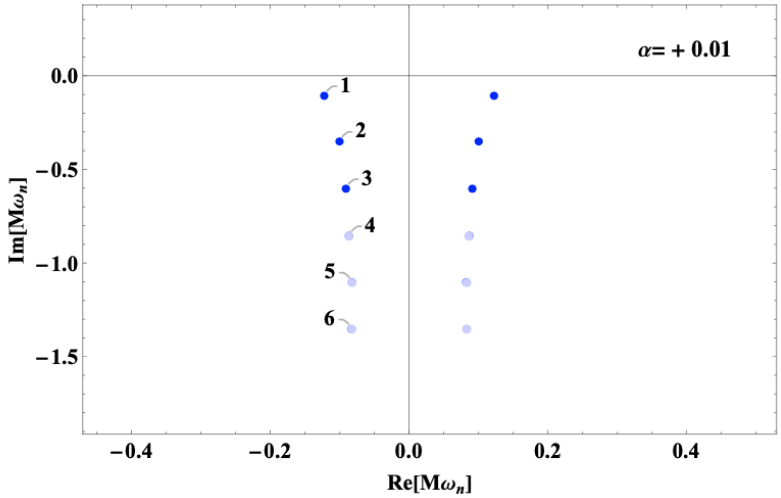
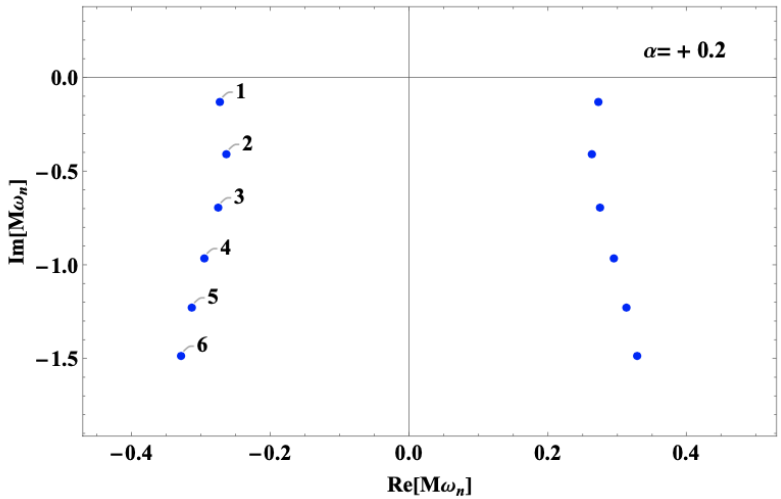


Figure 6: QNMs with positive values of α for $\varphi_h = 0.01$. Blue dots are tolerable less than 10^{-3}, whereas lighter blue dots have a tolerance of 10^{-2}.

G_{QNM} : Symmetry-Broken Phase

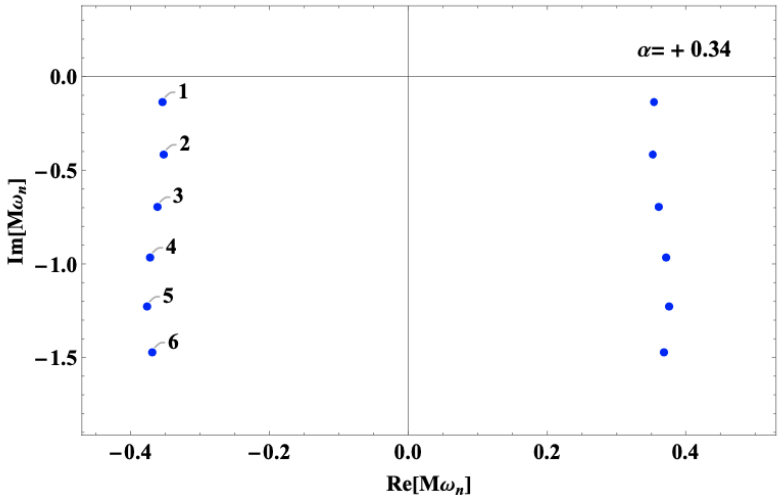


	$\mathrm{Re}[M\omega_n]$	$\mathrm{Im}[M\omega_n]$
1	± 0.12229	-0.10701
2	± 0.10025	-0.35077
3	± 0.09104	-0.60312
4	± 0.08662	-0.85448
5	± 0.08226	-1.10205
6	± 0.08295	-1.35246

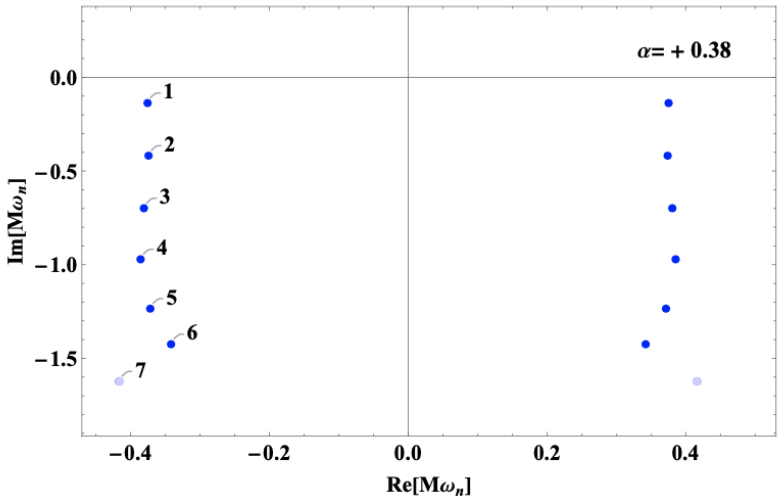


	$\mathrm{Re}[M\omega_n]$	$\mathrm{Im}[M\omega_n]$
1	± 0.27261	-0.13084
2	± 0.26325	-0.40969
3	± 0.27500	-0.69474
4	± 0.29492	-0.96587
5	± 0.31297	-1.22847
6	± 0.32848	-1.4865

G_{QNM} : Symmetry-Broken Phase



	$\mathrm{Re}[M\omega_n]$	$\mathrm{Im}[M\omega_n]$
1	± 0.35393	-0.13610
2	± 0.35217	-0.41623
3	± 0.36116	-0.69570
4	± 0.37179	-0.96582
5	± 0.37610	-1.22716
6	± 0.36866	-1.47193

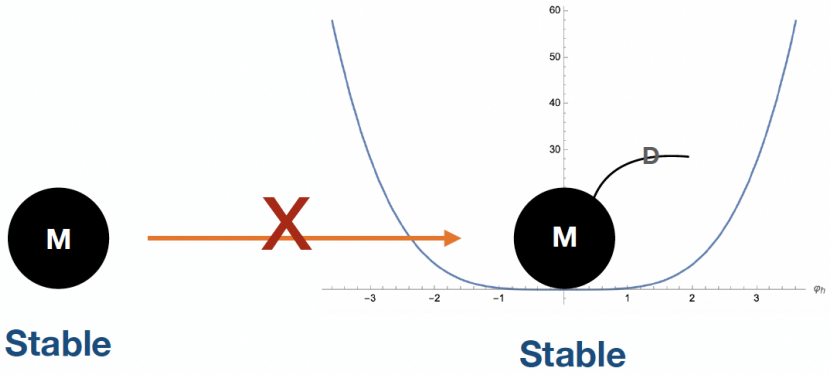


	$\mathrm{Re}[M\omega_n]$	$\mathrm{Im}[M\omega_n]$
1	± 0.37538	-0.13731
2	± 0.37399	-0.41842
3	± 0.38069	-0.69878
4	± 0.38549	-0.97137
5	± 0.37156	-1.23482
6	± 0.34159	-1.42465
7	± 0.41624	-1.62298

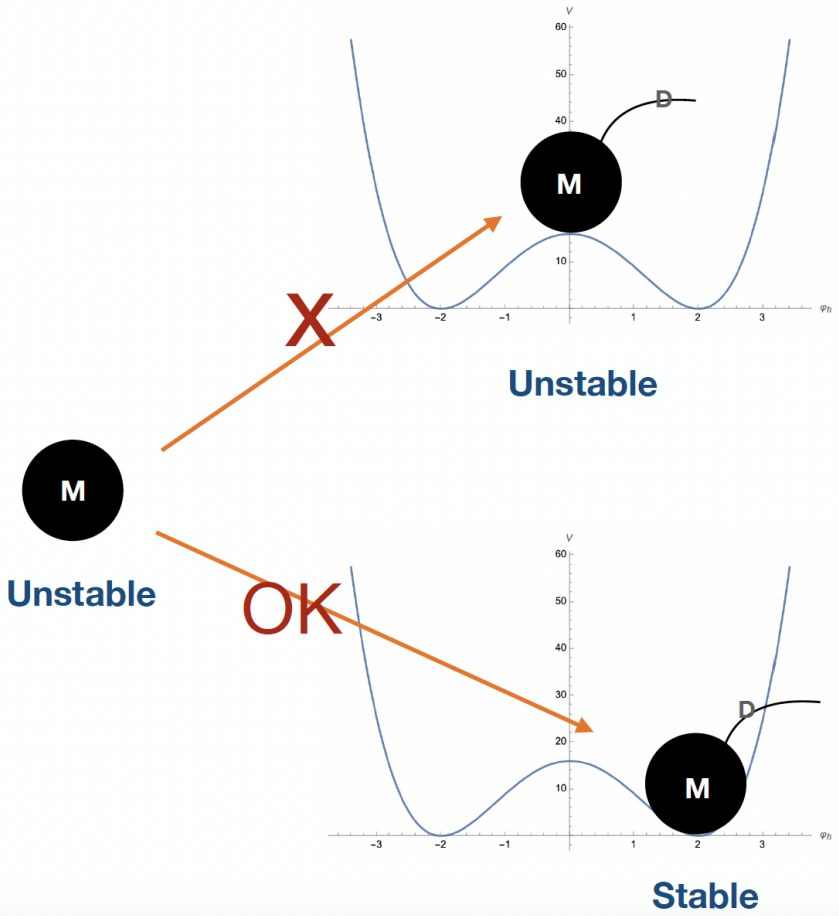
Figure 11 & Table 3: QNMs with various values of α for $\sigma_h = 0.01$. Blue dots are tolerable less than 10^{-3}, whereas lighter blue dots have a tolerance of 10^{-2}.

Summary

When $\alpha < 0$

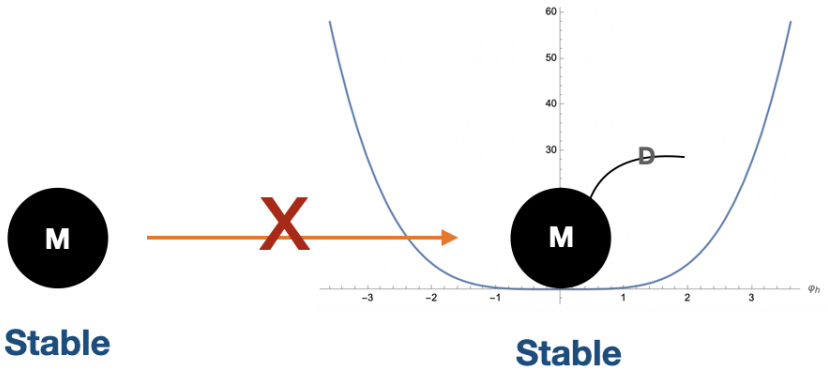


When $\alpha > 0$

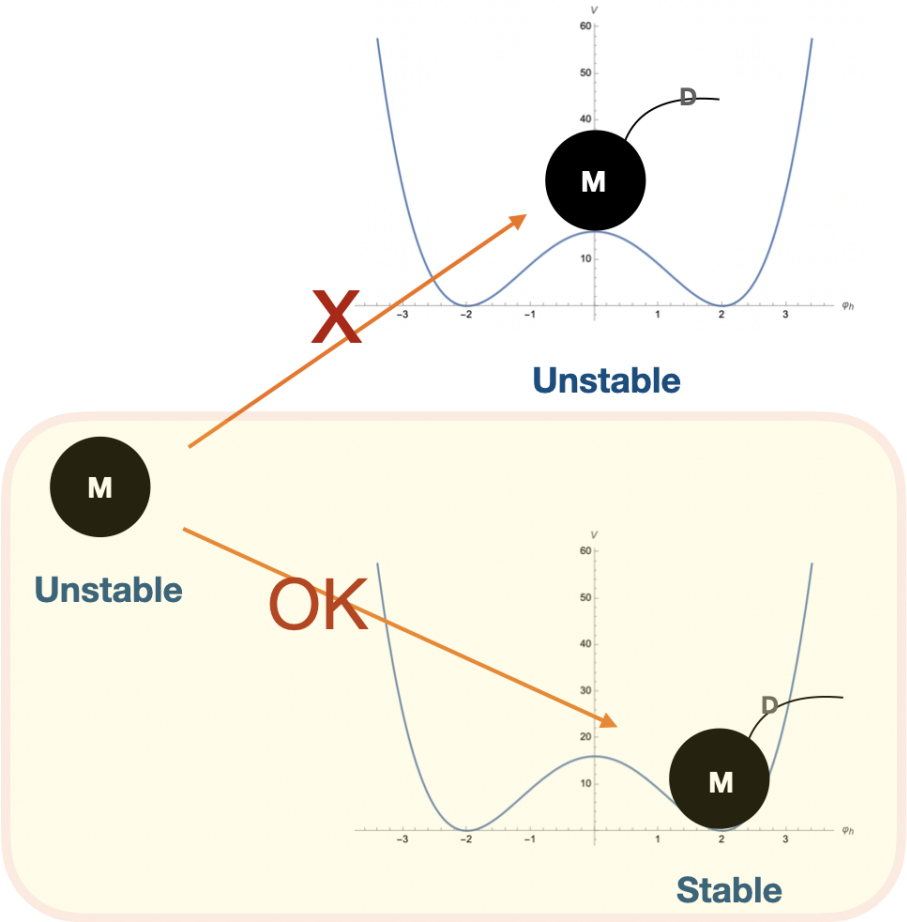


Summary

When $\alpha < 0$



When $\alpha > 0$



Our Claim :

Phase transition will occur and form black hole's hairs in symmetry-broken phase!