# Phenomenology of Rotating pNGBs:
## *Baryogenesis & Dark Matter*

*7th Cubes Workshop*

**Suruj Jyoti Das**

Institute for Basic Science

Based on:

2406.04180 (with Eung Jin Chun, Minxi He, Tae Hyun Jung, Jin Sun)

2502.08720 (with Chao Chen, Konstantinos Dimopoulos, Anish Ghoshal)

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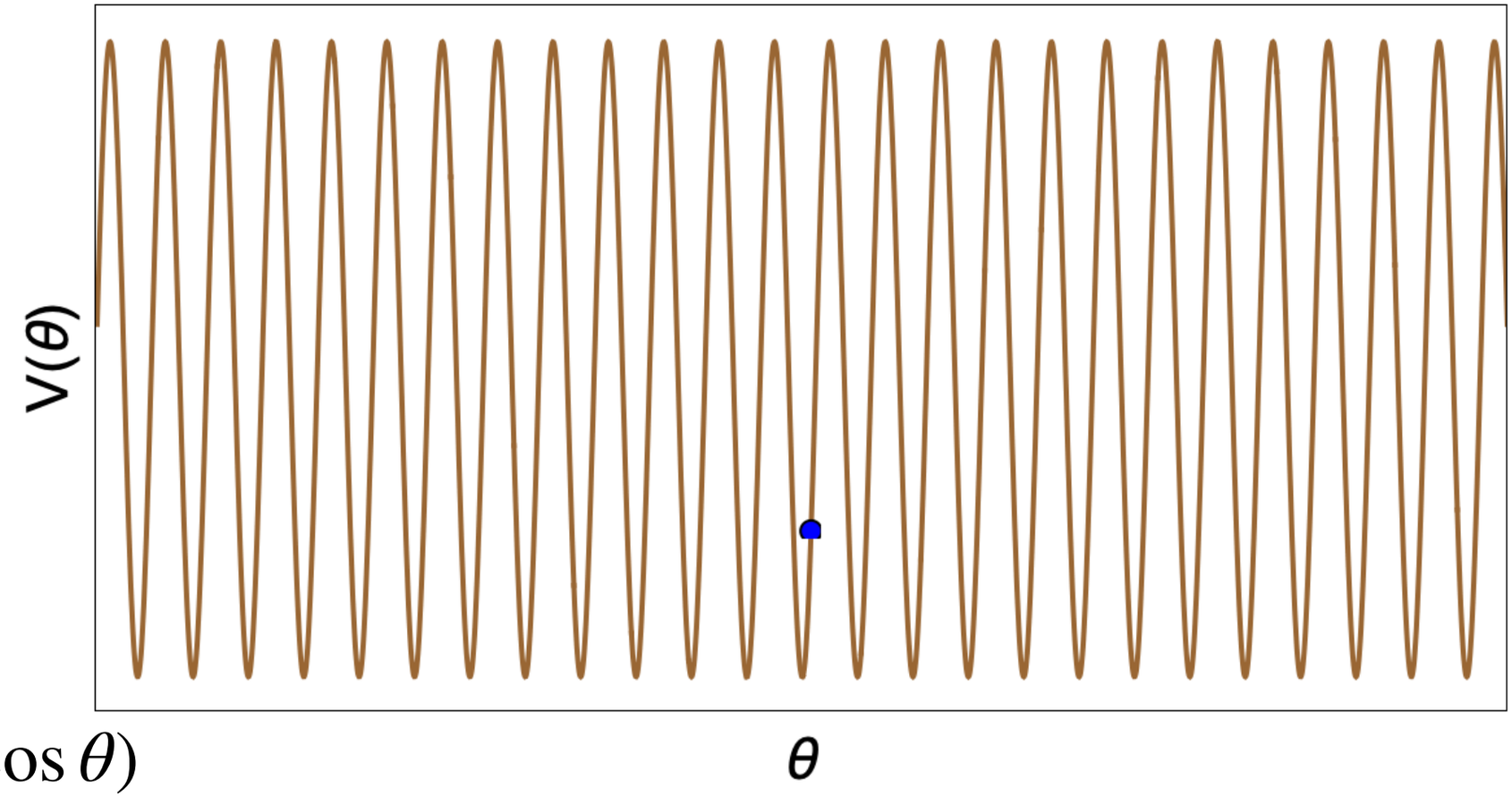
**Non-Rotating pNGBs**

$$\theta = \frac{a}{f_a}$$

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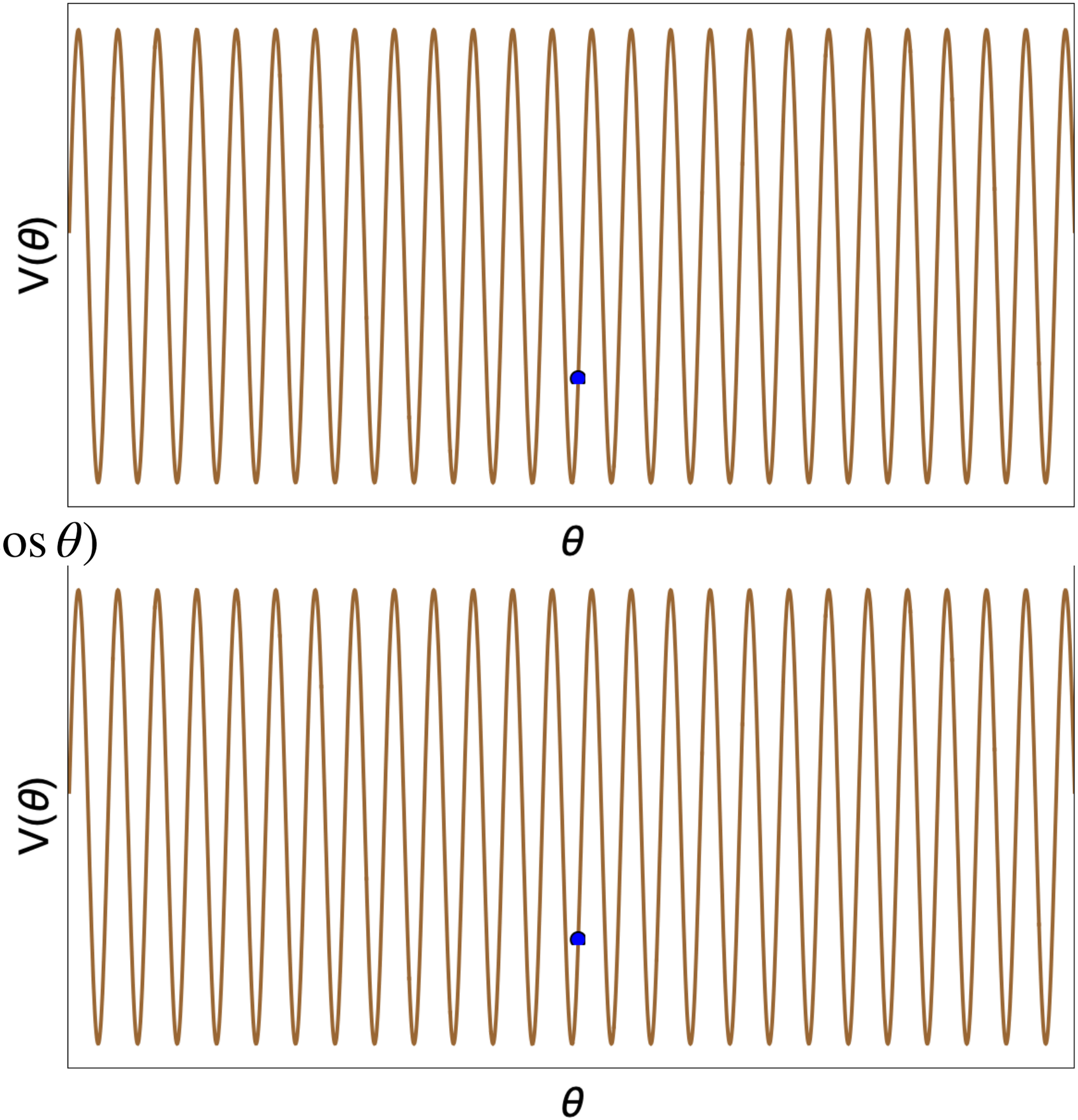


**Rotating/sliding pNGBs**

# Why Rotation ?

- Delays oscillation, more parameter space for axion experiments..
- Can source particle/antiparticle asymmetry.

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+ cogenesis of
baryon and dark matter .

# Today's talk

- Motivation.
- Introduction.
- Rotation via *Symmetry non-restoration*.
- Rotation via *Non-Minimal Coupling*.
- Remarks & Summary.

# Conditions for Baryogenesis :

Sakharov ’67

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- Background dynamics of scalar field (pNGB): spontaneously breaks CPT.

Source: $\frac{c}{f_a}(\partial_\mu a)\ J^\mu_X \qquad X = B, L \ldots \qquad \bar{\psi}\gamma^\mu\psi$

- Energy shift  of $\psi(\bar{\psi})$ by $\Delta E_{\psi(\bar{\psi})} \sim \pm\, c\dot{\theta}$

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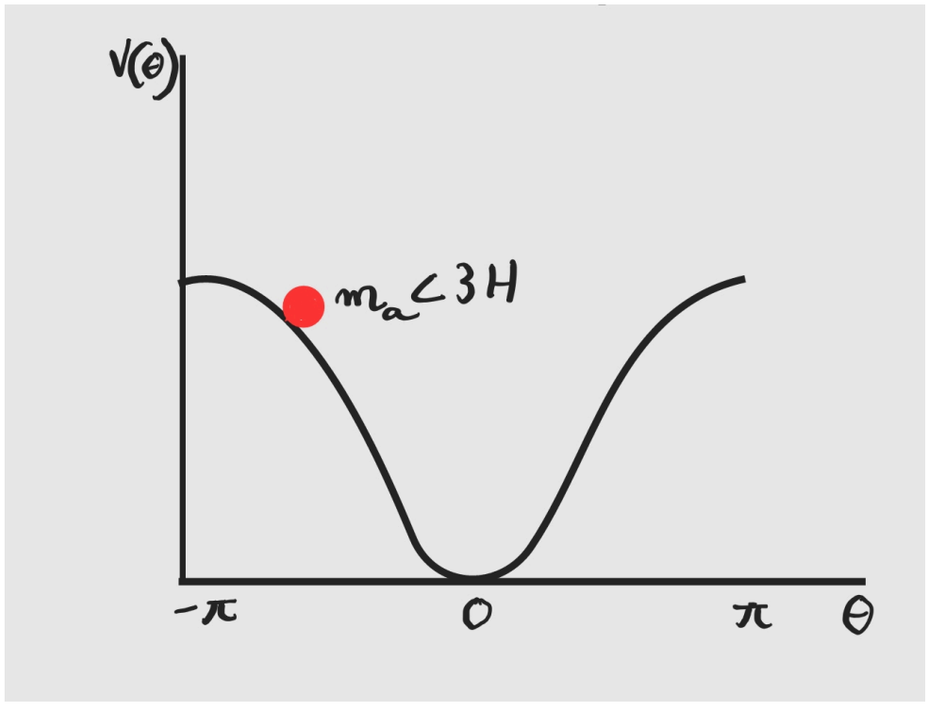
$$\frac{dn_i}{dt} = \text{Creation terms - Annihilation terms + bias terms} \rightarrow \textit{proportional to } \dot{\theta}$$

$$\text{Chemical potential} \Longrightarrow \text{Asymmetry}\ \frac{n_B}{s} \simeq \mathcal{O}(1)\frac{\dot{\theta}}{g_* T_{\mathrm{B-L}}}$$

# How to generate $\dot{\theta}$?

Recall: Conventional misalignment



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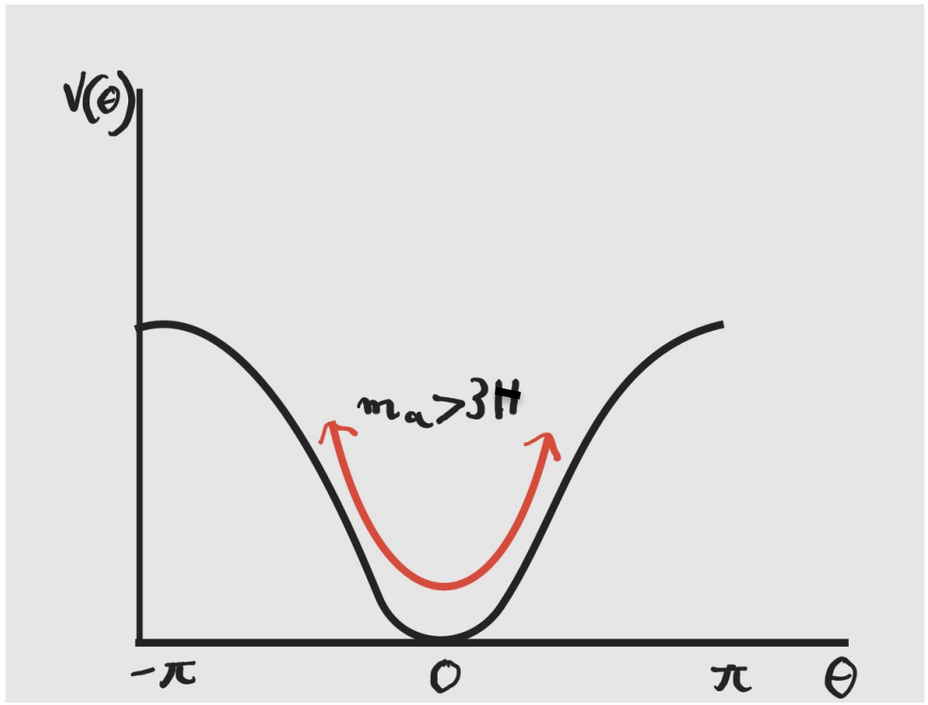
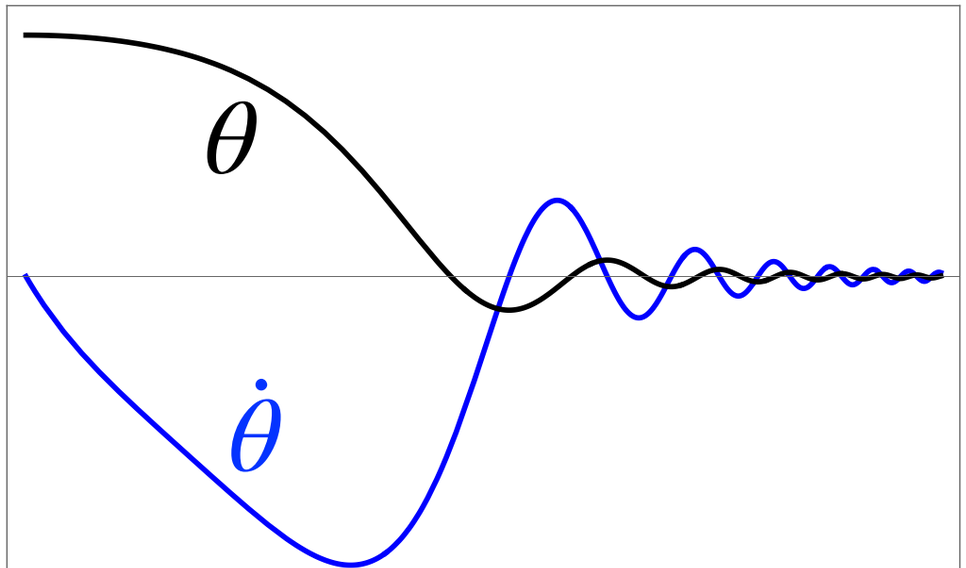



$\dot{\theta}$ = from 0 to $m_a$



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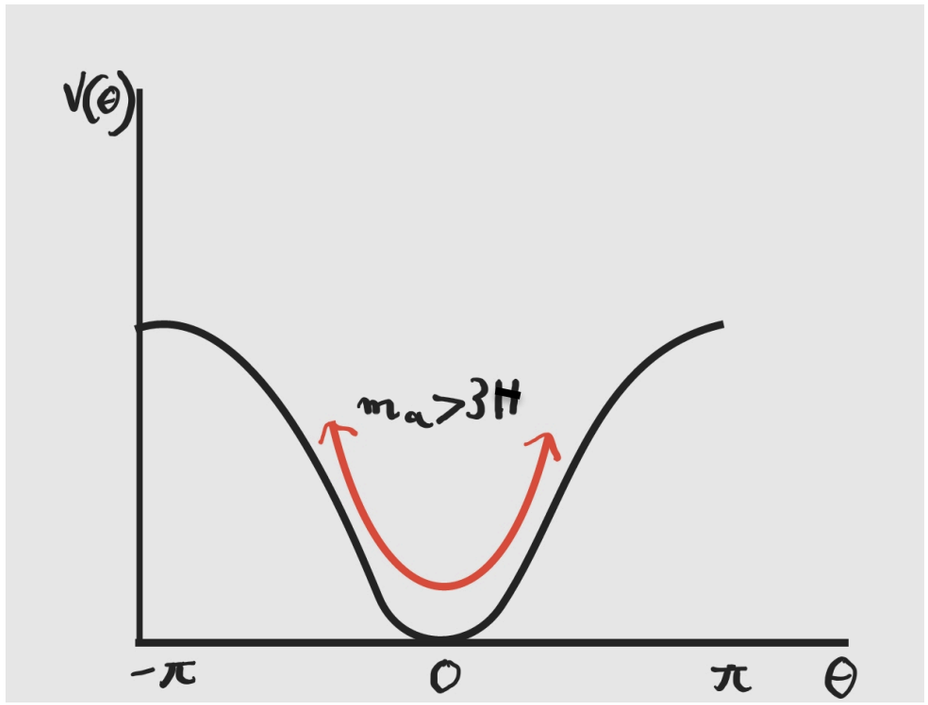


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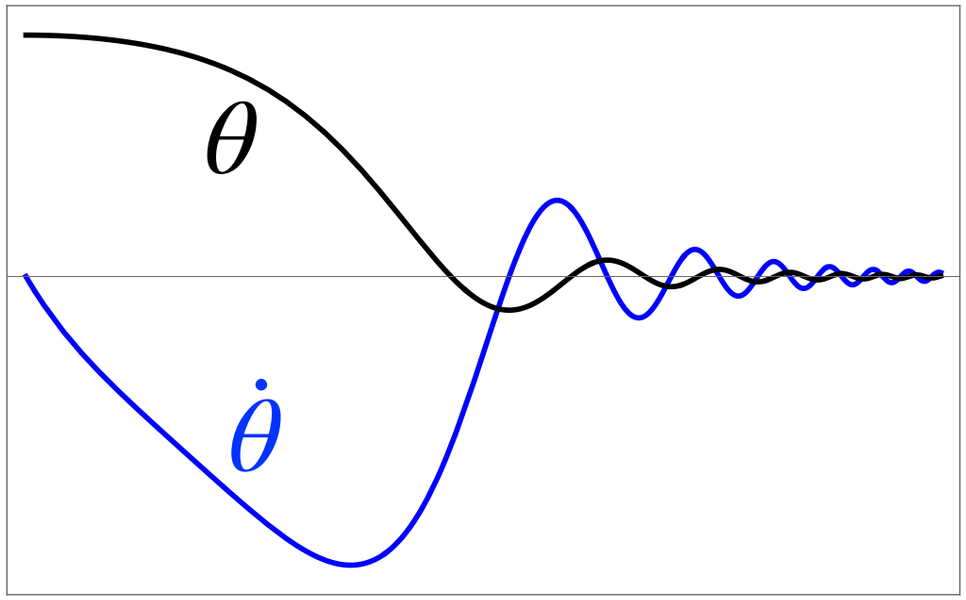


## Baryon Asymmetry:

$$T_{\mathrm{B-L}} \simeq T_{\mathrm{osc}} \simeq \sqrt{m_a M_P}$$

For $Y_B^{\mathrm{observed}} \sim \dfrac{\dot{\theta}}{g_* T_{\mathrm{osc}}} \sim 10^{-10}$

$$m_a \sim O(10^2)\ \mathrm{GeV}$$

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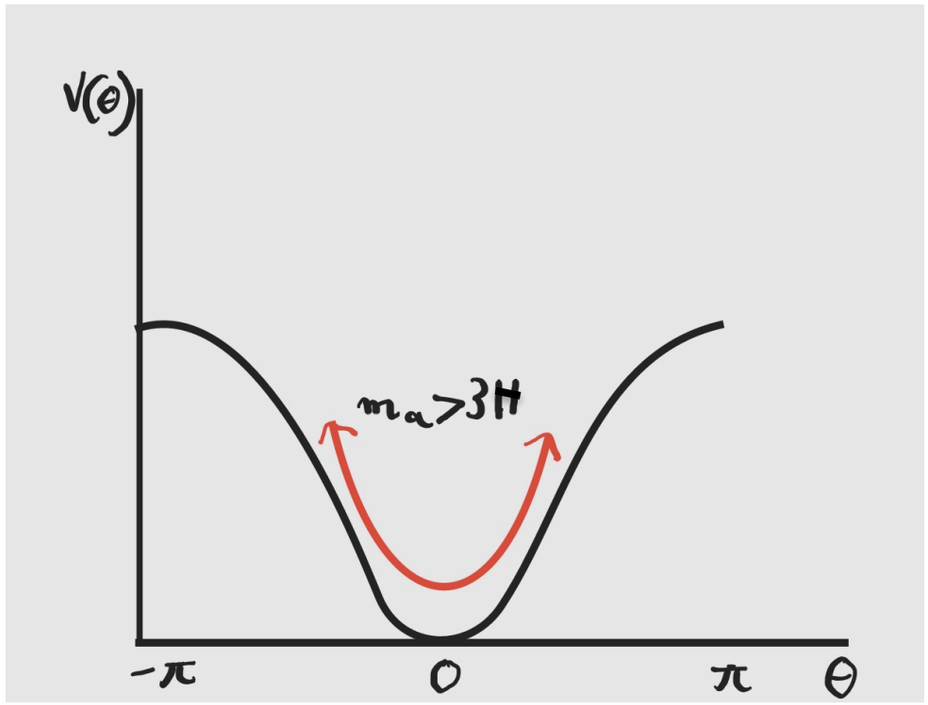


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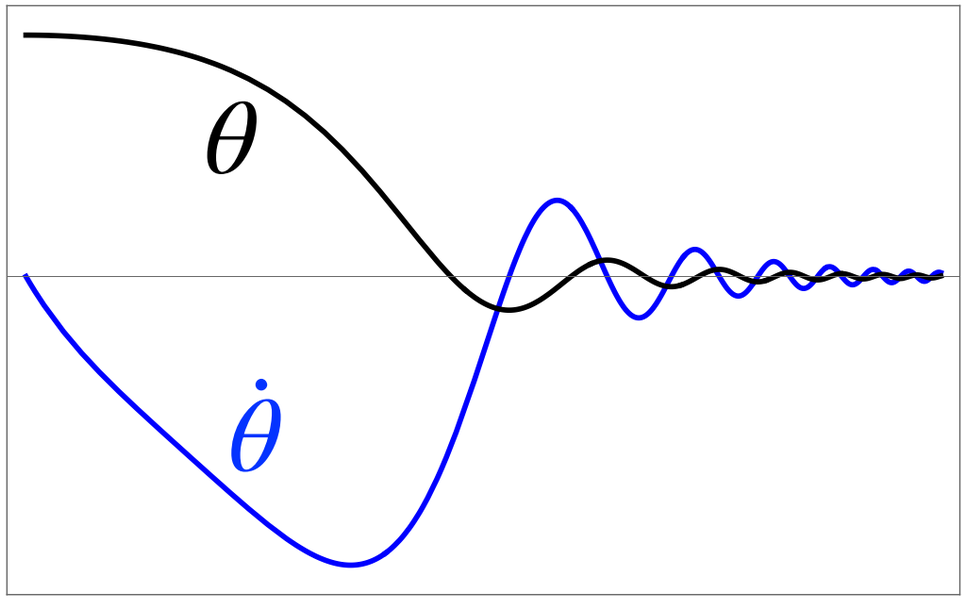


## Dark Matter:

$$\rho_{\rm DM} \sim m_a \times \theta_i^2 m_a f_a^2 \text{ (Mass X No. density)}$$

$$\frac{\rho_{\rm DM}}{s} \sim \frac{m_a^2 f_a^2}{s} \sim \frac{m_a^2 T_{\rm osc}^2}{g_* T_{\rm osc}^3} \gg 0.44 \text{ eV (observed)}$$

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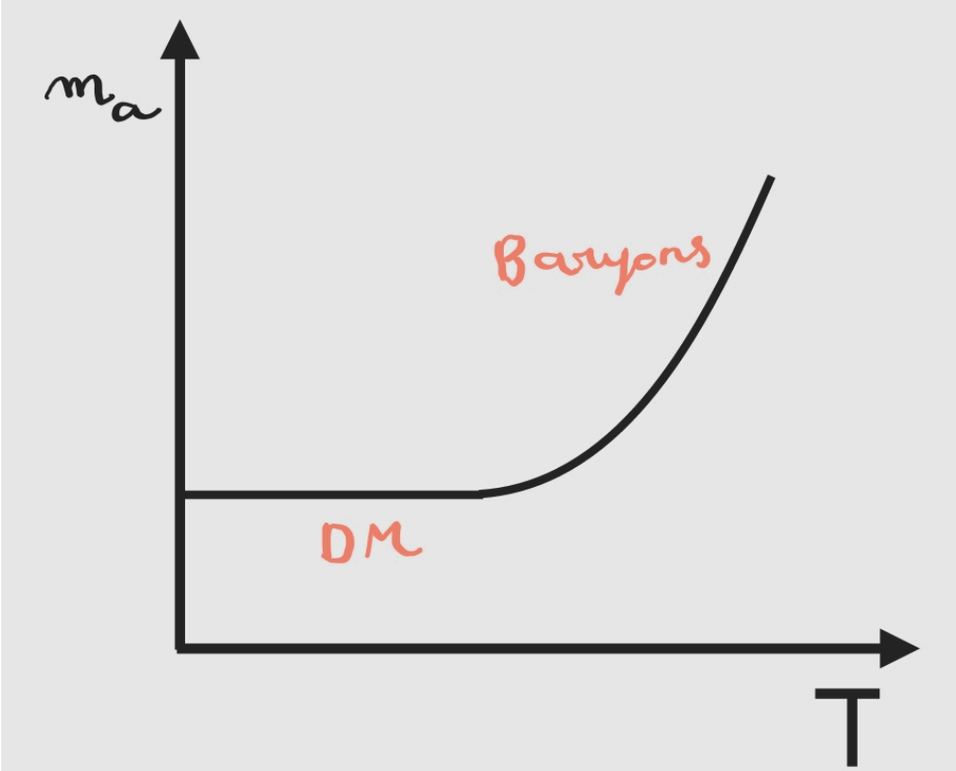
$\Longleftarrow$ $m_a \sim O(10^2)$ GeV

## Possible resolution:

Early dynamics with $m_a(T) \gg m_a^{(0)}$

Separate out $T_{\mathrm{B-L}} \gg T_{\mathrm{osc}}$




- Time-dependent $m_a$.
- $\dot{\theta}$ large enough before $T_{osc.}$
- **Baryogenesis** at $T_{B\text{-}L} > T_{osc.}$
- Oscillation at low temperature: **DM**.

# Today's talk

- Motivation.
- Introduction.
- **Rotation via *Symmetry non-restoration*.**
- Rotation via *Non-Minimal Coupling*.
- Discussion & Summary.

Explicit breaking of U(1):

$$\frac{\Phi^n}{\Lambda^{n-4}} \Rightarrow V_a(a) \simeq \frac{f_a^n}{\Lambda^{n-4}}\left(1-\cos\left(\frac{na}{f_a}\right)\right)$$

$$\langle \phi \rangle_T = f_a(T)$$

$$\Phi = \frac{1}{\sqrt{2}}\phi\, e^{ia/f_a}$$

pNGB

Mass of pNGB:

$$m_a^2(T) \sim \left(\frac{f_a(T)}{\Lambda}\right)^{n-4} f_a(T)^2 .$$

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## Symmetry non-restoration (S. Weinberg 1974)

Thermal corrections with **negative** contribution:

$$\Delta V = -2\lambda_{h\phi}|H|^2|\Phi|^2 \quad \text{or} \quad \Delta V = -\lambda_{\phi s_i}|\Phi|^2 s_i^2$$

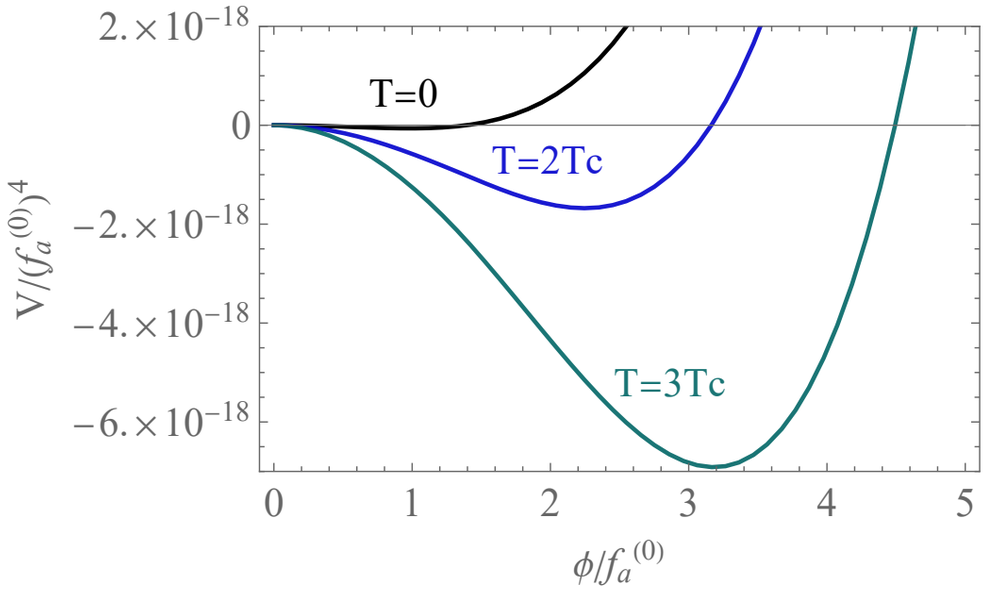
SM Higgs

## Temp. Dependent V:

$$V_T(\phi) \simeq \frac{\lambda_\phi}{4}\phi^4 - \frac{1}{2}(m_0^2 + c\,T^2)\phi^2$$

$$f_a(T) = \sqrt{f_a^{(0)2} + c_\lambda T^2}$$

$$c_\lambda \simeq \lambda_{\rm mix}/\lambda_\phi$$




For

$$T > T_c \equiv f_a^{(0)} \,/\, \sqrt{c_\lambda}$$

$$\rightarrow \quad f_a(T) \propto T \qquad m_a(T) \propto T^{(n-2)/2}$$

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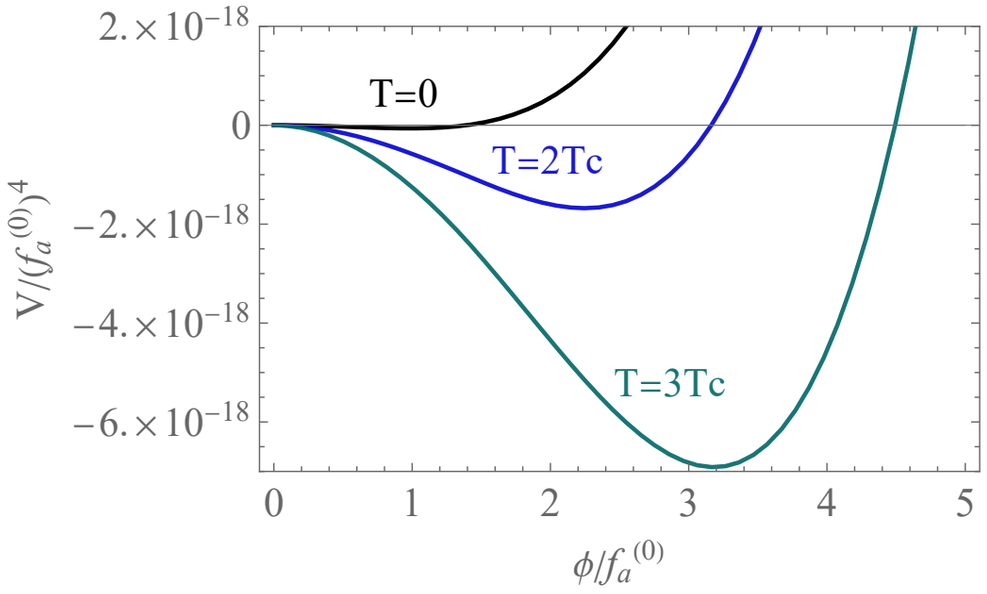


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$$\phi\phi \leftrightarrow aa \propto \frac{T^2}{f_a^4} \implies c_\lambda \gtrsim 10^7$$

# pNGB Dynamics (n=5)

**Modified E.O.M. :**

$$\ddot{\theta} + \left( 3H + 2\frac{\dot{f}_a}{f_a} \right) \dot{\theta} = -\frac{1}{n} m_a^2(T) \sin(n\theta)$$

$\frac{\dot{f}_a}{f_a} \to -H$

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**1st epoch: H(T) > m(T)**

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$H \to$ constant

**pNGB is frozen**

*until*

H(T) = m(T) $\Longrightarrow$ $T_0$

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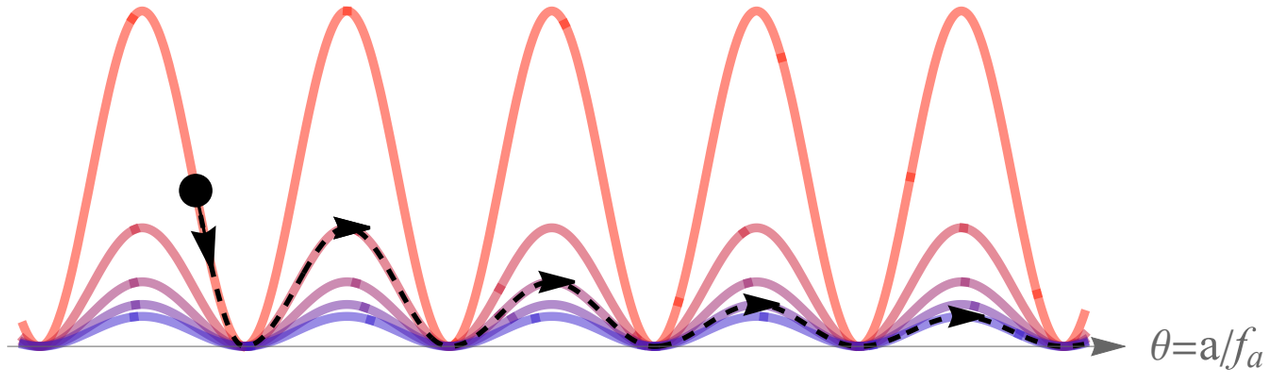
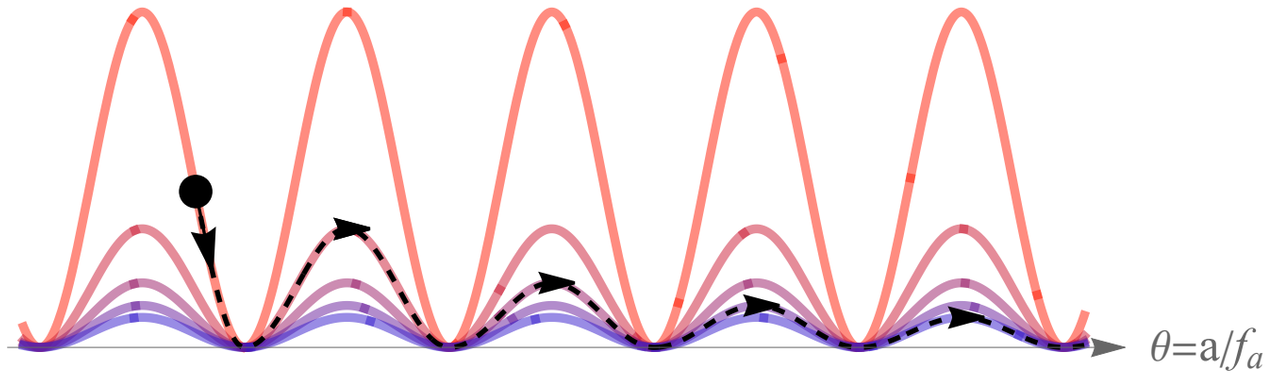
**2nd epoch: T < $T_0$** **Oscillation?**

pNGB slides



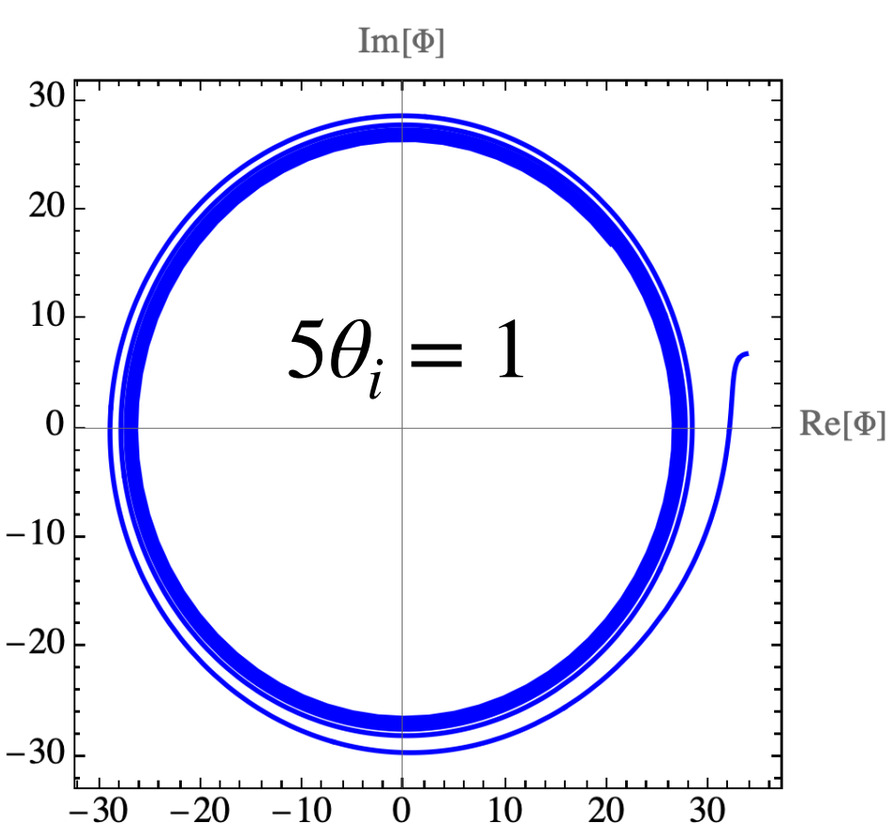

K.E. = Barrier → $\dot{\theta}(T_{\rm slide}) \simeq \frac{2}{5} m_a(T_{\rm slide})$

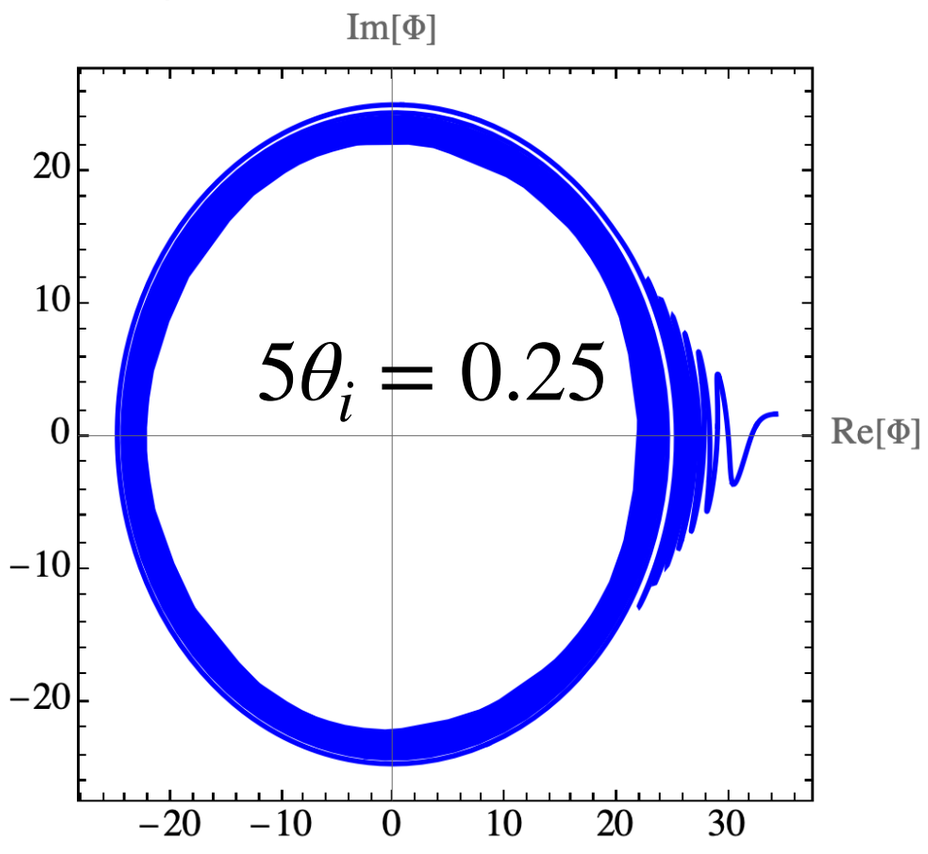
## pNGB slides




**K.E. = Barrier** → $\dot{\theta}(T_{\rm slide}) \simeq \frac{2}{5} m_a(T_{\rm slide})$

$$T_{\rm slide} \sim T_0(1-\cos(5\theta_i))^2$$



$$\ddot{\theta} + H\dot{\theta} = -\frac{1}{n} m_a^2(T) \sin(n\theta)$$

**Gives asymmetry**

$$\frac{\dot{\theta}}{T} = \text{constant}$$

From Spontaneous Baryo.

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**3rd epoch: $T < T_c$**

$$\ddot{\theta} + \left(3H + 2\frac{\dot{f}_a}{f_a}\right)\dot{\theta} = -\frac{1}{n} m_a^2(T) \sin(n\theta)$$

**$f_a(T)$ saturates**

$$\dot{\theta} \propto T^3$$

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$f_a(T)$ **saturates**

$$\dot{\theta} \propto T^3$$

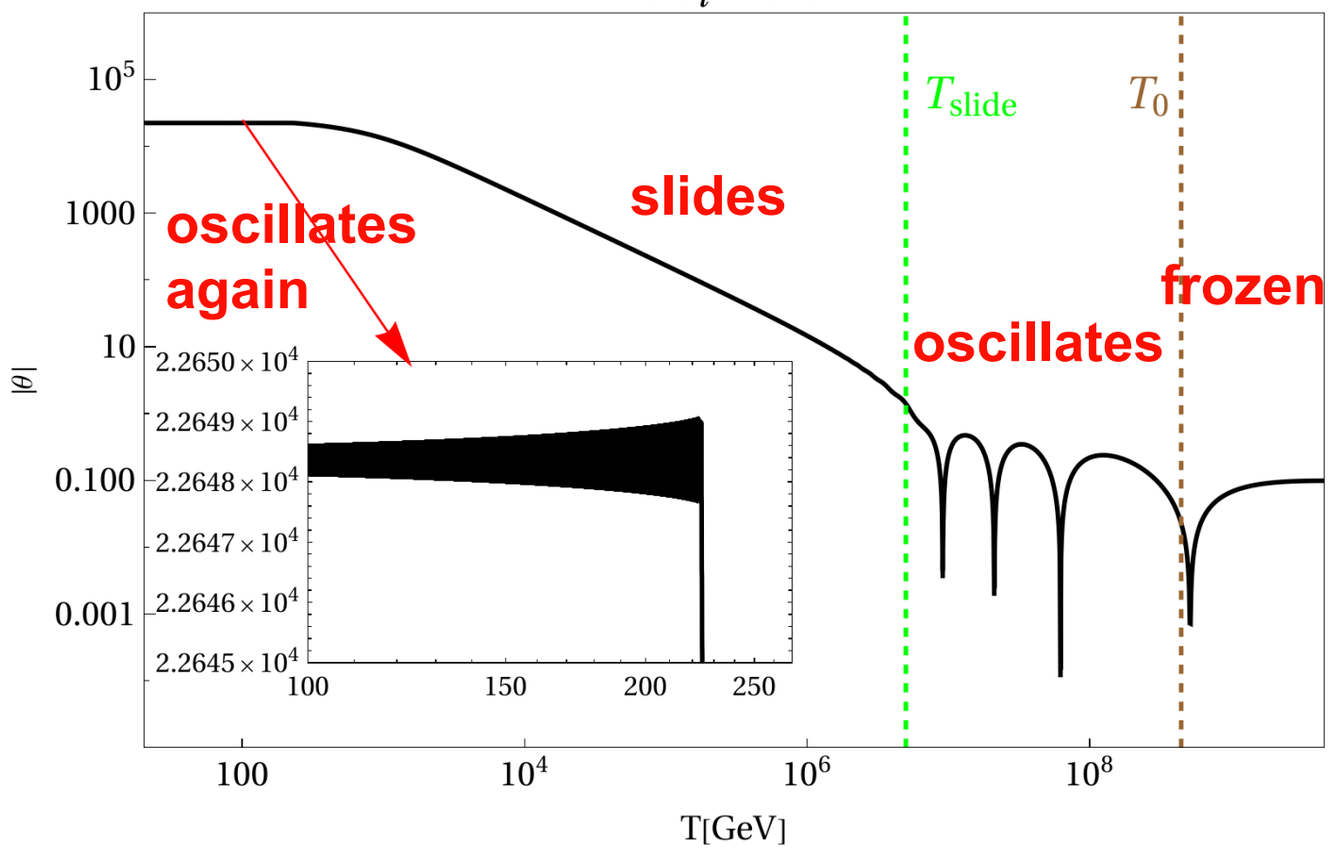
**4th epoch: $T < T_{osc}$**

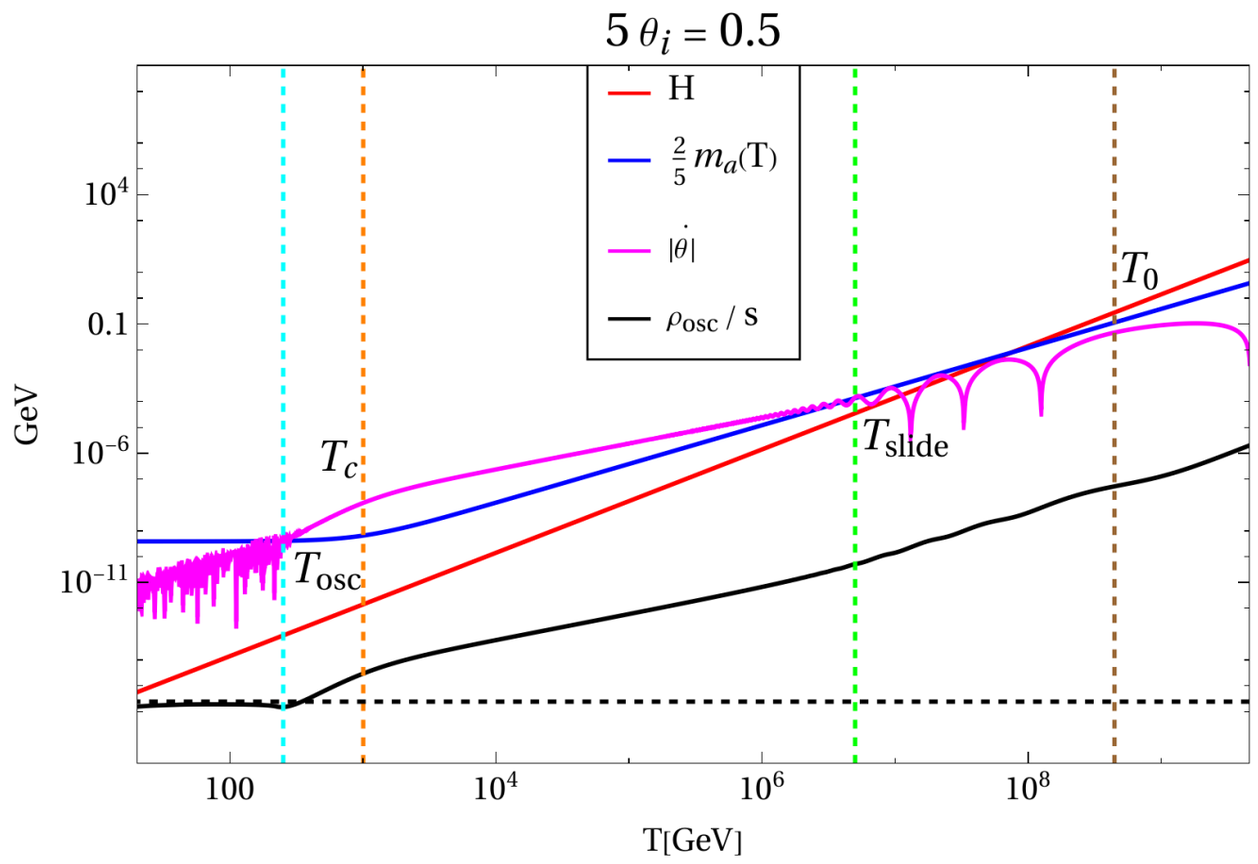
$$\ddot{\theta} + 3H\dot{\theta} = -\frac{1}{n} m_a^{(0)2} \sin(n\theta)$$

Final oscillation: $\Longrightarrow$ $\dot{\theta}(T_{\rm osc}) \simeq \frac{2}{5} m_a^{(0)}$

$$\frac{\rho_{\rm osc}}{s} \sim \frac{\left(m_a^{(0)} f_a^{(0)}\right)^2}{s(T_{\rm osc})}$$

**pNGB oscillates, gives DM**


$5\,\theta_i = 0.5$
$10^5$
1000
10
0.100
0.001
$|\theta|$
$T_{\rm slide}$
$T_0$
oscillates again
slides
oscillates
frozen
$2.2650\times10^4$
$2.2649\times10^4$
$2.2648\times10^4$
$2.2647\times10^4$
$2.2646\times10^4$
$2.2645\times10^4$
100
150
200
250
100
$10^4$
$10^6$
$10^8$
T[GeV]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H
$\frac{2}{5}\,m_a$(T)
$|\dot{\theta}|$
$\rho_{\rm osc}$ / s
$10^4$
0.1
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GeV
$T_c$
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# An explicit example: Majoron

$$-\Delta\mathscr{L} = (y\Phi N^c N^c + Y_D \bar{l}_\alpha \tilde{H} N^c + h\,.\,c.) + V(\Phi)$$

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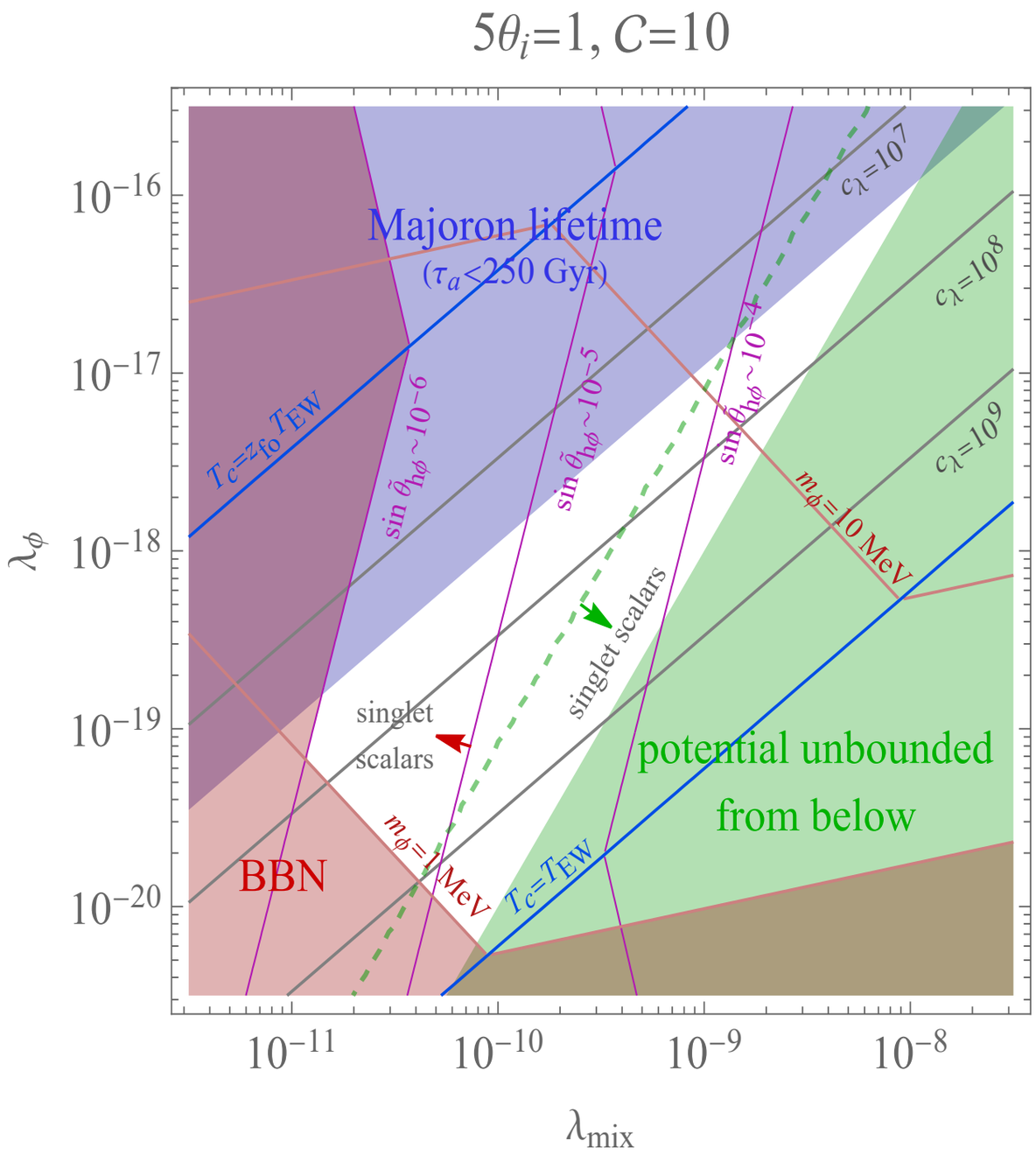


## The Cogenesis Window

$$m_a^{(0)} = \frac{5\ \text{eV}}{C^{1/9}(5\theta_i)^{4/9}}\left(\frac{g_*}{100}\right)^{1/3}\left(\frac{10^8}{c_\lambda}\right)$$

$$f_a^{(0)} = 3\times 10^6\ \text{GeV}\, C^{1/18}(5\theta_i)^{2/9}\left(\frac{100}{g_*}\right)^{1/6}\left(\frac{c_\lambda}{10^8}\right)^{5/18}$$

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- **Rotation via *Non-Minimal Coupling*.**
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## Periodic non-minimal coupling to gravity

$$\mathscr{L} = \frac{1}{2}m_P^2 R - \frac{1}{2}(\partial\phi)^2 - \left(M^4 - \frac{1}{2}\xi m_P^2 R\right)\left[1-\cos(\phi/f)\right]$$

$m_\phi^2 f^2 \qquad R = 3(1-3w)H^2$

**Inflation followed by a Kination era?**

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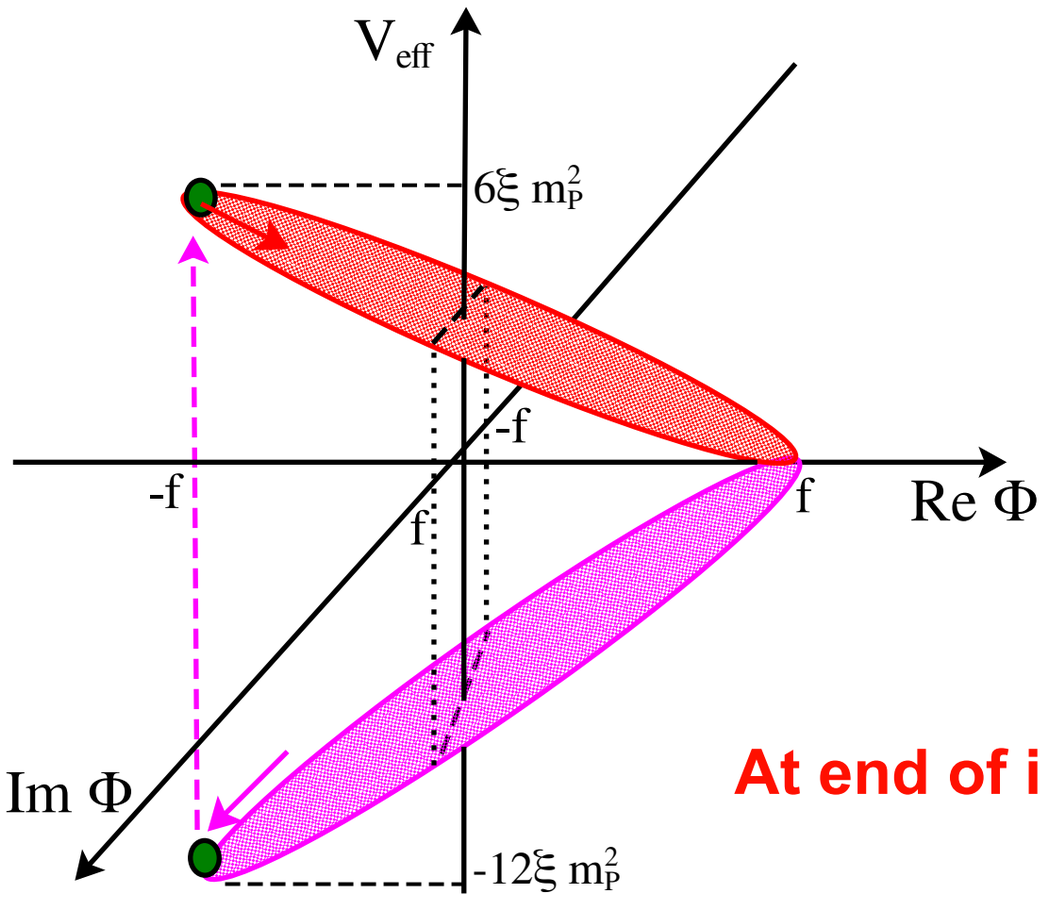
$M^4 \to m_\phi^2 f^2$ $\quad R = 3(1-3w)H^2$

**Inflation followed by a Kination era?**

$$V_{\rm eff}(\phi) \simeq -6\xi m_P^2 H^2[1-\cos(\phi/f)] \longrightarrow V_{\rm eff}(\phi) \simeq 3\xi m_P^2 H^2[1-\cos(\phi/f)]$$




**Kinetic energy =**

$$\frac{1}{2}f^2\dot{\theta}_{\rm m}^2 \simeq 6\xi m_P^2 H_{\rm end}^2$$

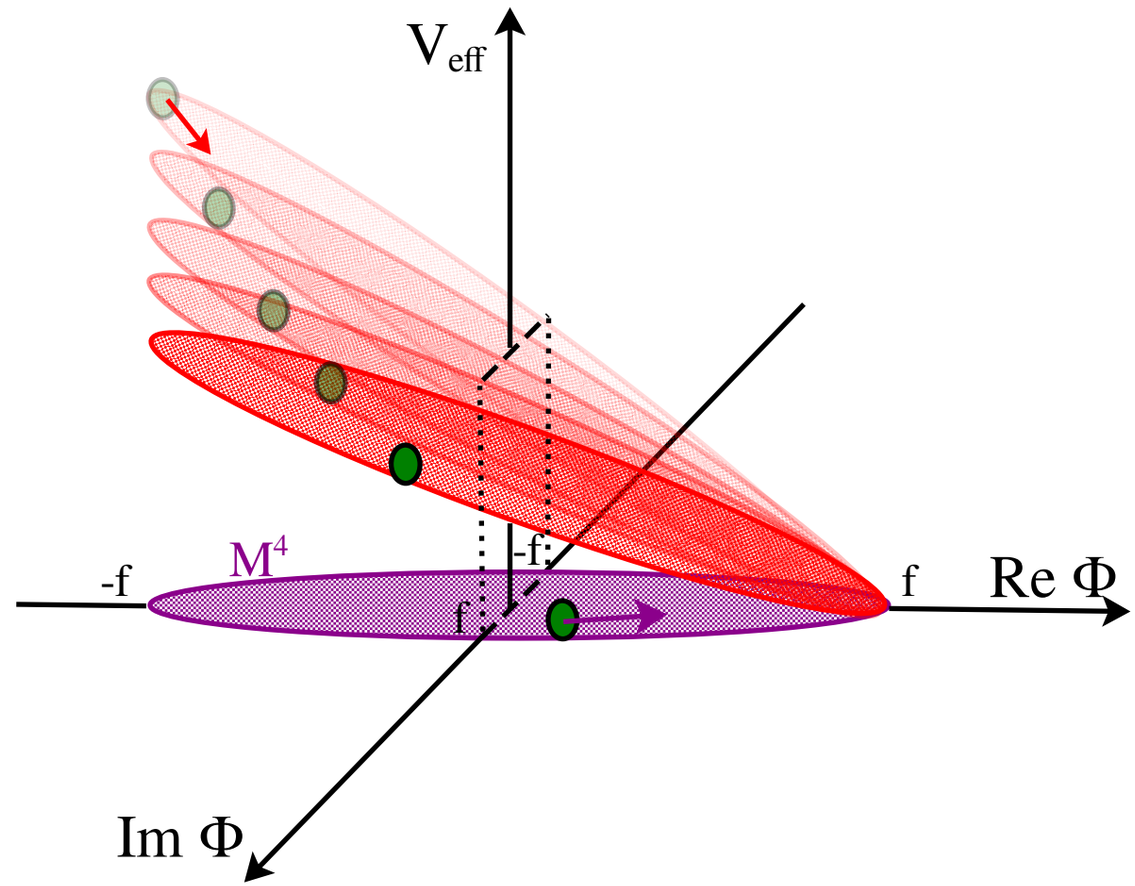
**At end of inflation**

# After inflation

**Barrier height:** $\propto H^2 \propto \frac{1}{t^2}$

*in tandem with*

**Kinetic Energy:** $\propto \dot{\theta}^2 \propto \frac{1}{a^6} \propto \frac{1}{t^2}$



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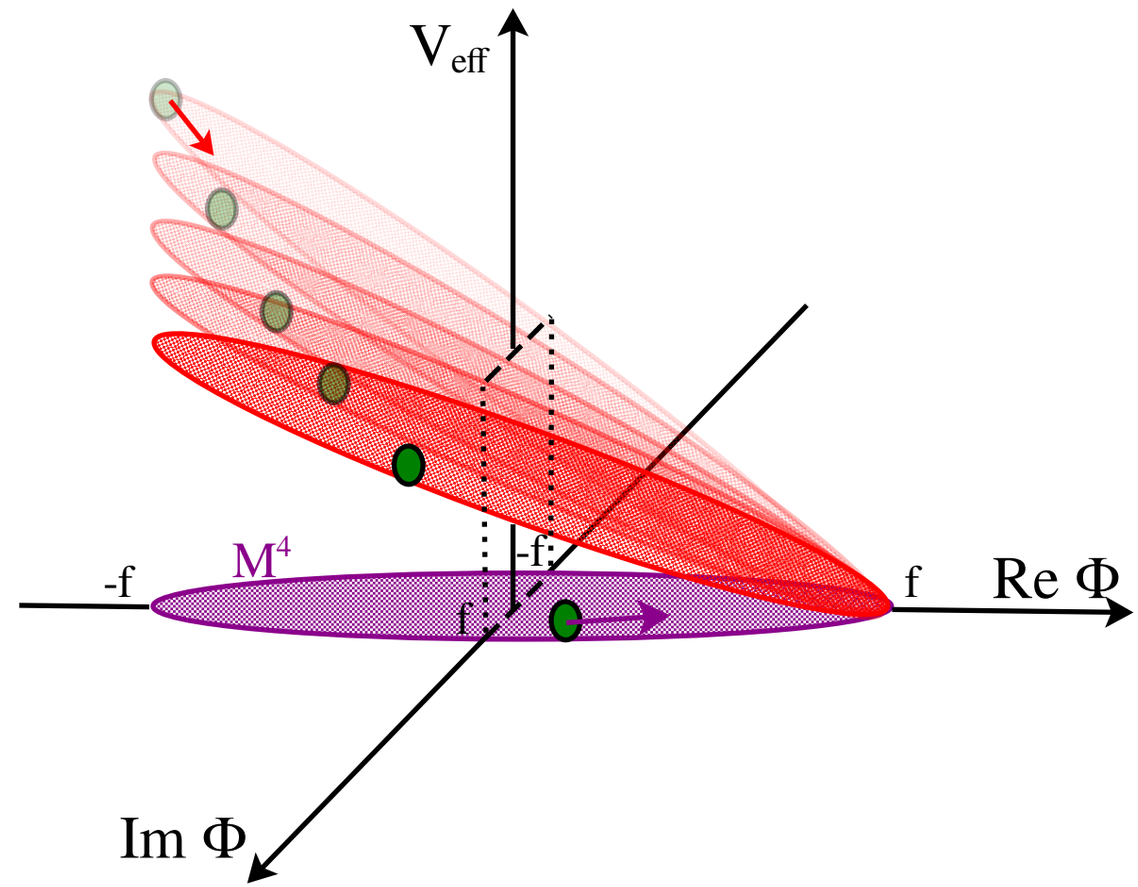
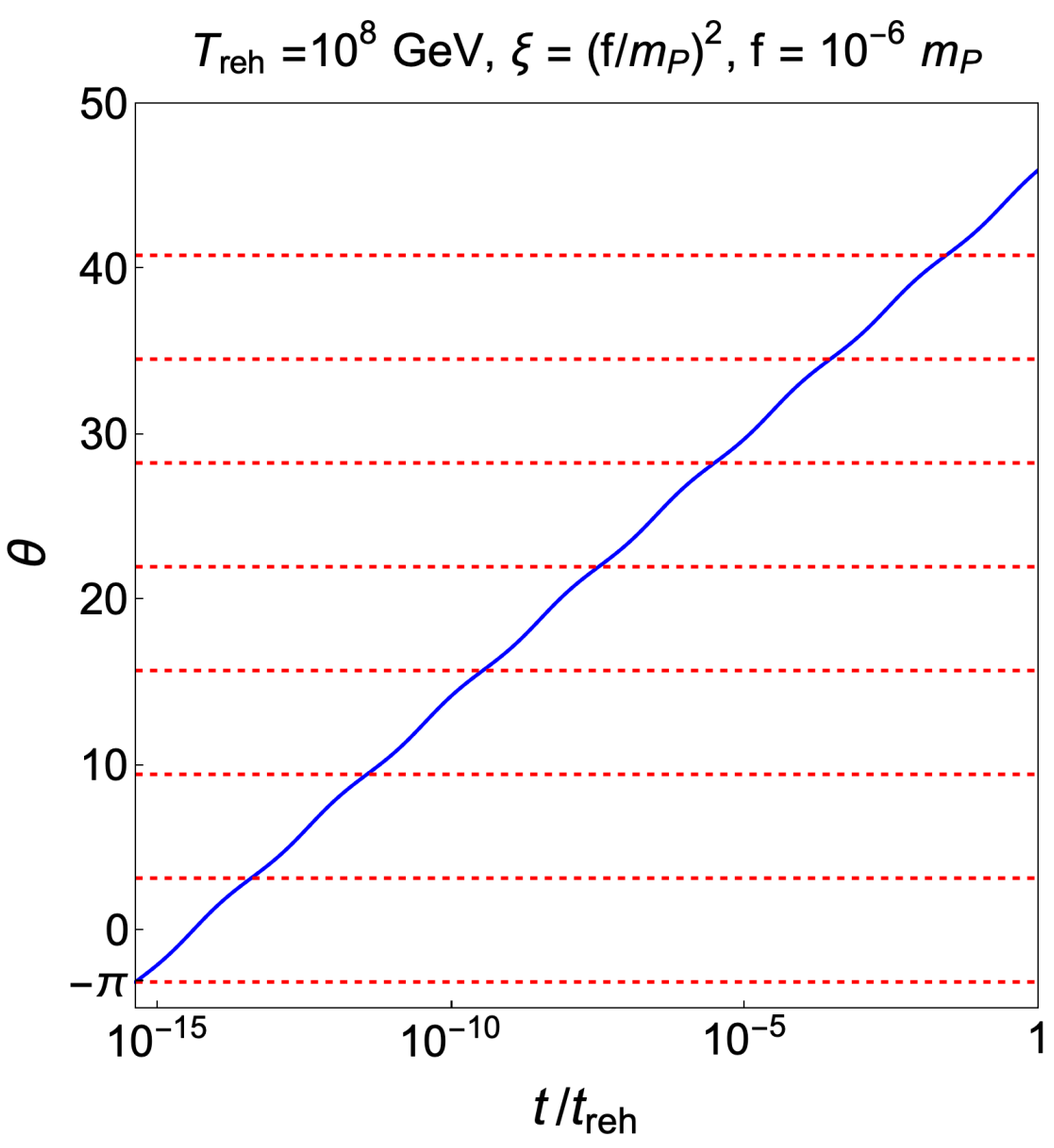
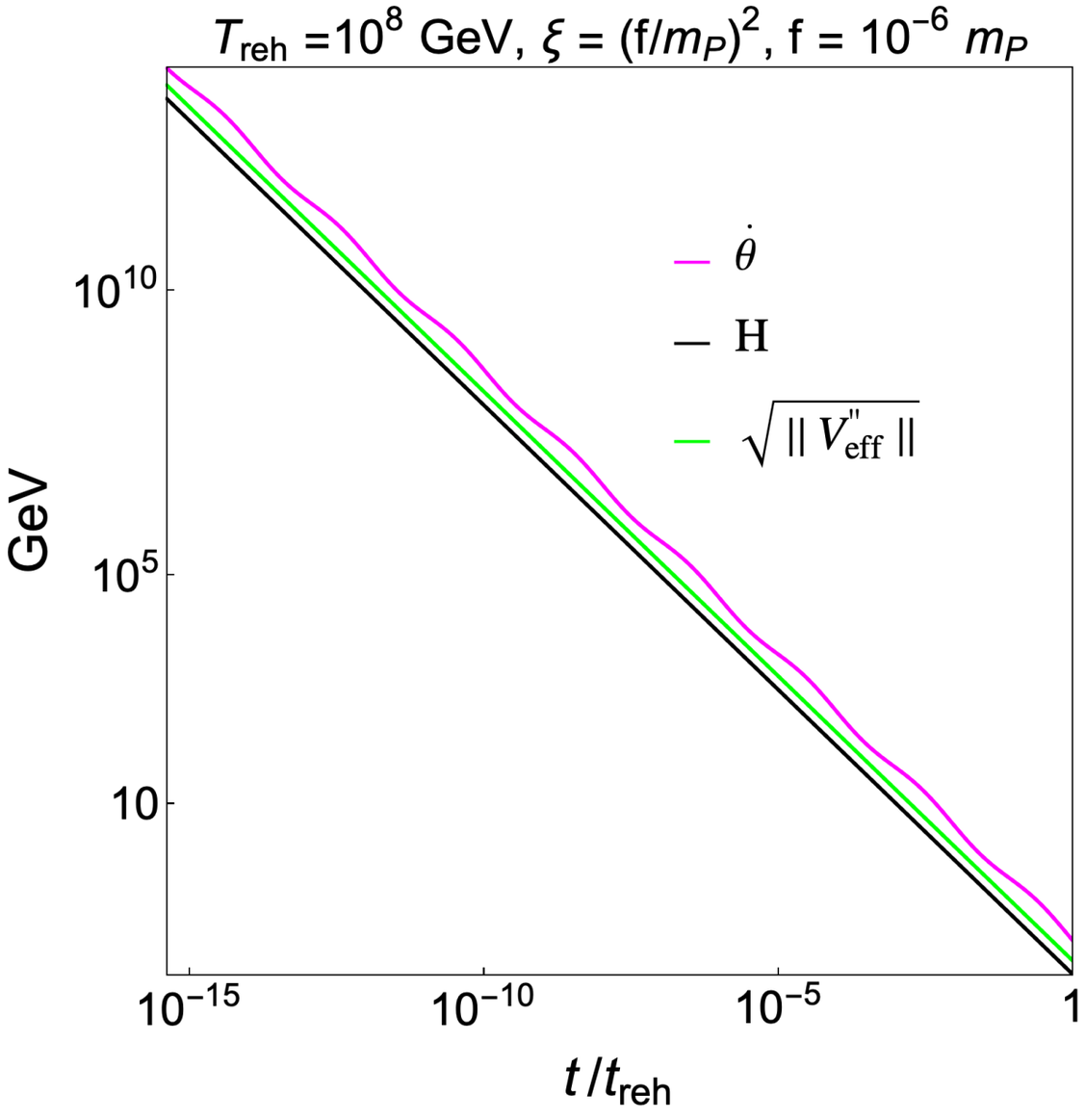
*in tandem with*

**Kinetic Energy:** $\propto \dot{\theta}^2 \propto \frac{1}{a^6} \propto \frac{1}{t^2}$




Range of $\xi$: $\Longrightarrow |\,|\,V''_{\rm eff}\,|\,| > H \longrightarrow \frac{3}{4}\left(\frac{f}{m_P}\right)^2 < \xi \ll \frac{1}{2} \longleftarrow$ pNGB remains sub-dominant


$T_{\rm reh}$ =10$^8$ GeV, $\xi$ = (f/$m_P$)$^2$, f = 10$^{-6}$ $m_P$
50
40
30
20
10
0
$-\pi$
$\theta$
$10^{-15}$
$10^{-10}$
$10^{-5}$
1
$t/t_{\rm reh}$


$T_{\rm reh}$ =10$^8$ GeV, $\xi$ = (f/$m_P$)$^2$, f = 10$^{-6}$ $m_P$
$10^{10}$
$10^5$
10
GeV
$\dot{\theta}$
H
$\sqrt{\| V''_{\rm eff} \|}$
$10^{-15}$
$10^{-10}$
$10^{-5}$
1
$t/t_{\rm reh}$

Spontaneous baryogengesis:

$$Y_B \sim \frac{\sqrt{\xi} c_B}{\sqrt{g_*}} \frac{T_{\mathrm{B-L}}^2}{f\ T_{\mathrm{reh}}}$$

## Spontaneous baryogengesis:

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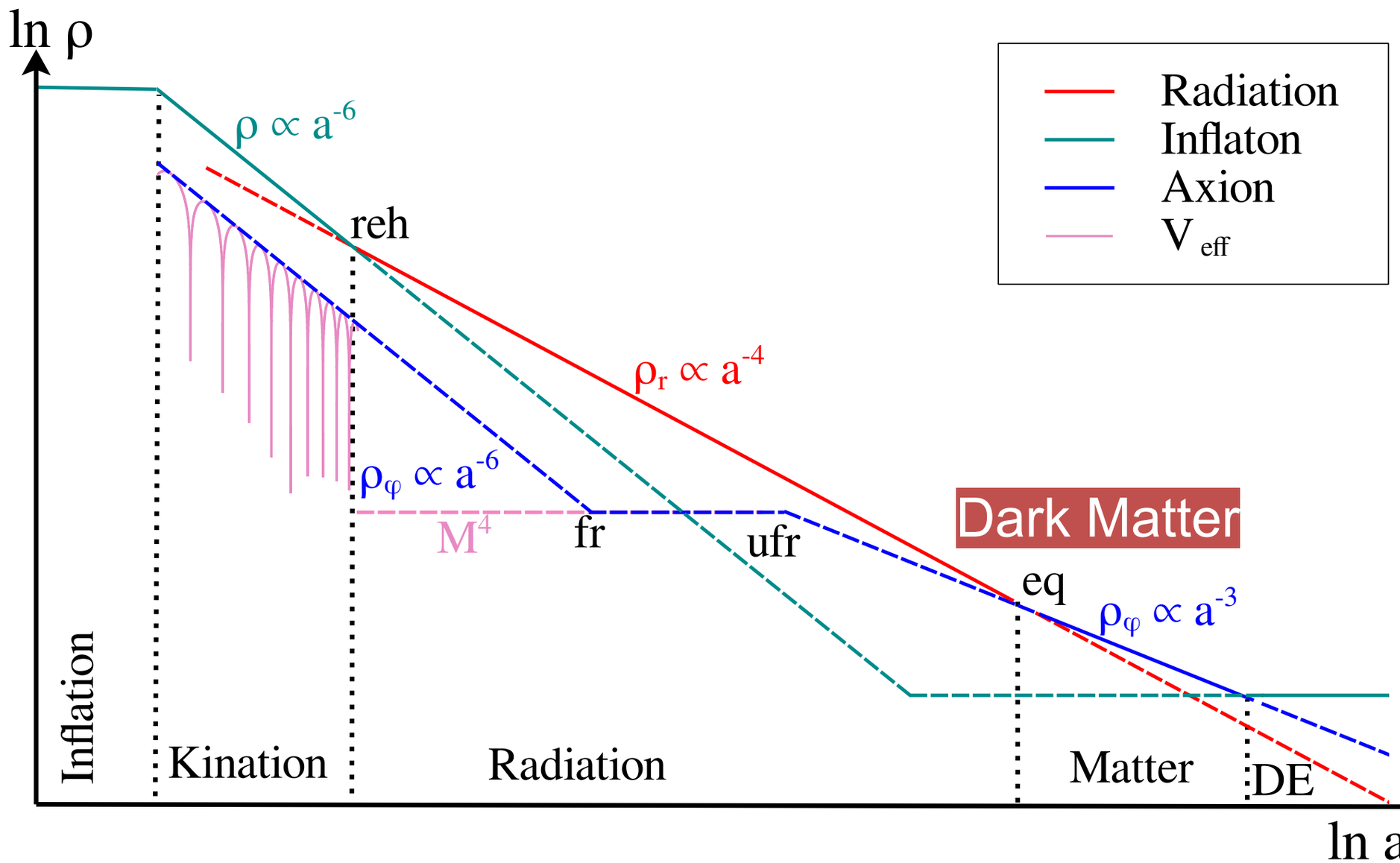
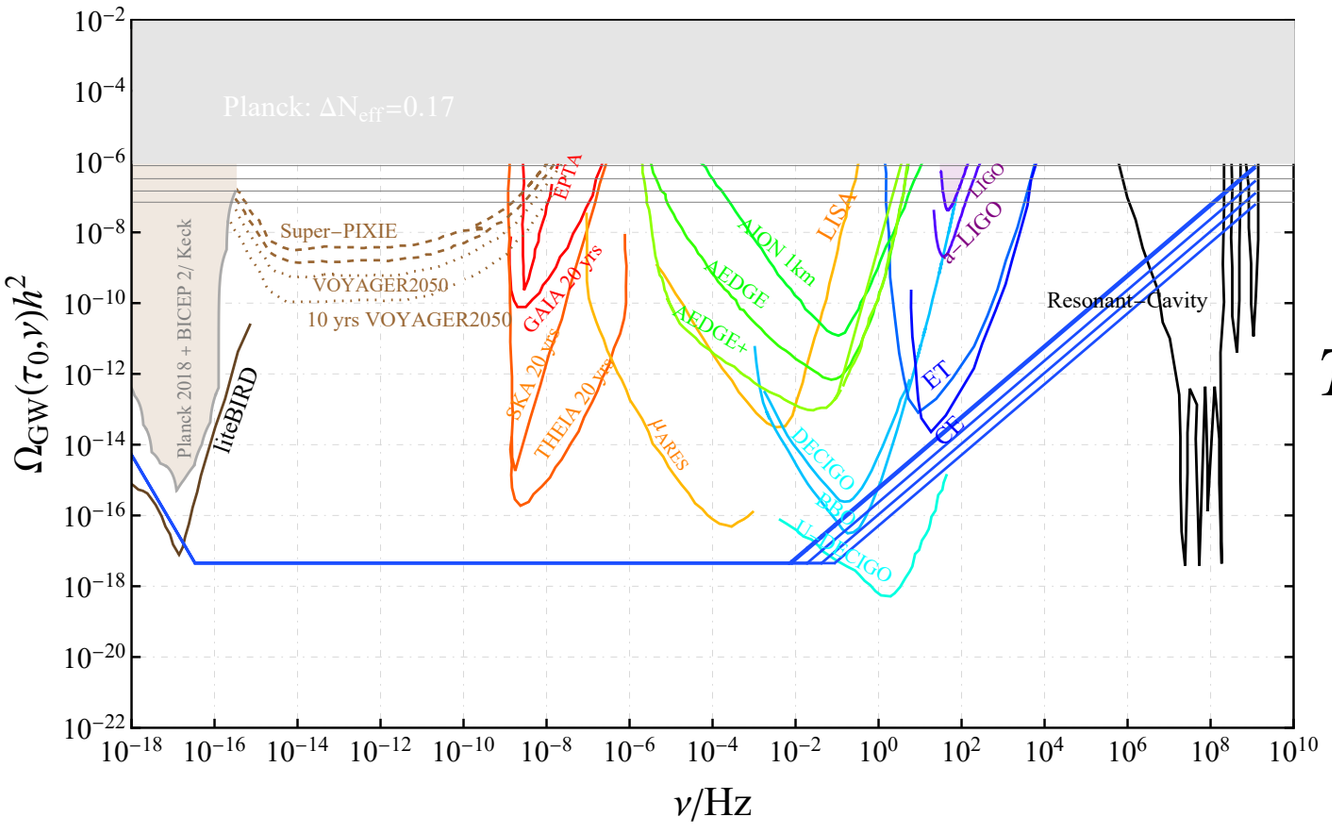
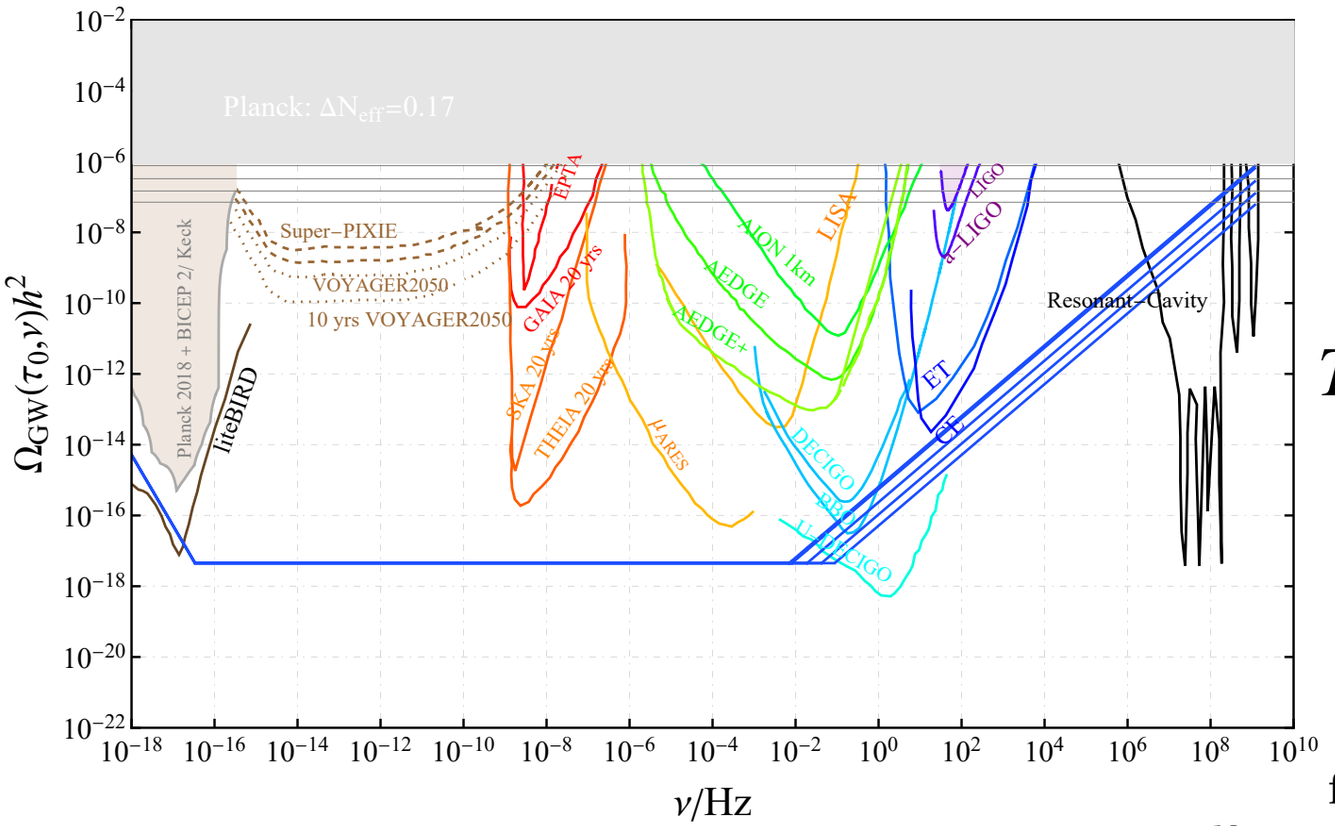


# Primordial gravitational waves:




$$T_{\rm reh} \gtrsim \frac{5.8\times10^6\ {\rm GeV}}{\Delta N_{\rm eff}^{3/4}}$$

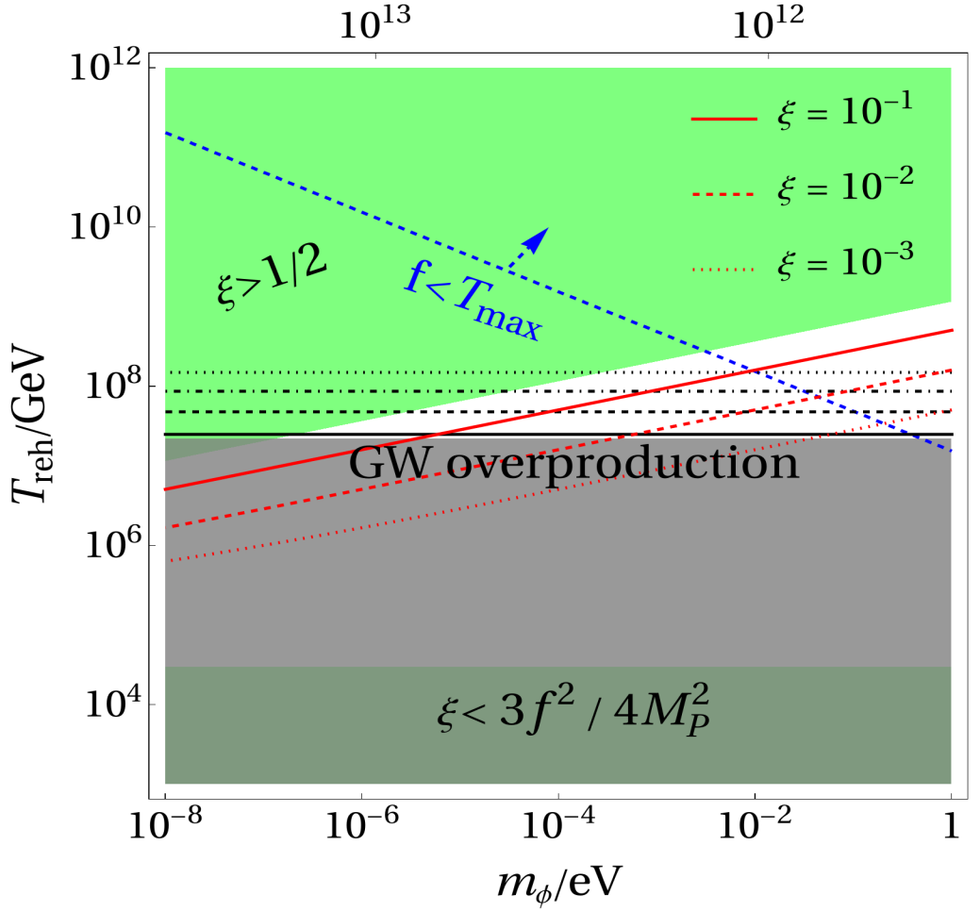
# Primordial gravitational waves:



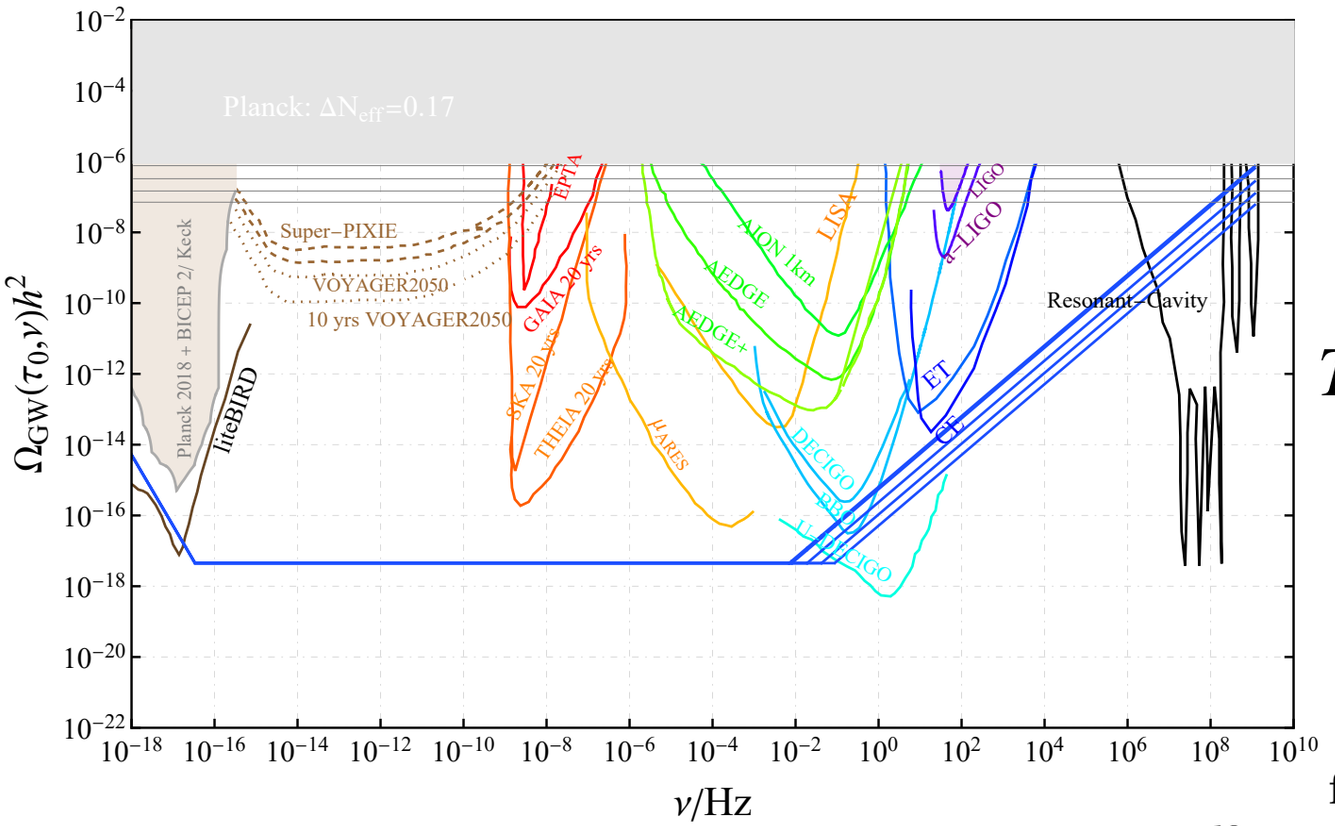

$$T_{\rm reh} \gtrsim \frac{5.8\times 10^6\ {\rm GeV}}{\Delta N_{\rm eff}^{3/4}}$$

## Predictions for Majoron:

*for* $M_N = 10^7\ GeV$



# Primordial gravitational waves:

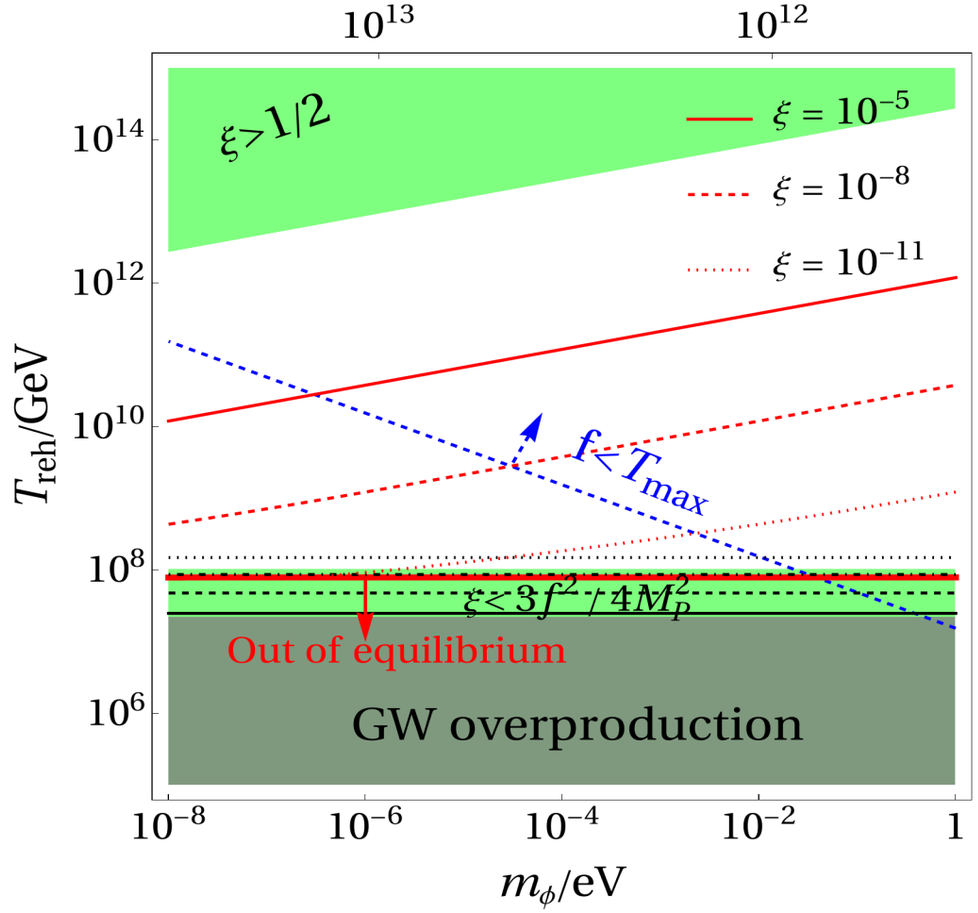



$$T_{\rm reh} \gtrsim \frac{5.8\times 10^6\ {\rm GeV}}{\Delta N_{\rm eff}^{3/4}}$$

f/GeV

## Predictions for Majoron:

*for* $M_N = 5\times 10^9\ GeV$



# Today's talk

- Motivation.
- Introduction.
- Rotation via *Symmetry non-restoration*.
- Rotation via *Non-Minimal Coupling*.
- **Remarks & Summary.**

# Remarks

- Delayed oscillations$\Longrightarrow$Growth of pNGB fluctuations due to

  parametric resonance $\left( \text{if } \dfrac{\text{Growth rate}}{\text{Hubble rate}} \sim \dfrac{m_a^2}{\dot{\theta} H} \gg 1 \right)$.

- (a) pNGB is fragmented just before “oscillation” (DM abundance almost unchanged).
- (b) Growth suppressed for $\xi \lesssim 10^3 \left( \dfrac{f}{m_P} \right)^2$.

# Remarks

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- (a) pNGB is fragmented just before “oscillation” (DM abundance almost unchanged).

- (b) Growth suppressed for $\xi \lesssim 10^3 \left( \dfrac{f}{m_P} \right)^2$.

- (b) Kibble issue (Rotation clock/anti-clock wise?), corrections to R

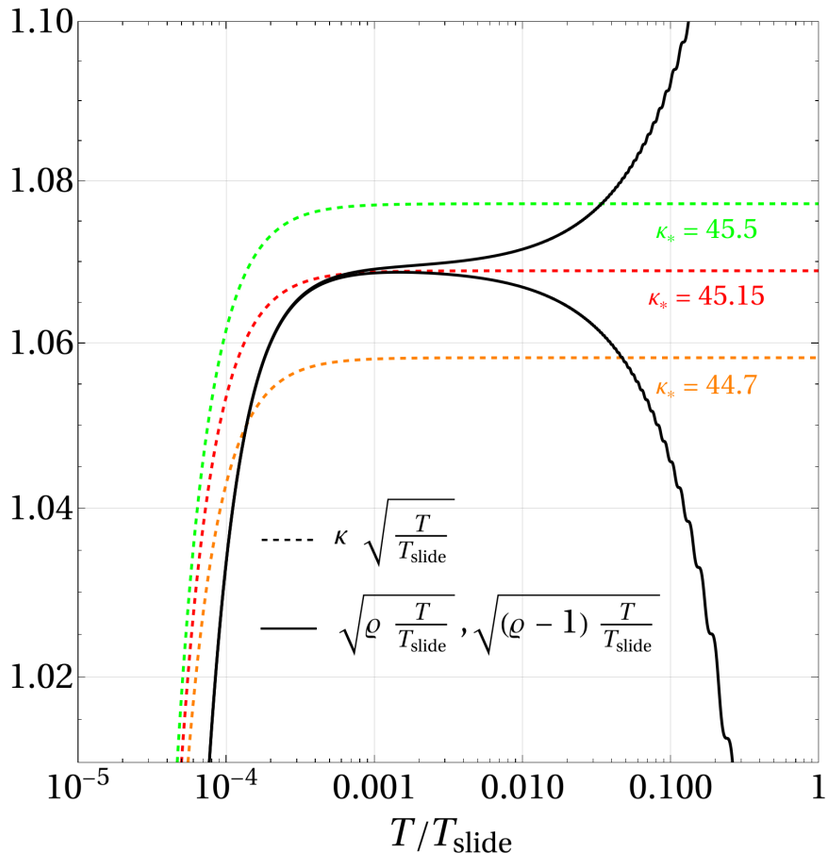
  $\Longrightarrow \ \xi \sim \left( \dfrac{f}{m_P} \right)^2$.

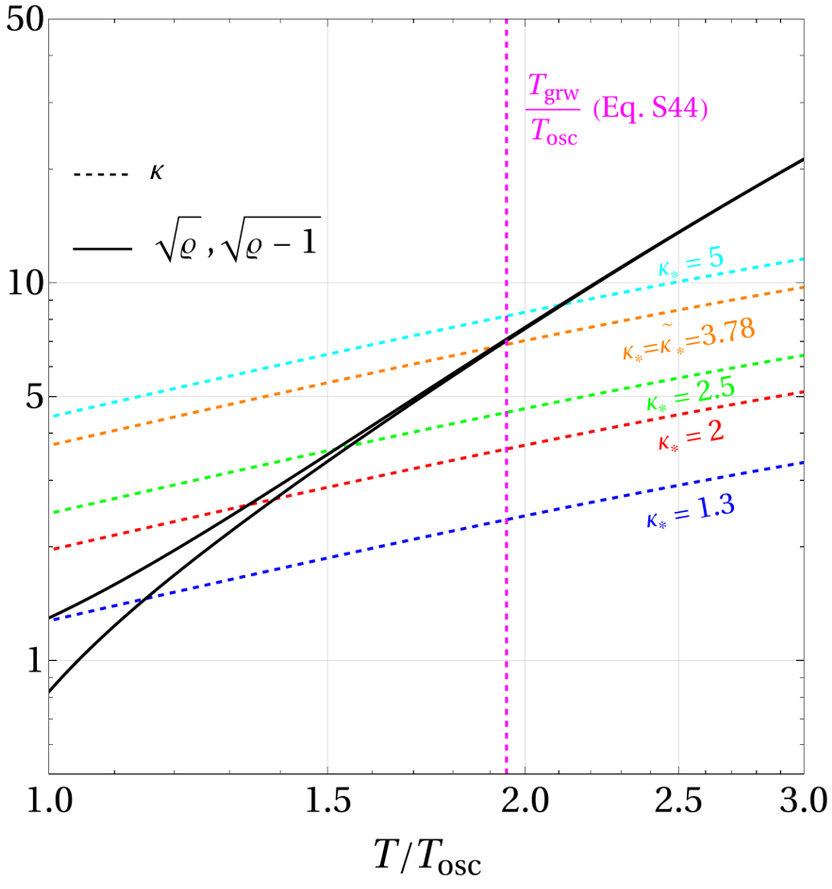
# Summary

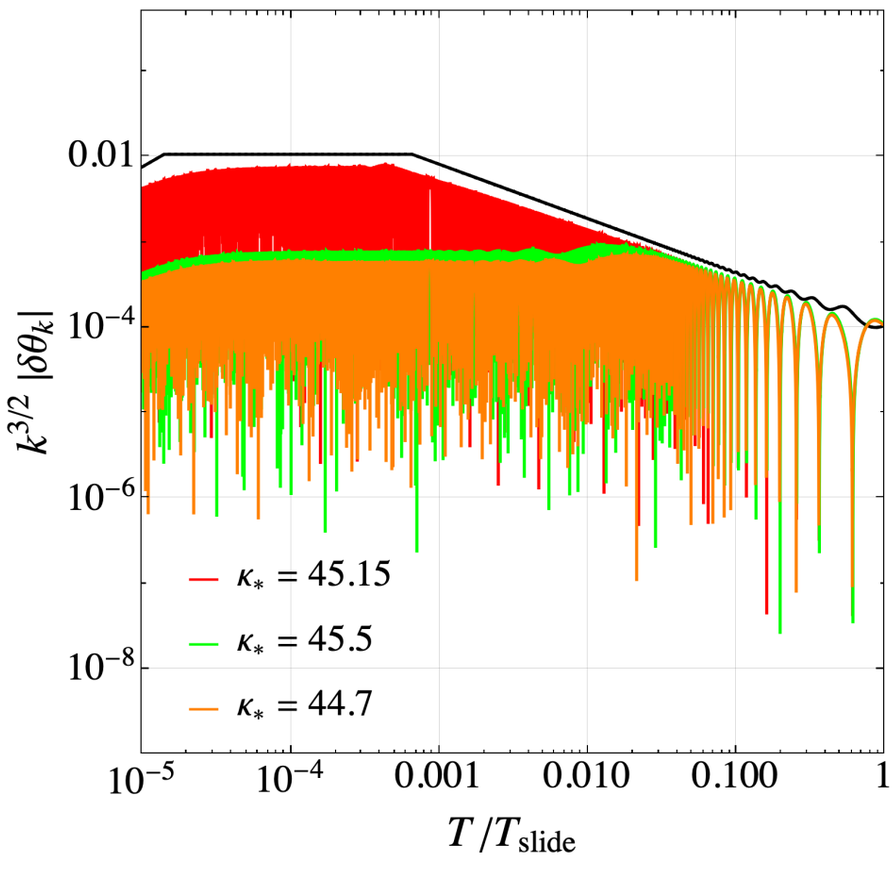
- $\dot{\theta}_i = 0$ can lead to rotation/sliding and baryon & DM abundance.
- Via (a) Symmetry non-restoration, (b) Non-minimal coupling to gravity.
- Can be realized for Majoron, testable at (a) kaon experiments, colliders (b) GW.
- Extension of other D-operators, other scenarios?

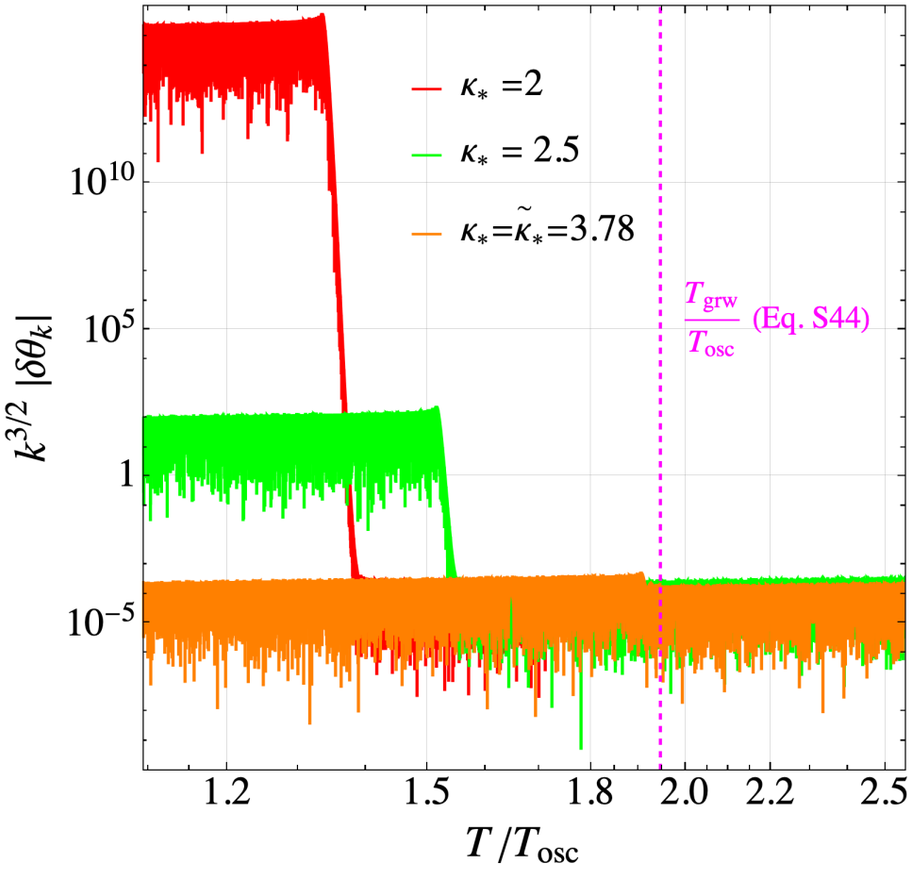
***THANK YOU***

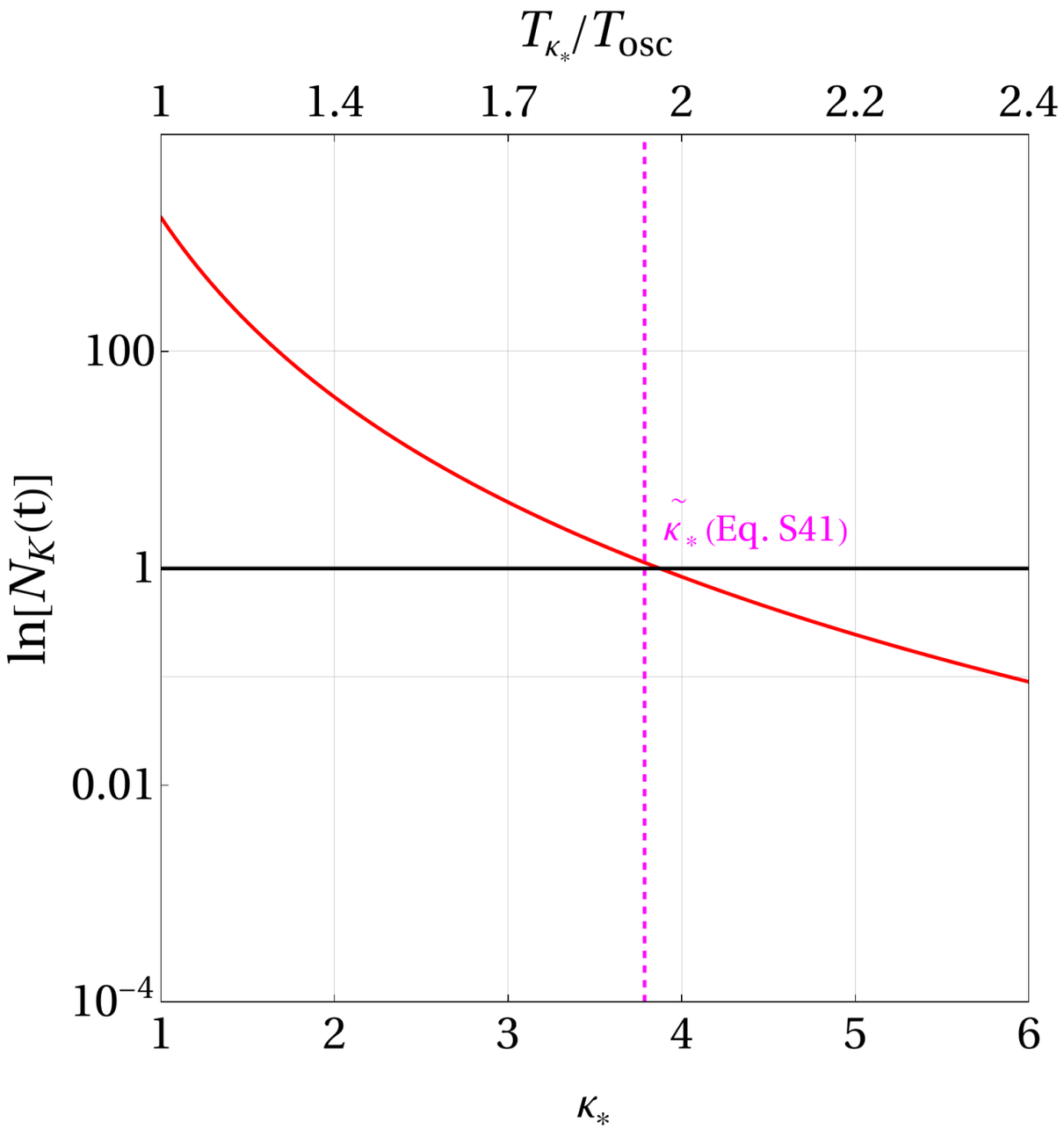
# Back-Up

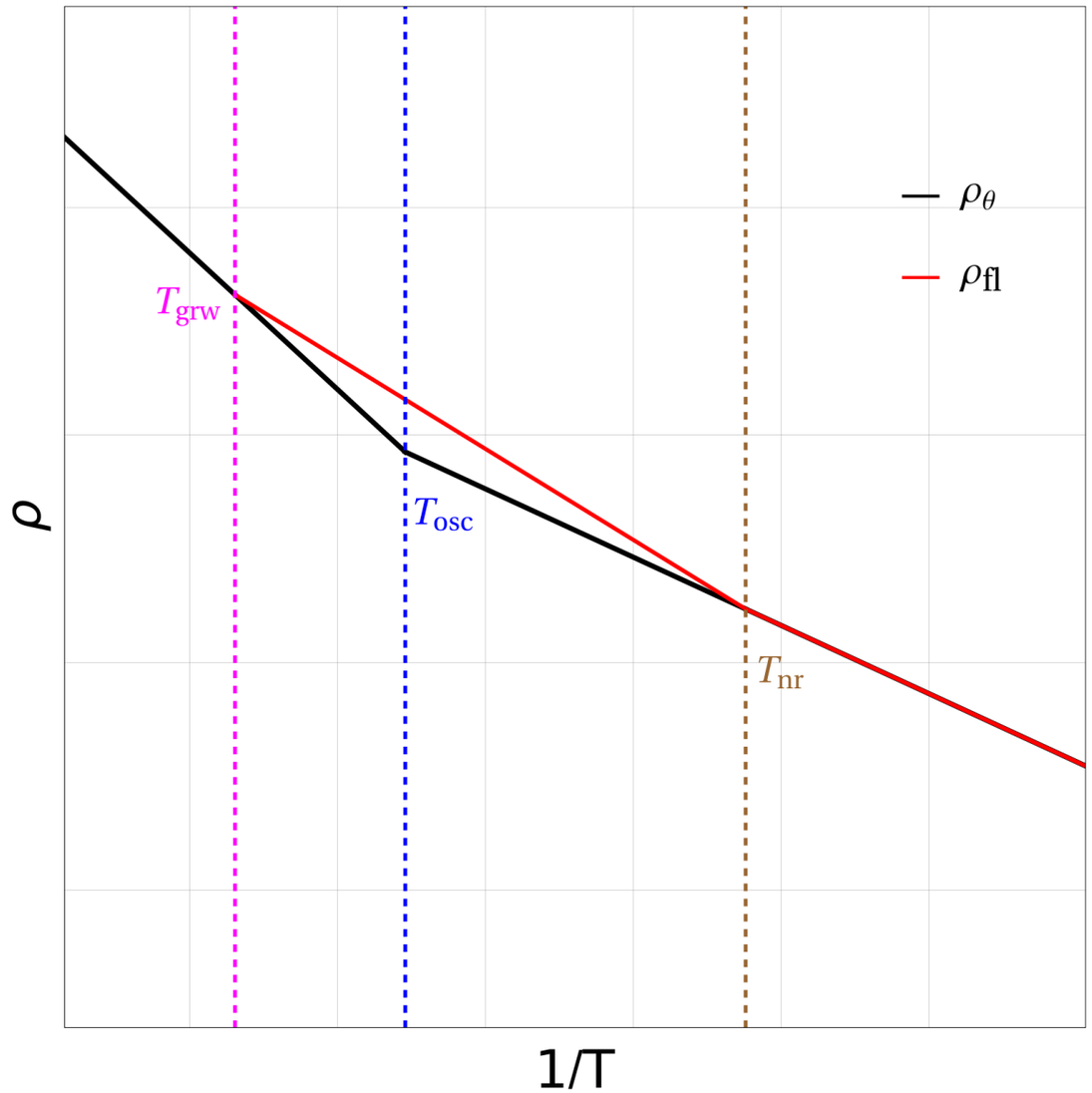

1.10
1.08
1.06
1.04
1.02
$\kappa_* = 45.5$
$\kappa_* = 45.15$
$\kappa_* = 44.7$
$\kappa \sqrt{\frac{T}{T_{\rm slide}}}$
$\sqrt{\varrho \frac{T}{T_{\rm slide}}}, \sqrt{(\varrho-1)\frac{T}{T_{\rm slide}}}$
$10^{-5}$
$10^{-4}$
0.001
0.010
0.100
1
$T/T_{\rm slide}$

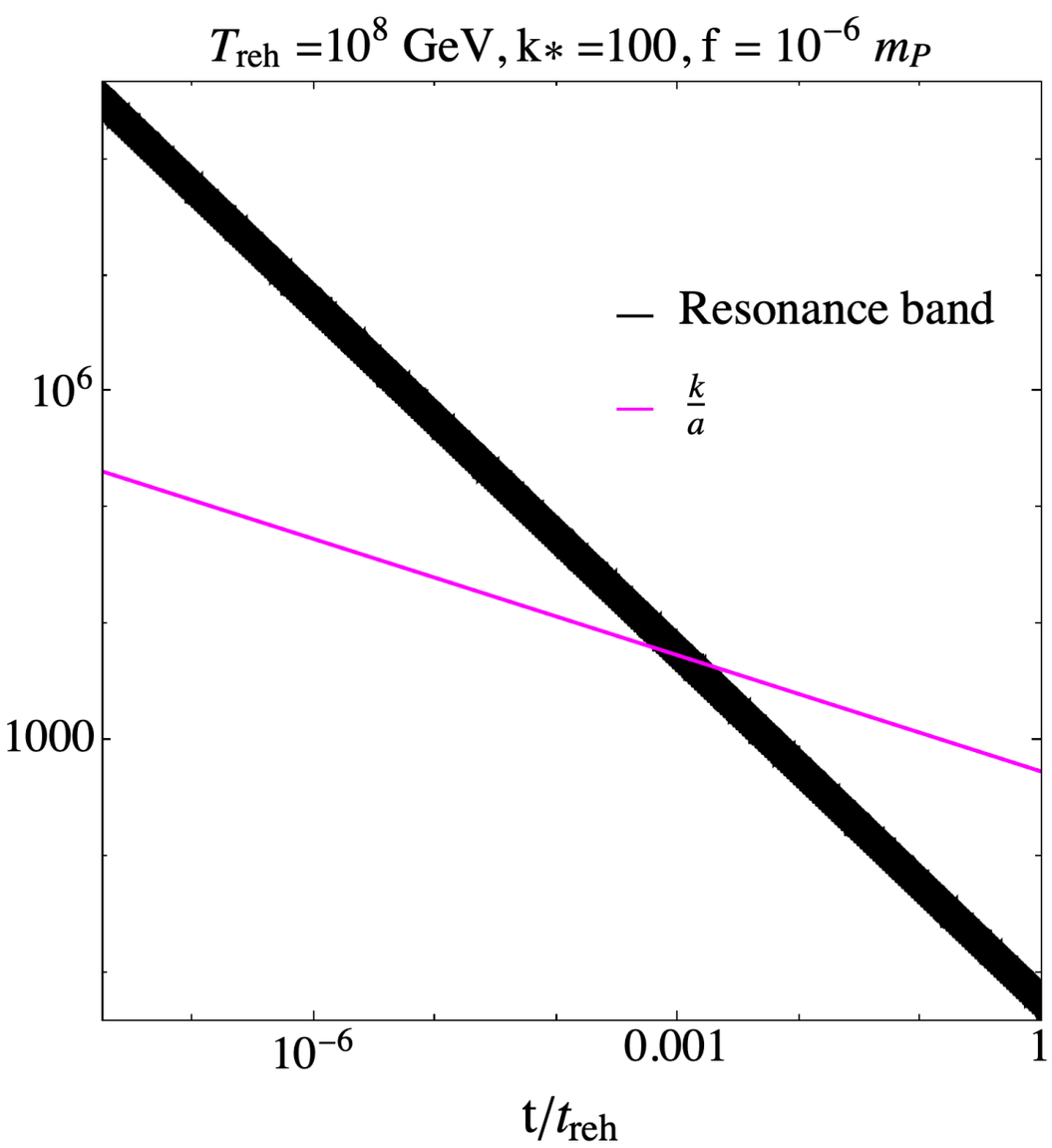

50
10
5
1
$\frac{T_{\rm grw}}{T_{\rm osc}}$ (Eq. S44)
$\kappa$
$\sqrt{\varrho}, \sqrt{\varrho-1}$
$\kappa_* = 5$
$\kappa_* = \tilde{\kappa}_* = 3.78$
$\kappa_* = 2.5$
$\kappa_* = 2$
$\kappa_* = 1.3$
1.0
1.5
2.0
2.5
3.0
$T/T_{\rm osc}$

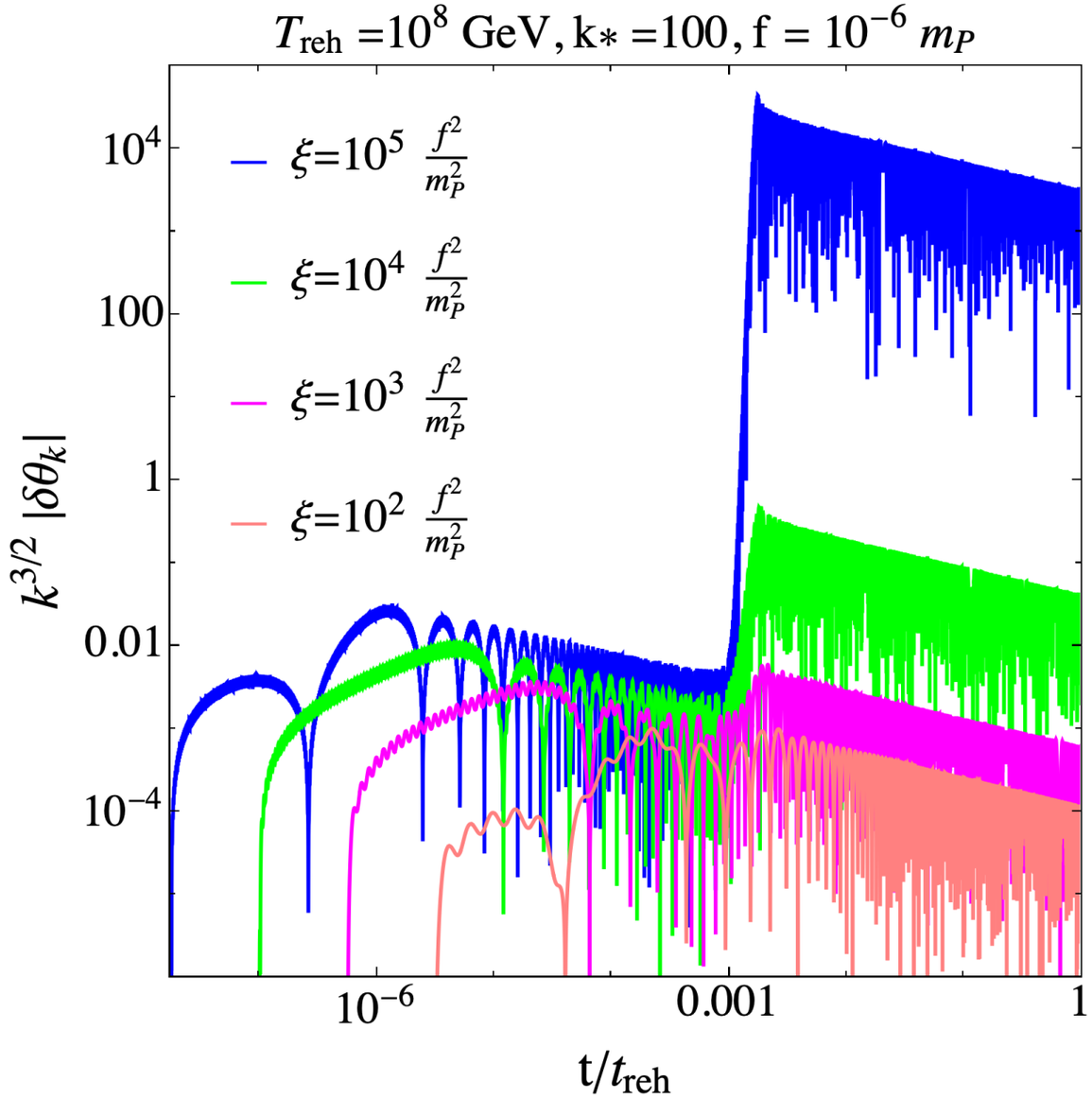

0.01
$10^{-4}$
$10^{-6}$
$10^{-8}$
$k^{3/2}\,|\delta\theta_k|$
$\kappa_* = 45.15$
$\kappa_* = 45.5$
$\kappa_* = 44.7$
$10^{-5}$
$10^{-4}$
0.001
0.010
0.100
1
$T/T_{\rm slide}$


$10^{10}$
$10^{5}$
1
$10^{-5}$
$k^{3/2}\,|\delta\theta_k|$
$\kappa_* = 2$
$\kappa_* = 2.5$
$\kappa_* = \tilde{\kappa}_* = 3.78$
$\frac{T_{\rm grw}}{T_{\rm osc}}$ (Eq. S44)
1.2
1.5
1.8
2.0
2.2
2.5
$T/T_{\rm osc}$


$T_{\kappa_*}/T_{\rm osc}$
1
1.4
1.7
2
2.2
2.4
100
1
0.01
$10^{-4}$
$\ln[N_K(t)]$
$\tilde{\kappa}_*$ (Eq. S41)
1
2
3
4
5
6
$\kappa_*$


$\rho_\theta$
$\rho_{\rm fl}$
$T_{\rm grw}$
$T_{\rm osc}$
$T_{\rm nr}$
$\rho$
1/T


$T_{\rm reh}$ =10$^8$ GeV, k∗ =100, f = 10$^{-6}$ $m_P$
Resonance band
$\frac{k}{a}$
10$^6$
1000
10$^{-6}$
0.001
1
t/$t_{\rm reh}$


$T_{\rm reh}$ =10$^8$ GeV, k∗ =100, f = 10$^{-6}$ $m_P$
$\xi$=10$^5$ $\frac{f^2}{m_P^2}$
$\xi$=10$^4$ $\frac{f^2}{m_P^2}$
$\xi$=10$^3$ $\frac{f^2}{m_P^2}$
$\xi$=10$^2$ $\frac{f^2}{m_P^2}$
$k^{3/2}$ $|\delta\theta_k|$
10$^4$
100
1
0.01
10$^{-4}$
10$^{-6}$
0.001
1
t/$t_{\rm reh}$