Neutrino inelastic scattering cross sections and non-perturbative structure functions

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Based on PRD 108, 113010 (2023)

7th Mini-Workshop on Chirality in the Universe
beyond the Electroweak Scale
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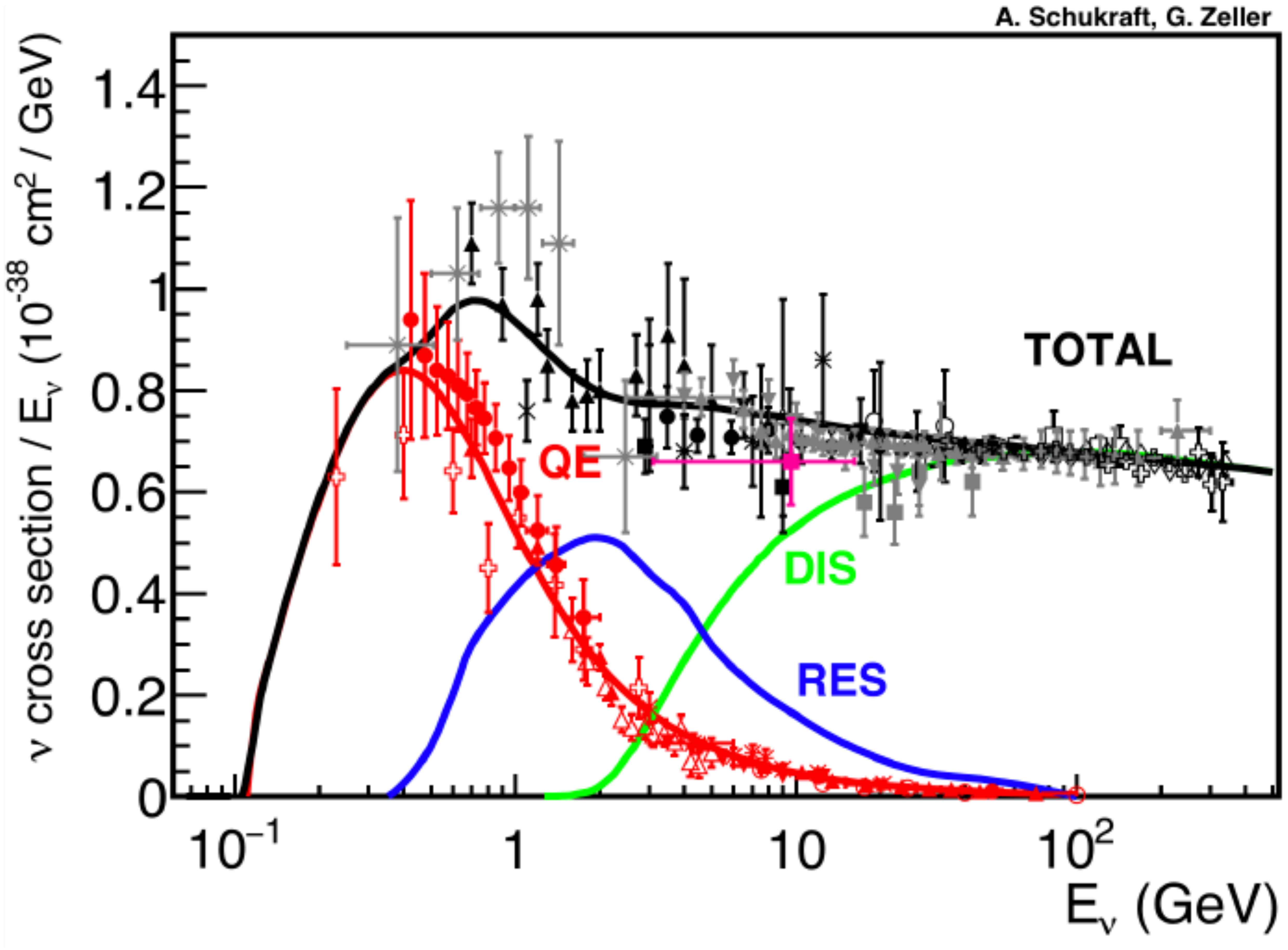
Neutrino-Nucleon interaction cross sections



- Quasi Elastic (QE)

$$\nu_\ell + n \to \ell^- + p$$

- Resonance production (RES)

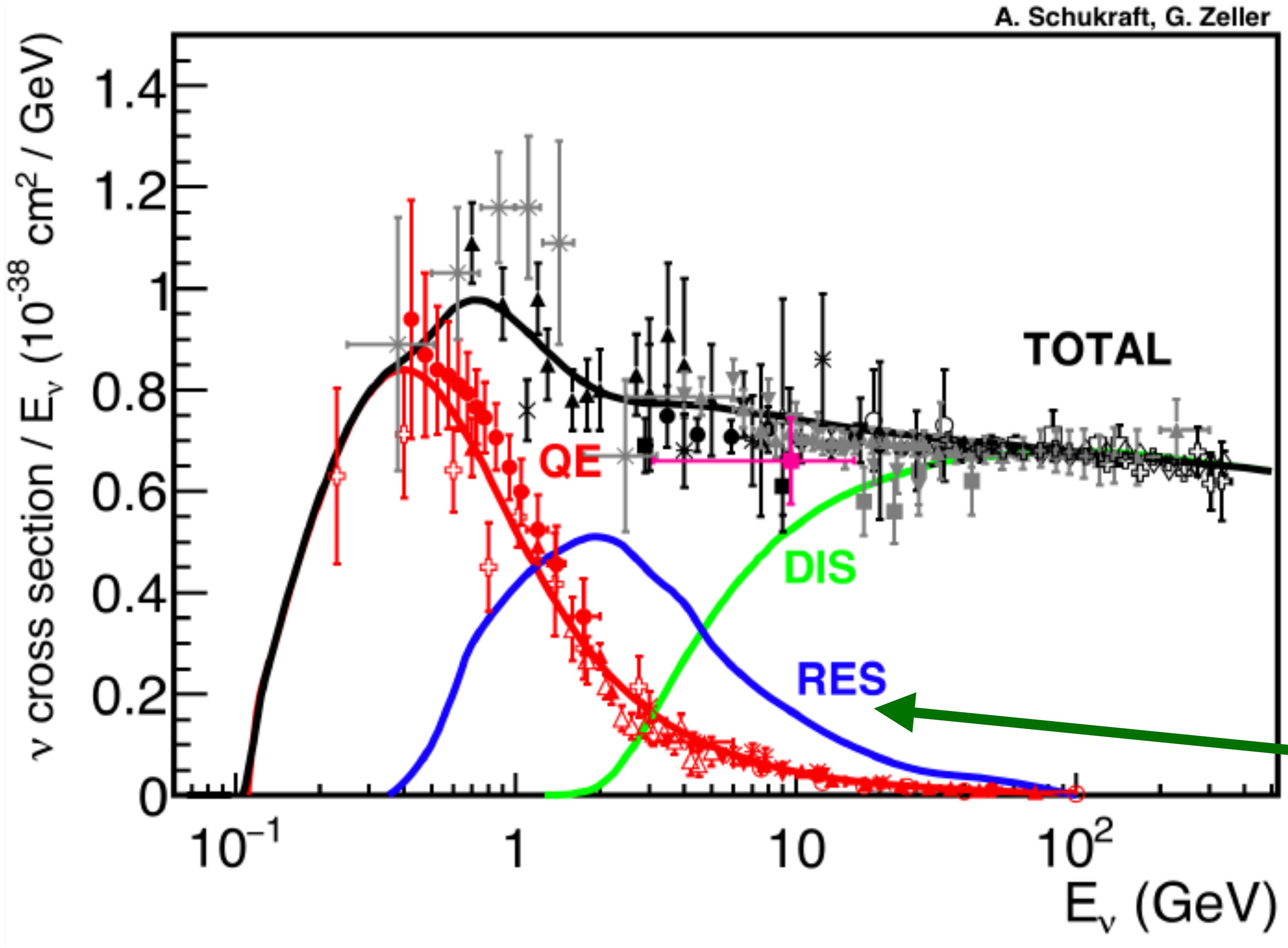
$$\nu_\ell + N \to \ell^- + \Delta \to \ell^- + N + \pi$$

- Deep Inelastic Scattering (DIS)

$$\nu_\ell + N \to \ell^- + X$$

Shallow inelastic scattering (SIS) region

- Interactions at transition region from RES to DIS



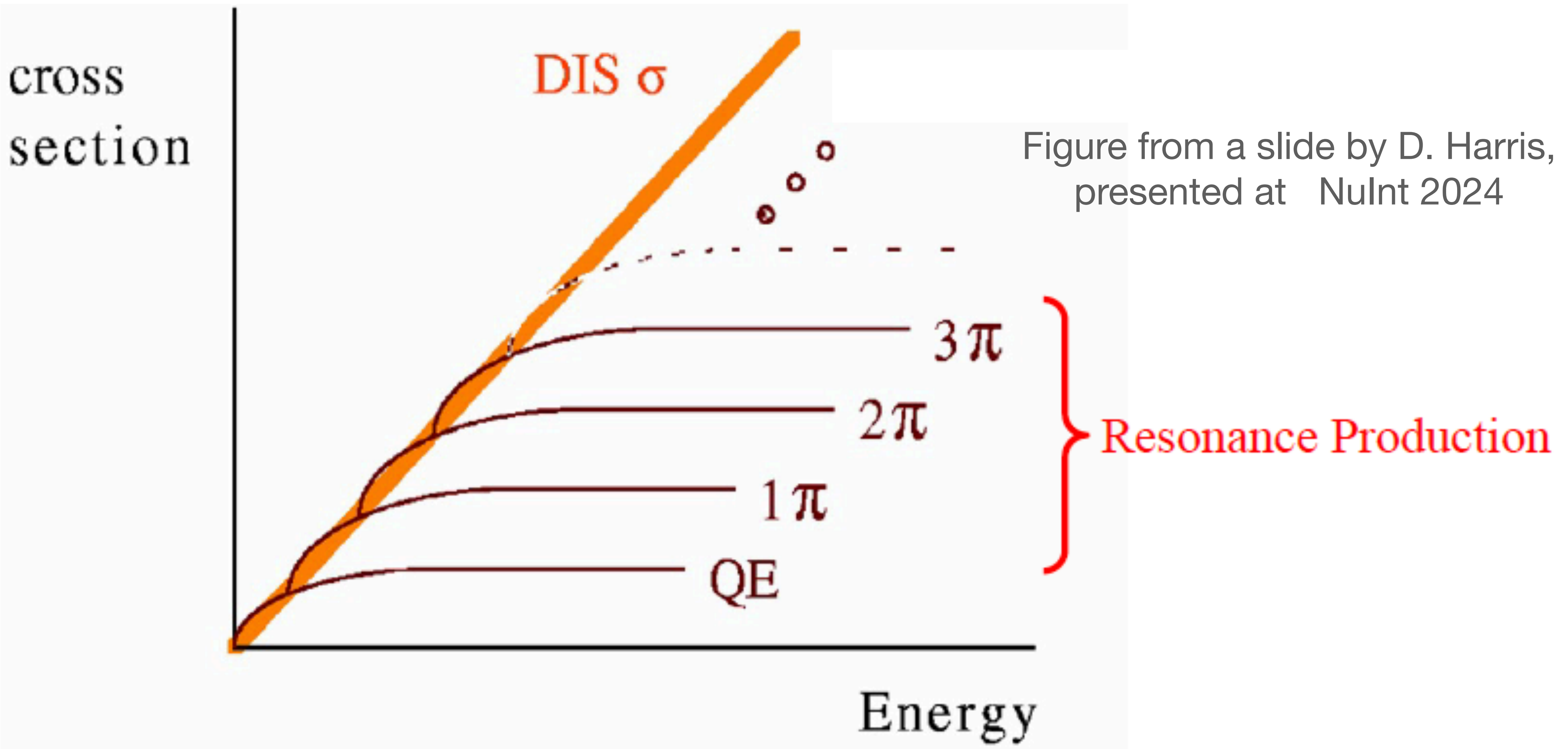
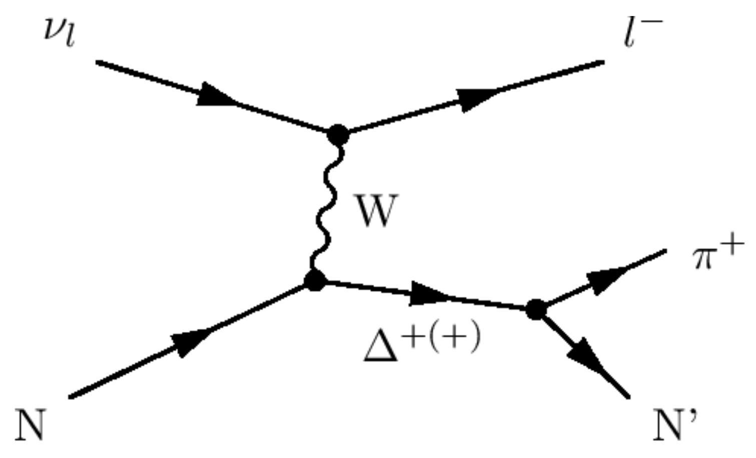


Figure from a slide by D. Harris, presented at NuInt 2024

Single pion production



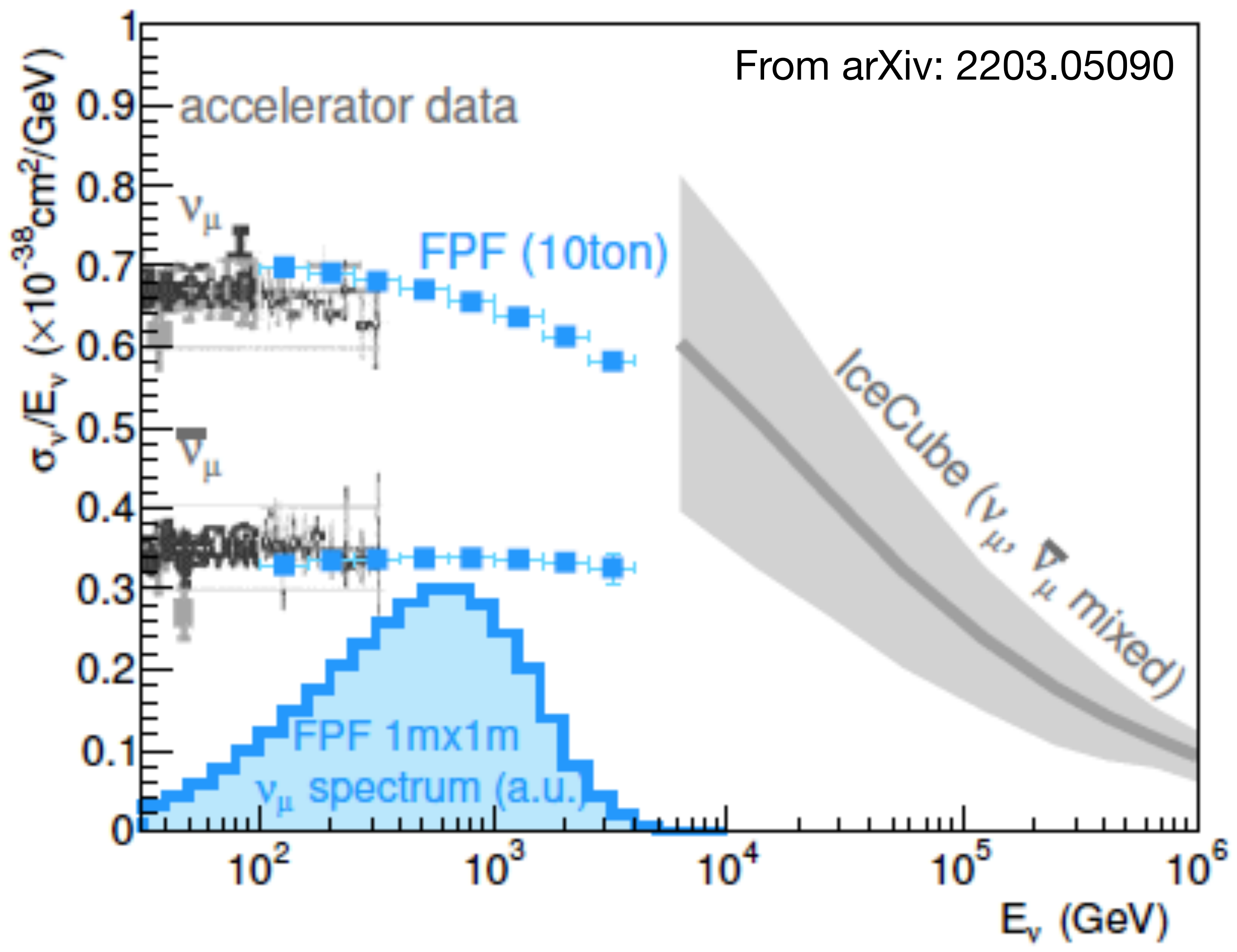
How to approach the SIS region

- Approach from the Resonance side
  - Start from single pion production and extend the model by adding multi-pions and heavier meson production channels.
  - Commonly adopted in low energy neutrino experiments and implemented in event generators.
  - Strong model dependence due to theoretical assumptions -> leads to large uncertainty.

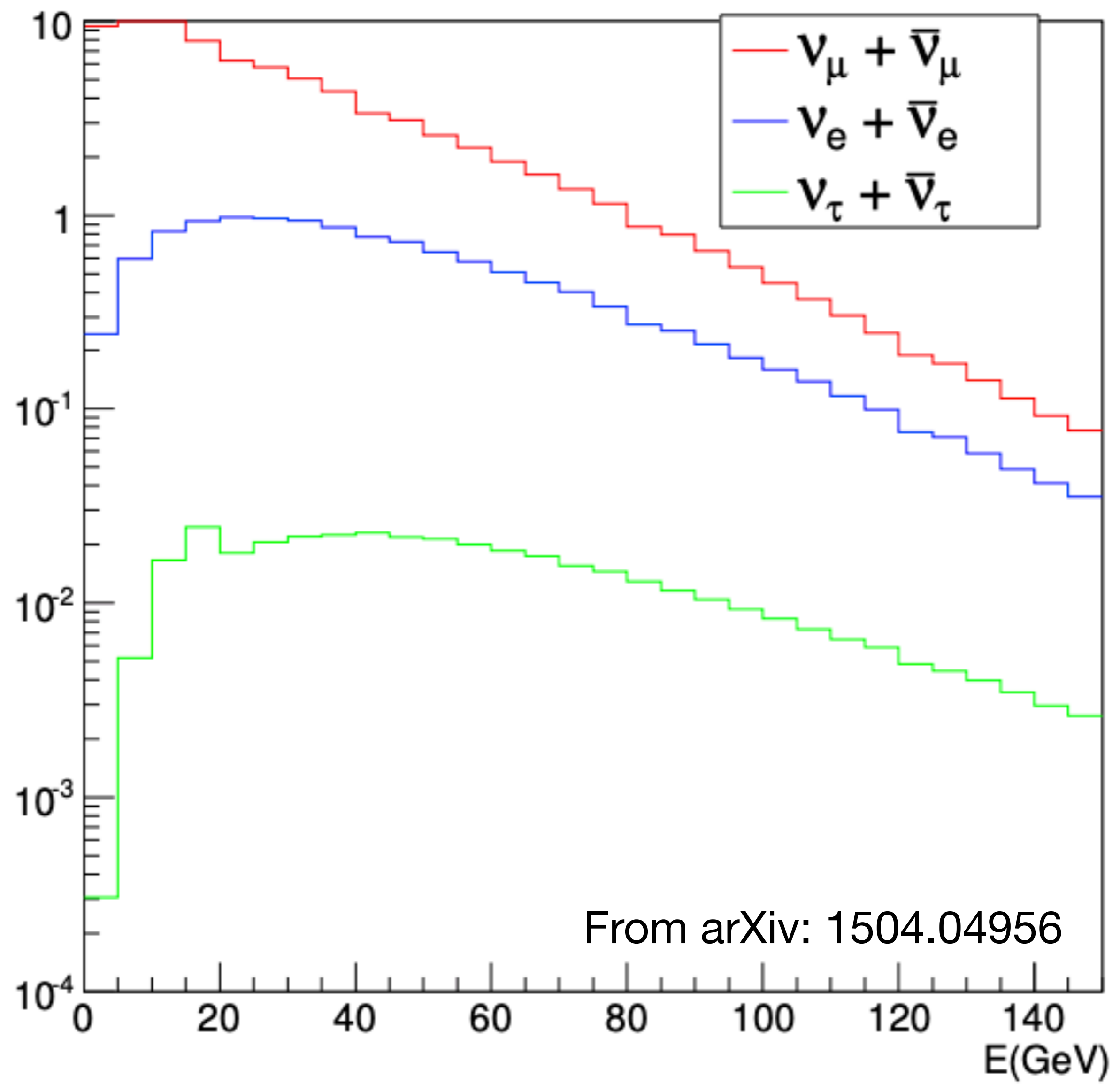
- Approach from the DIS side
  - Use the better defined DIS cross section, and extrapolate into the transition region.
  - Less model dependent than resonant-based approaches.
  - Helps constrain the SIS region by isolating the non-DIS contributions.

Neutrino experiments at CERN

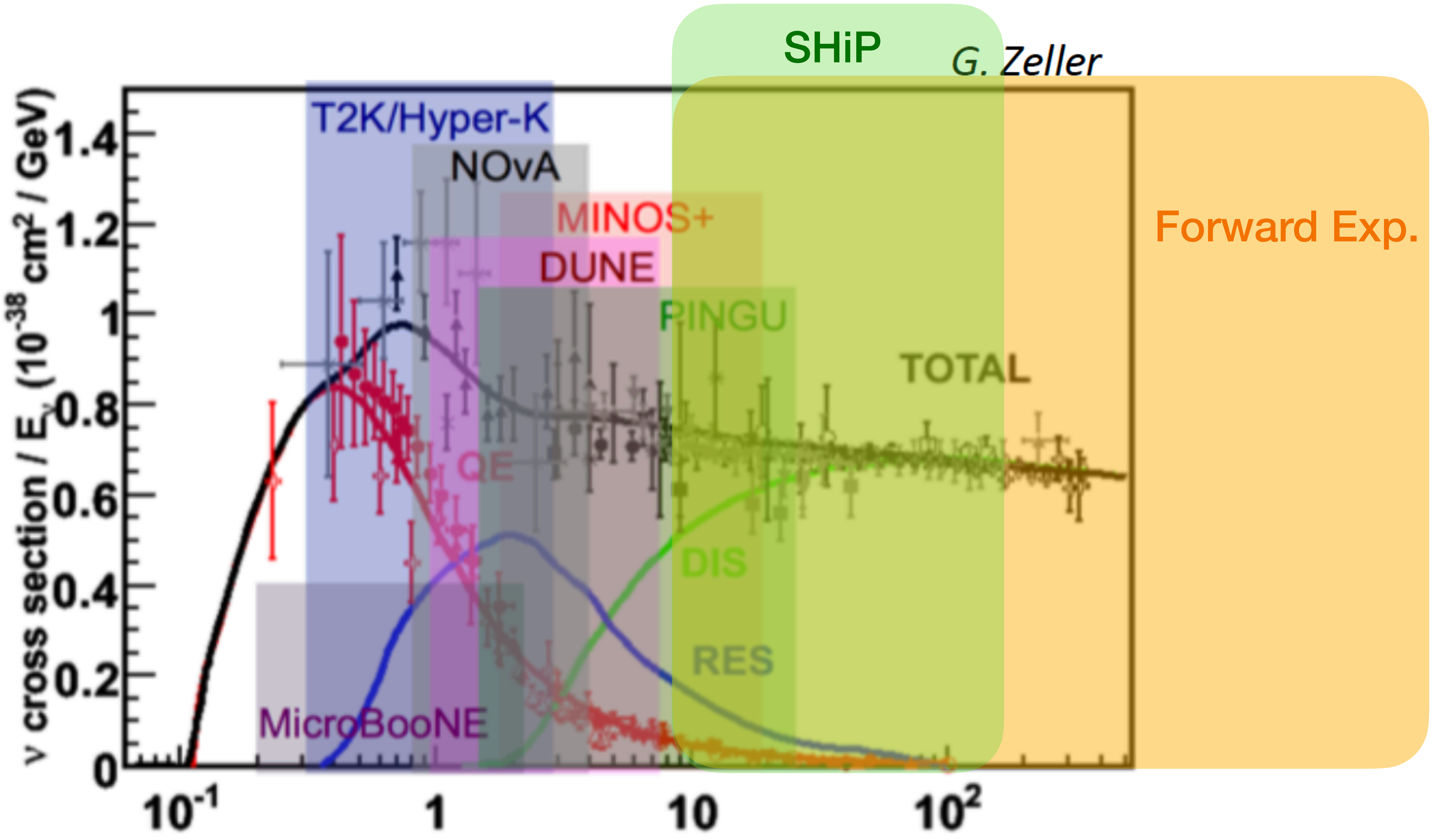
- Forward Experiments (FASERν, SND@LHC, FPF)
  - Detect LHC neutrinos in the forward region
  - Energy range: from a few GeV up to several TeV (mainly above 100 GeV).
  - Large statistics - e.g. about 10^6 for muon neutrinos at FPF.



- SHiP (beam dump @ SPS):
  - Use 400 GeV proton beam on a heavy target (e.g. Tungsten).
  - Energy range: ~10 to a few hundred GeV
  - Provide complimentary coverage in 10s GeV region, where LHC neutrino statistics are relatively low.



Neutrino experiments at CERN



Deep inelastic scattering cross sections

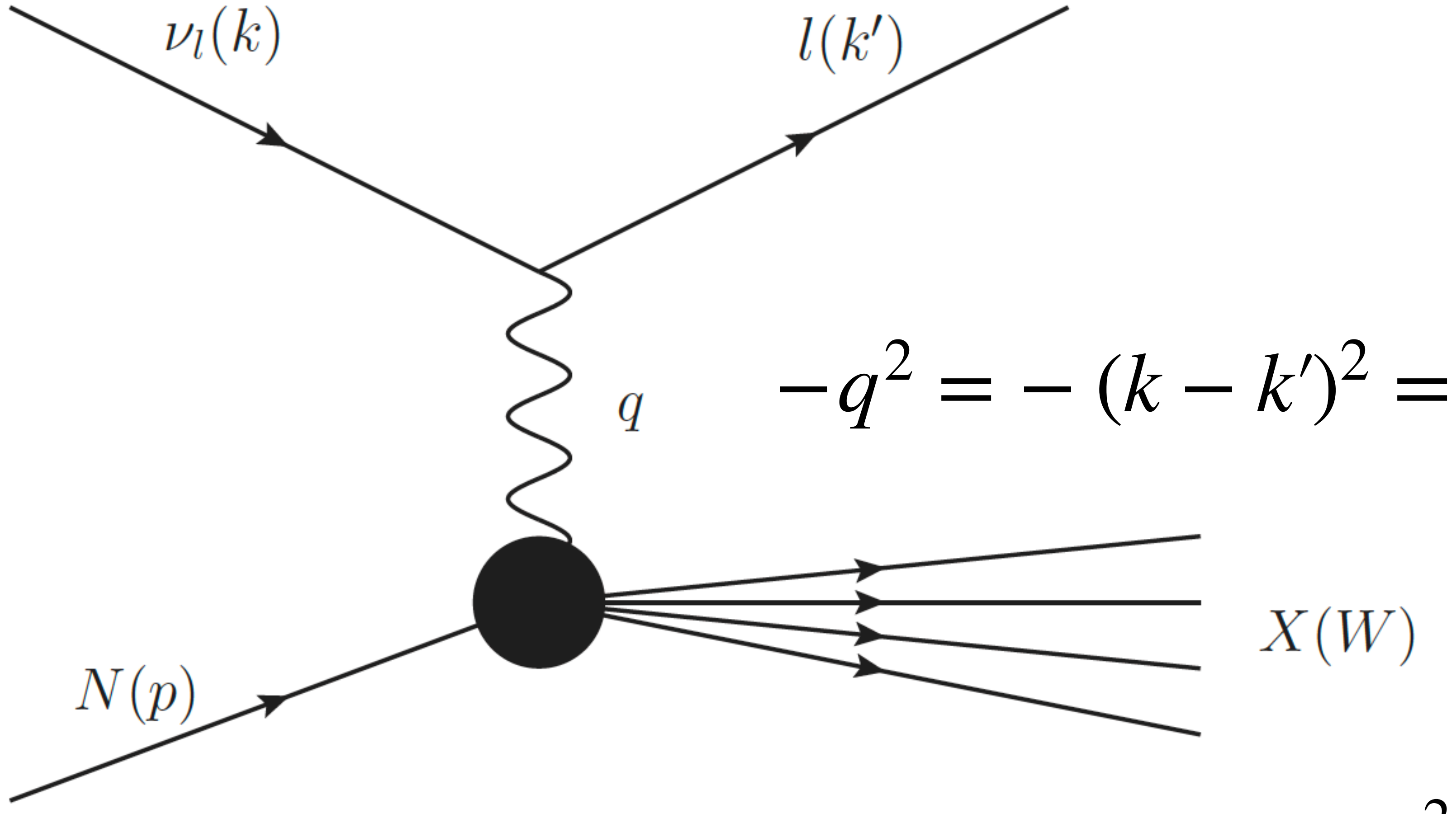
- Neutrino-nucleon charged-current (CC) cross section for deep inelastic scattering (DIS)

$$\frac{d^2\sigma^{\nu(\bar{\nu})}}{dx\,dy} = \frac{G_F^2 m_N E_\nu}{\pi(1+Q^2/M_W^2)^2}\Bigg(\left(y^2 x + \frac{m_\ell^2 y}{2E_\nu m_N}\right)F_{1,\mathrm{CC}}(x,Q^2) + \left[\left(1-\frac{m_\ell^2}{4E_\nu^2}\right) - \left(1+\frac{m_N x}{2E_\nu}\right)y\right]F_{2,\mathrm{CC}}(x,Q^2)$$

$$\pm\left[xy\left(1-\frac{y}{2}\right) - \frac{m_\ell^2 y}{4E_\nu m_N}\right]F_{3,\mathrm{CC}}(x,Q^2) + \frac{m_\ell^2(m_\ell^2+Q^2)}{4E_\nu^2 m_N^2 x}F_{4,\mathrm{CC}}(x,Q^2) - \frac{m_\ell^2}{E_\nu m_N}F_{5,\mathrm{CC}}(x,Q^2)\Bigg)$$

$y = (E - E')/E$

Y. S. Jeong and M. H. Reno, *Phys. Rev. D 81 (2010) 114012*



$$-q^2 = -(k-k')^2 = Q^2$$

$$W^2 = Q^2\left(\frac{1}{x}-1\right)+m_N^2$$

- Kinematic region for DIS:

  (e.g.) $W > 2$ GeV and $Q^2 > 1$ GeV^2

Structure functions for DIS cross section

- Neutrino-nucleon charged-current (CC) cross section for deep inelastic scattering (DIS)

$$\frac{d^2\sigma^{\nu(\bar{\nu})}}{dx\,dy} = \frac{G_F^2 m_N E_\nu}{\pi(1+Q^2/M_W^2)^2}\left(\left(y^2x + \frac{m_\ell^2 y}{2E_\nu m_N}\right)F_{1,\mathrm{CC}}(x,Q^2) + \left[\left(1-\frac{m_\ell^2}{4E_\nu^2}\right) - \left(1+\frac{m_N x}{2E_\nu}\right)y\right]F_{2,\mathrm{CC}}(x,Q^2)\right.$$

$$\left. \pm \left[xy\left(1-\frac{y}{2}\right) - \frac{m_\ell^2 y}{4E_\nu m_N}\right]F_{3,\mathrm{CC}}(x,Q^2) + \frac{m_\ell^2(m_\ell^2+Q^2)}{4E_\nu^2 m_N^2 x}F_{4,\mathrm{CC}}(x,Q^2) - \frac{m_\ell^2}{E_\nu m_N}F_{5,\mathrm{CC}}(x,Q^2)\right)$$

Y. S. Jeong and M. H. Reno, *Phys. Rev. D 81 (2010) 114012*

- F2 and F3 are the dominant components:
  - F2 dominantly contributes to the total cross sections.
  - F3 is responsible for difference between neutrino and antineutrino.

- The terms related with F4 and F5 are considerable for tau neutrinos, otherwise suppressed.

- Some structure functions are related with F2 by theoretical constraints.

$$2xF_1 = F_2$$

(Callan-Gross relation at LO)

$$F_4 \approx 0,\; F_5 \approx F_2/2x$$

Structure functions at low Q^2

- At large Q^2, structure functions can be expressed with parton distribution functions (PDFs) using perturbative QCD.

(e.g.) $$F_{2,\mathrm{CC}}(x,Q^2) = \sum_{q,q'} 2x\left(q(x,Q^2) + \bar{q}'(x,Q^2)\right)$$ (at leading order in pQCD)

- For low Q2 region ($Q^2 < 1\ \mathrm{GeV}^2$), perturbative QCD is no longer applicable, so PDF-based structure functions become unreliable.

- In the non-perturbative region, structure functions must be constructed phenomenologically by fitting to experimental data.

Low Q^2 structure function models

Model	Fitting data	Features	Notes
Bodek-Yang (BY)	Charged lepton DIS data	- Extend SF derived from GRV98 LO PDF to low Q^2 - Tuned to data using a scaling variable and K-factor $F_2(x, Q^2 < 0.8\,\text{GeV}^2) = K(Q^2) \times F_2\left(\xi_\omega, Q^2 = 0.8\,\text{GeV}^2\right)$	Implemented in most neutrino event generators
CKMT	Charged Lepton DIS data	- Theory-motivated, based on Regge theory - Developed for EM structure functions	Need adjustment to use for neutrino interactions
NNSFν	Neutrino DIS data	- Obtained using machine learning techniques - Fully data-driven model	

CKMT parameterization

A. Capella, A. Kaidalov, C. Merino and J. Tran Thanh Van,
Phys. Lett. B 337, 358 (1994)

- CKMT parameterization was constructed as electromagnetic structure function.

$$F_{2,EM}^{\mathrm{CKMT}}(x,Q^2) = Ax^{-\Delta(Q^2)}(1-x)^{n(Q^2)+4}\left(\frac{Q^2}{Q^2+a}\right)^{1+\Delta(Q^2)} \rightarrow F_2^{sea}$$

$$+Bx^{1-\alpha_R}(1-x)^{n(Q^2)}\left(\frac{Q^2}{Q^2+b}\right)^{\alpha_R}\left(1+f(1-x)\right) \rightarrow F_2^{valence}$$

$$n(Q^2) = \frac{3}{2}\left(1+\frac{Q^2}{Q^2+c}\right)$$

$$\Delta(Q^2) = \Delta_0\left(1+\frac{2Q^2}{Q^2+d}\right)$$

- To apply for neutrino-nucleon charged-current scattering, the same functional form is used for F2 and xF3, and normalization parameters are modified.

M. H. Reno, *Phys. Rev. D74 (2006) 033001*
Y. S. Jeong and M. H. Reno, *arXiv:2307.09241*

Δ_0	α_R	a [GeV^2]	b [GeV^2]	c [GeV^2]	d [GeV^2]
0.07684	0.4150	0.2631	0.6452	3.5489	1.1170

Process	A	B	f
EM F_2	0.1502	1.2064	0.15
νN F_2	0.5967	2.7145	0.5962
νN xF_3	9.3955×10^{-3}	2.4677	0.5962
$\bar{\nu} N$ xF_3	9.3955×10^{-3}	−2.4677	0.5962

PCAC

- PCAC (partially conserved axial-vector current)
  - Axial vector current is not completely conserved due to chiral symmetry breaking.
  - PCAC relates divergence of axial vector current to the pion field:

$$\partial^{\mu} A_{\mu}^{a}(x) \approx f_{\pi} m_{\pi}^{2} \phi^{a}(x)$$

  - It leads to a non-zero structure function in low Q^2 limit through longitudinal component F_L.

- Why do we need PCAC?
  - CKMT structure functions are constructed based on EM data, thus vanish as $Q^2 \to 0$,

$$\sigma_{\gamma p}^{\text{tot}} = \left[\frac{4\pi^2 \alpha_{\text{EM}}}{Q^2} F_{2,\text{EM}}(x, Q^2)\right]_{Q^2 \to 0}$$

  - Weak interactions have additional contributions from the axial-vector current.

PCAC corrections: Kulagin-Petti approach

- The PCAC contributes to F2 through longitudinal component:

$$F_2 = (F_L + F_T)/(1+\gamma^2) \qquad \text{where} \quad \gamma^2 = 4x^2M^2/Q^2$$

- PCAC contribution to F_L (Kulagin-Petti):

$$F_L^{\mathrm{PCAC}} = \frac{f_\pi^2 \sigma_\pi(W^2)}{\pi} f_{\mathrm{PCAC}}(Q^2), \qquad f_{\mathrm{PCAC}}(Q^2) = \left(1 + \frac{Q^2}{M_{\mathrm{PCAC}}^2}\right)^{-2}$$

$$f_\pi = 0.93\, m_\pi\,, \quad \sigma_\pi \simeq X(W^2)^\epsilon + Y(W^2)^{-\eta_1}, \quad M_{\mathrm{PCAC}} = 0.8\,\mathrm{GeV}$$

- PCAC correction is approximately incorporated into CKMT parameterizations using $f_{\mathrm{PCAC}}(Q^2)$, so that F_2 remains non-zero in the low Q^2 limit.

S. A. Kulagin, and R. Petti,
Phys. Rev. D 76, 094023 (2007)

Structure function used in evaluation

- Structure functions are obtained using nCTEQ PDFs, NLO QCD and target mass correction (TMC) for $Q^2 > Q_0^2$, and patched with a phenomenological structure functions for $Q^2 < Q_0^2$.

- The low Q structure functions:

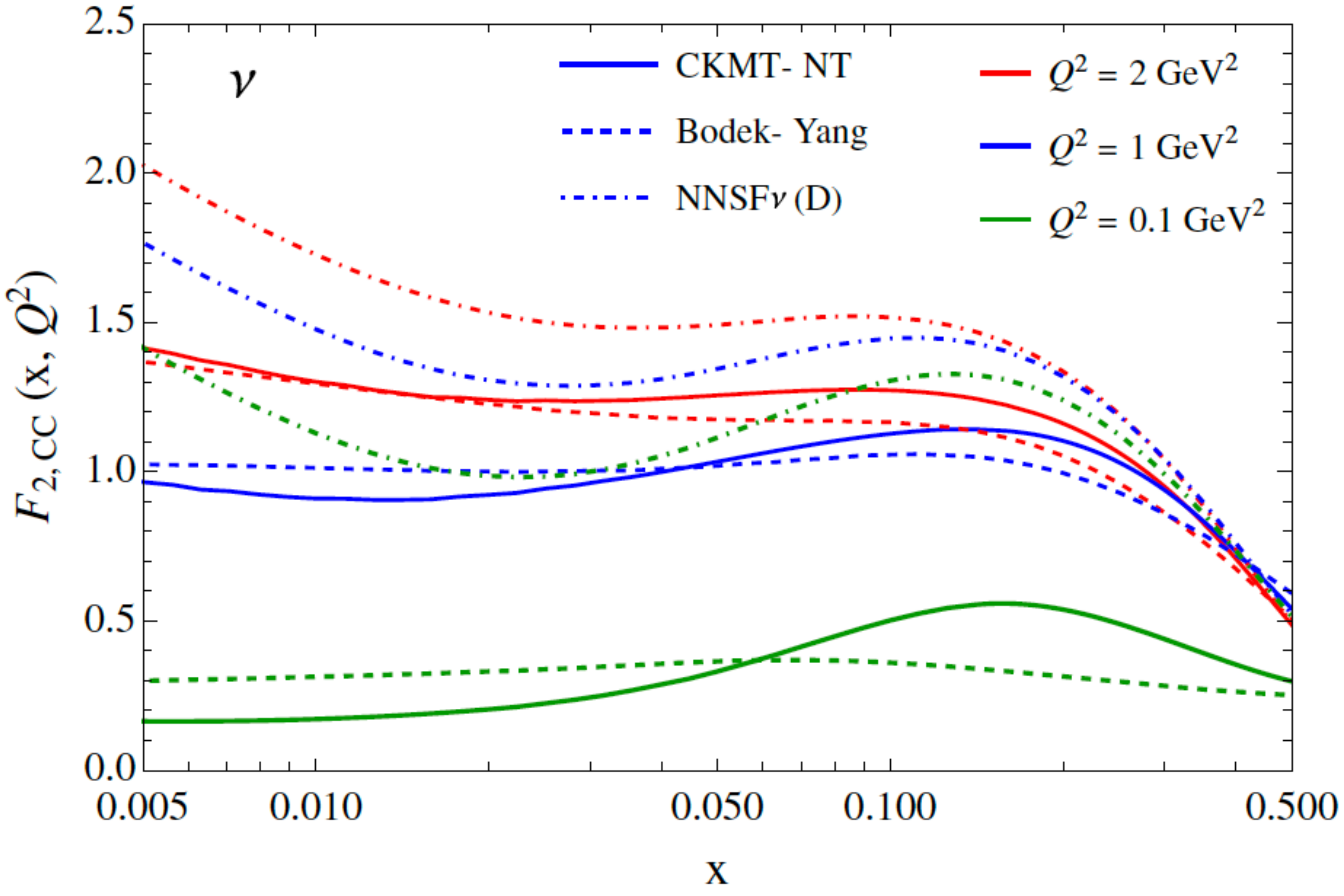
$$F_{2,\mathrm{CC}}(x, Q^2) = F_{2,\mathrm{CC}}^{\mathrm{CKMT}}(x, Q^2) + F_{2,\mathrm{CC}}^{\mathrm{PCAC}}(x, Q^2)$$

$$F_{3,\mathrm{CC}}(x, Q^2) = F_{3,\mathrm{CC}}^{\mathrm{CKMT}}(x, Q^2)$$

F_2 structure functions from different models

Y. S. Jeong and M. H. Reno
Phys. Rev. D 108, 113010 (2023)

✳ NT: NLO+TMC



- Discrepancy becomes large at low Q^2.
- CKMT vs. Bodek-Yang:

  Comparable at high Q^2, but the difference becomes noticeable at Q^2 = 0.1 GeV^2.
- NNSFν:

  Shows distinct behavior in both shape and magnitude.

Effect of PCAC on F_2

Y. S. Jeong and M. H. Reno
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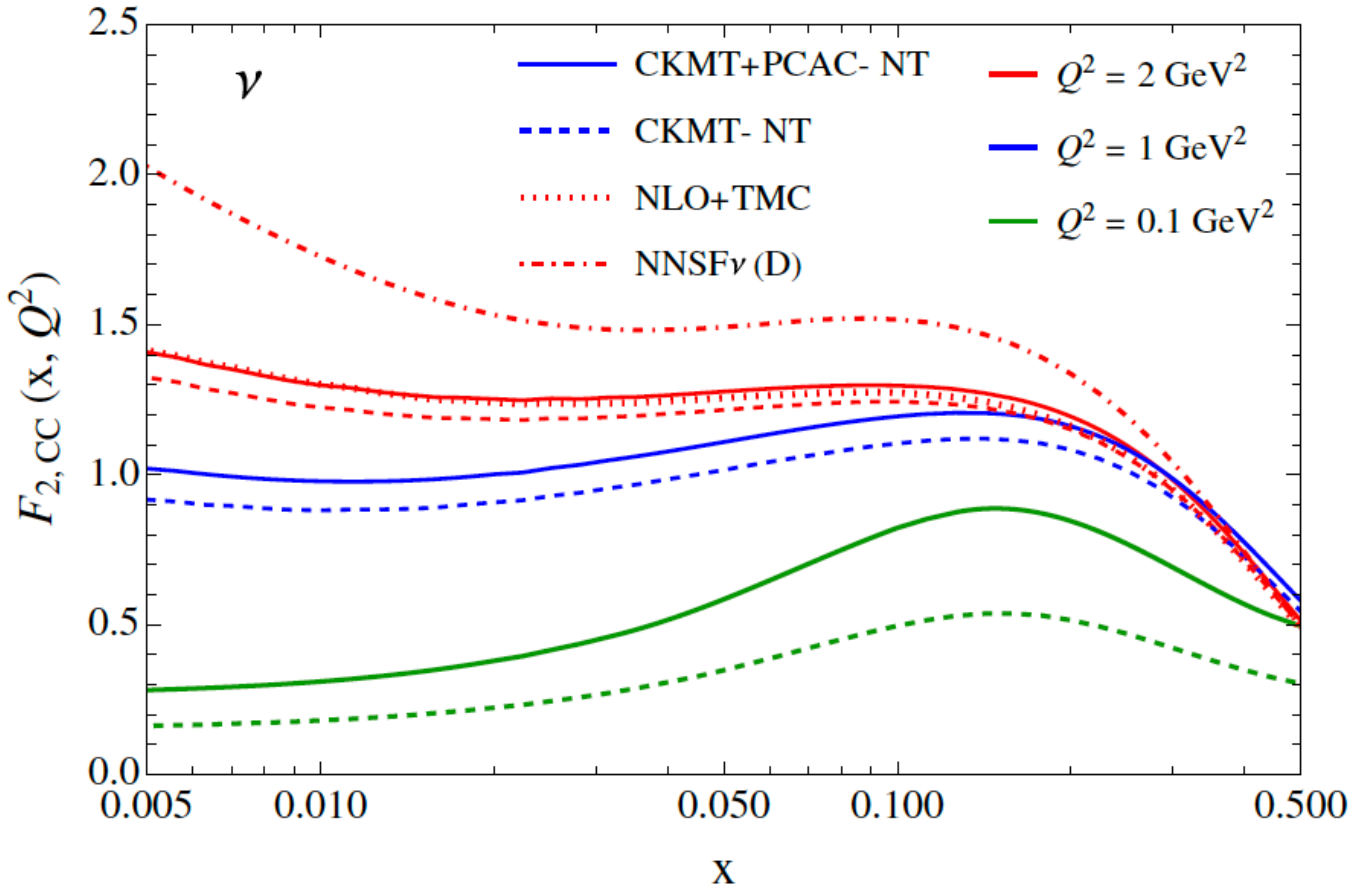

- CKMT F_2 with and without PCAC correction
- PCAC adds a non-zero contribution to F2.
- Correction becomes more visible at low Q^2.

DIS charge current cross sections

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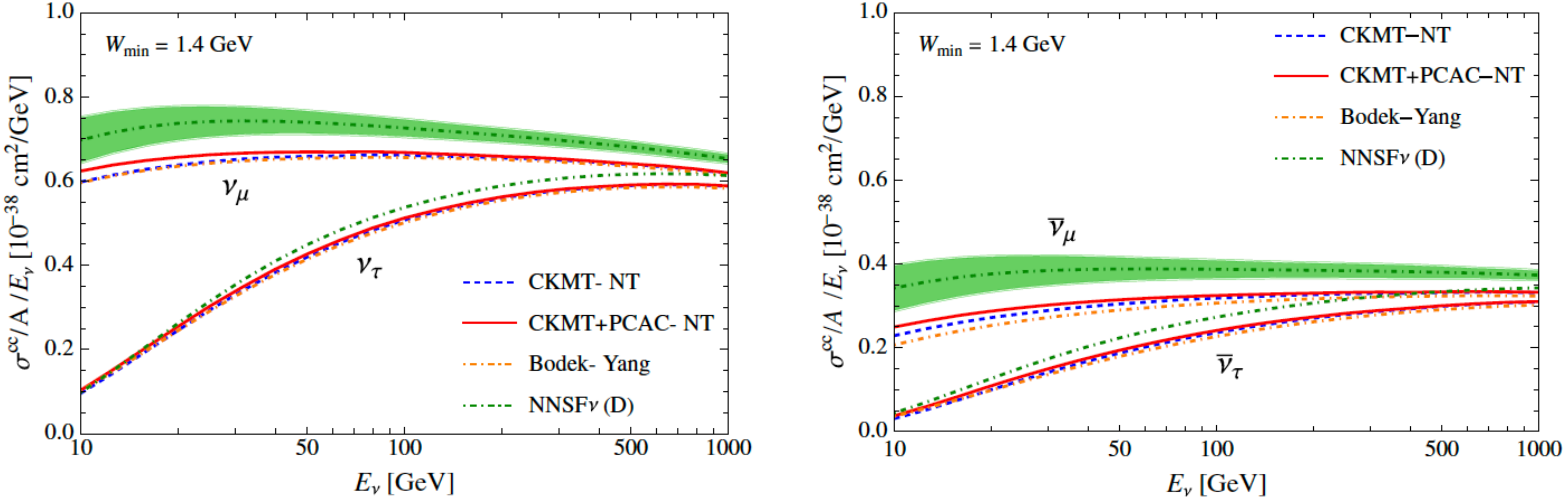
✳ NT: NLO+TMC



- CKMT-based cross sections are very close to the Bodek-Yang results across the full energy range, with differences within about 10% (20%) even at 10 GeV for muon neutrinos (tau neutrinos).
- NNSFν predictions are consistently higher than both CKMT and BY over the presented energy range.

Comparison with data

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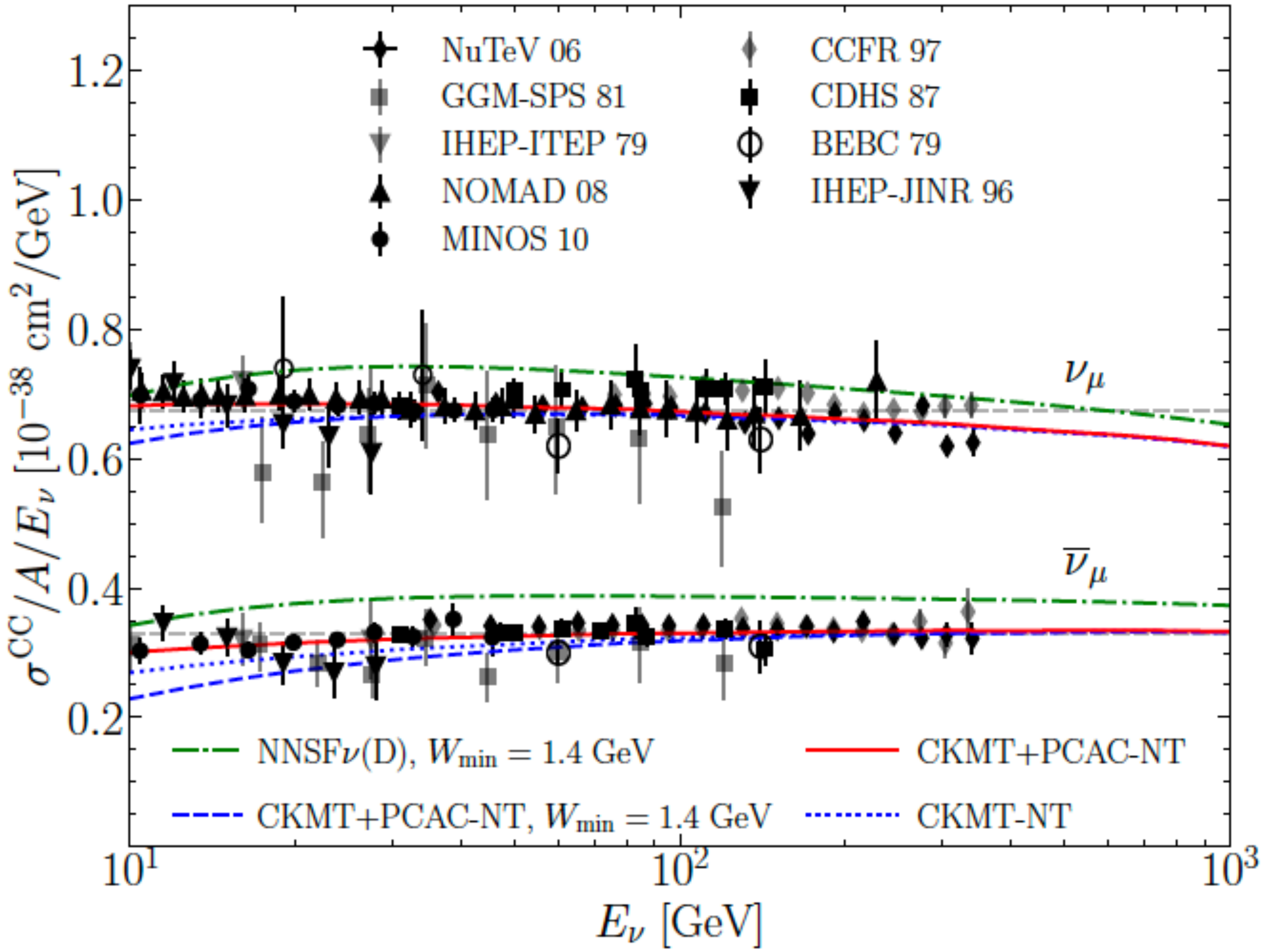


- Cross sections per nucleon, scaled by E_ν
- Red: $W_{\rm min} = m_N + m_\pi$
  - Includes all inelastic CC interactions beyond quasi-elastic (QE) scattering.
  - Shows good agreement with experimental data.
- Red vs. Blue dotted:
  - PCAC correction improves agreement with the data, especially at low energies

Summary and Outlook

- Neutrino interactions at intermediate energies (10–100 GeV) are crucial for current and future accelerator-based neutrino experiments.

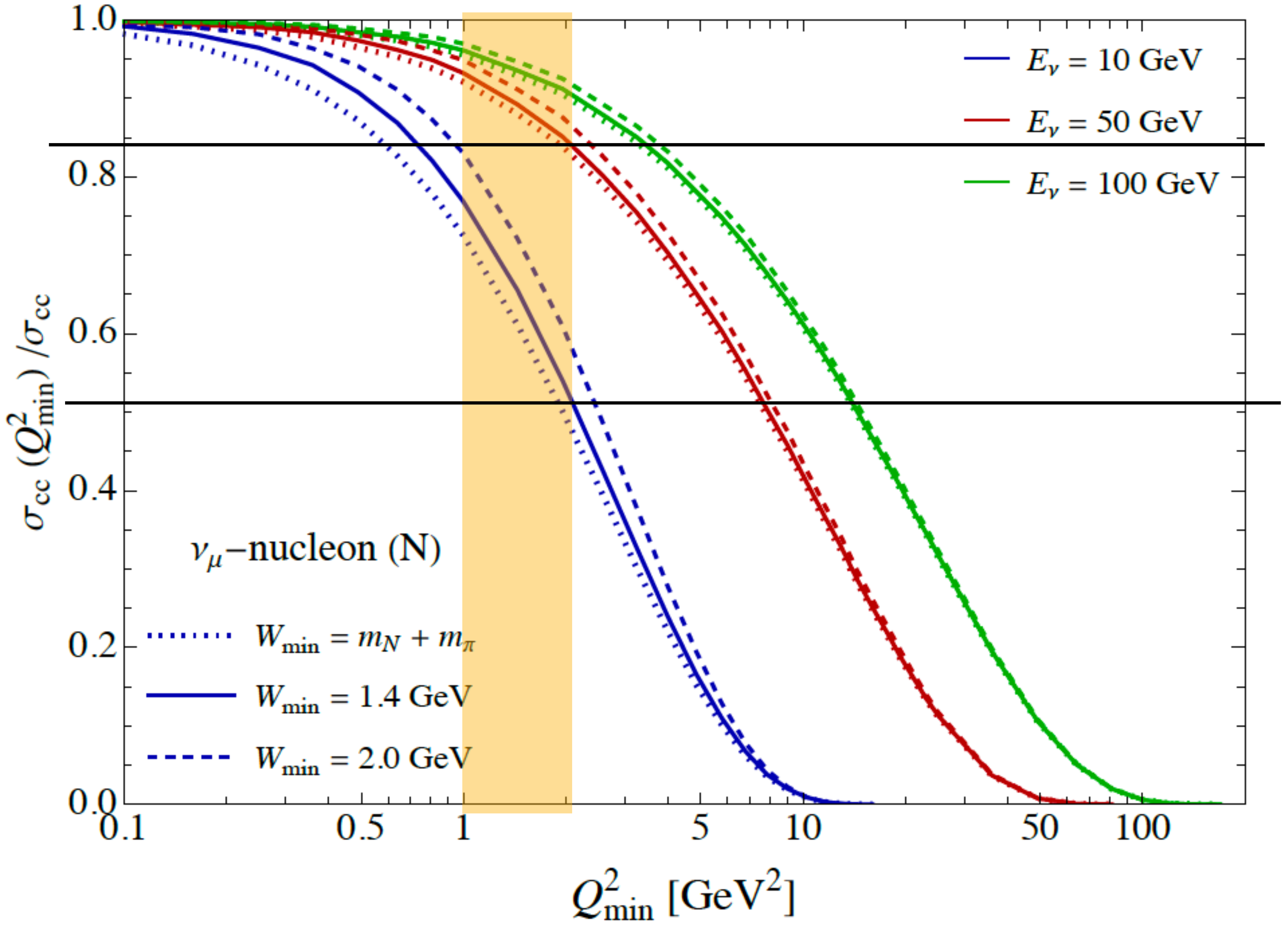
- Low-Q^2 structure functions plays an important role in neutrino cross section prediction in the SIS region, where clear model dependence leads to significant discrepancies.

- Our evaluation using CKMT model with PCAC correction is in good agreement with existing data, emphasizing the role of axial current contribution. However:
  - CKMT was developed decades ago using electromagnetic scattering data.
  - The recent model NNSFν based on neutrino data still shows deviations.

- Ongoing efforts aim to improve structure function model for reliable cross section predictions, which can have implications for low energy oscillation experiments.

Thank you for your attention

W and Q^2 dependence (muon neutrinos)



- For E>100 GeV, the contribution of low Q, is small regardless of W, but it becomes larger at lower energy, and depends on W.

- (e.g.) when E = 50 GeV and W_{min} = 1.4 GeV, cross section for $Q^2 < 2$ GeV^2 is about 15% of the total CC cross section for muon neutrinos, and ~30% for antineutrinos.

Comparison with NNSFν for Fe target and W > 2 GeV

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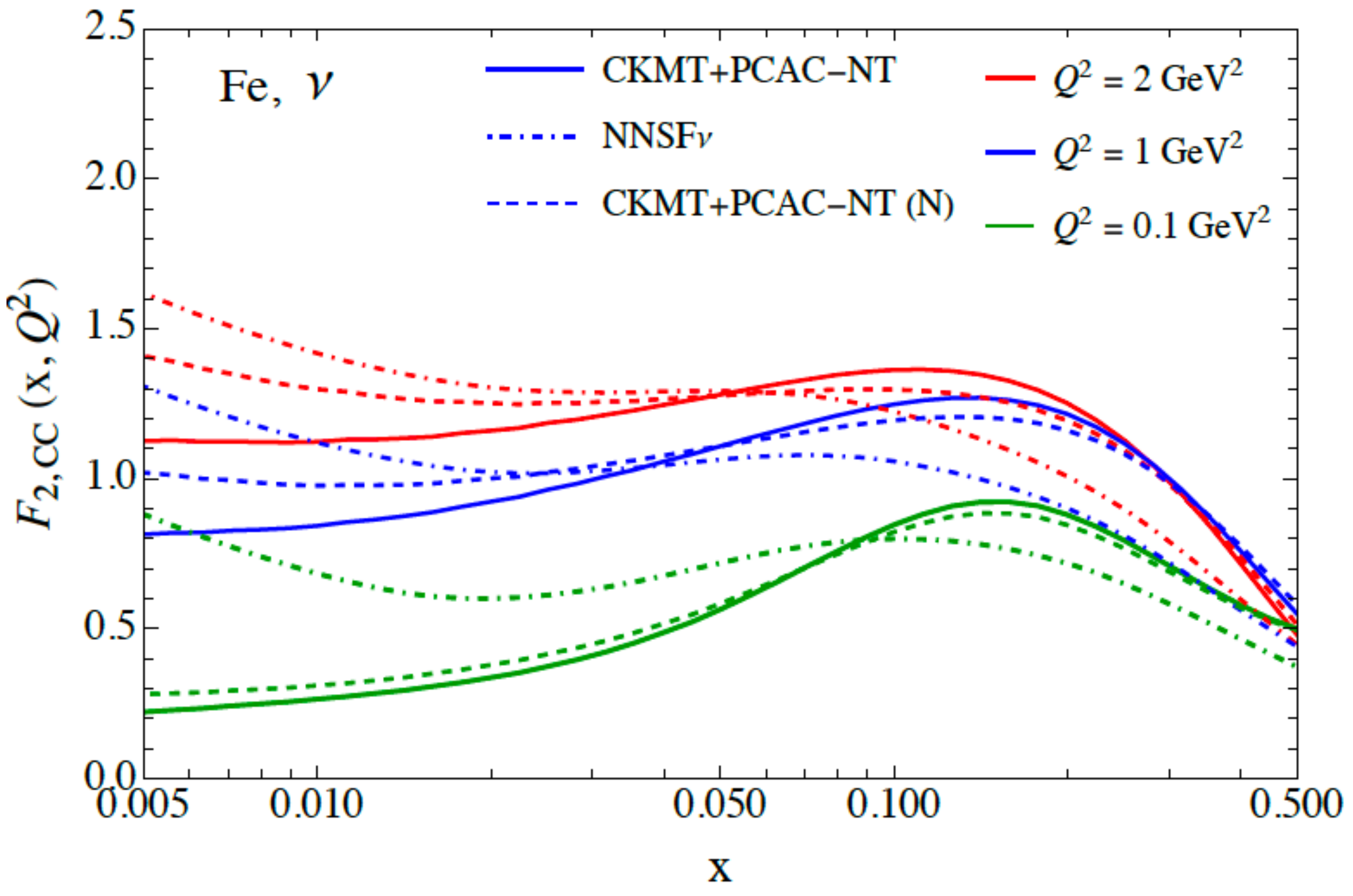

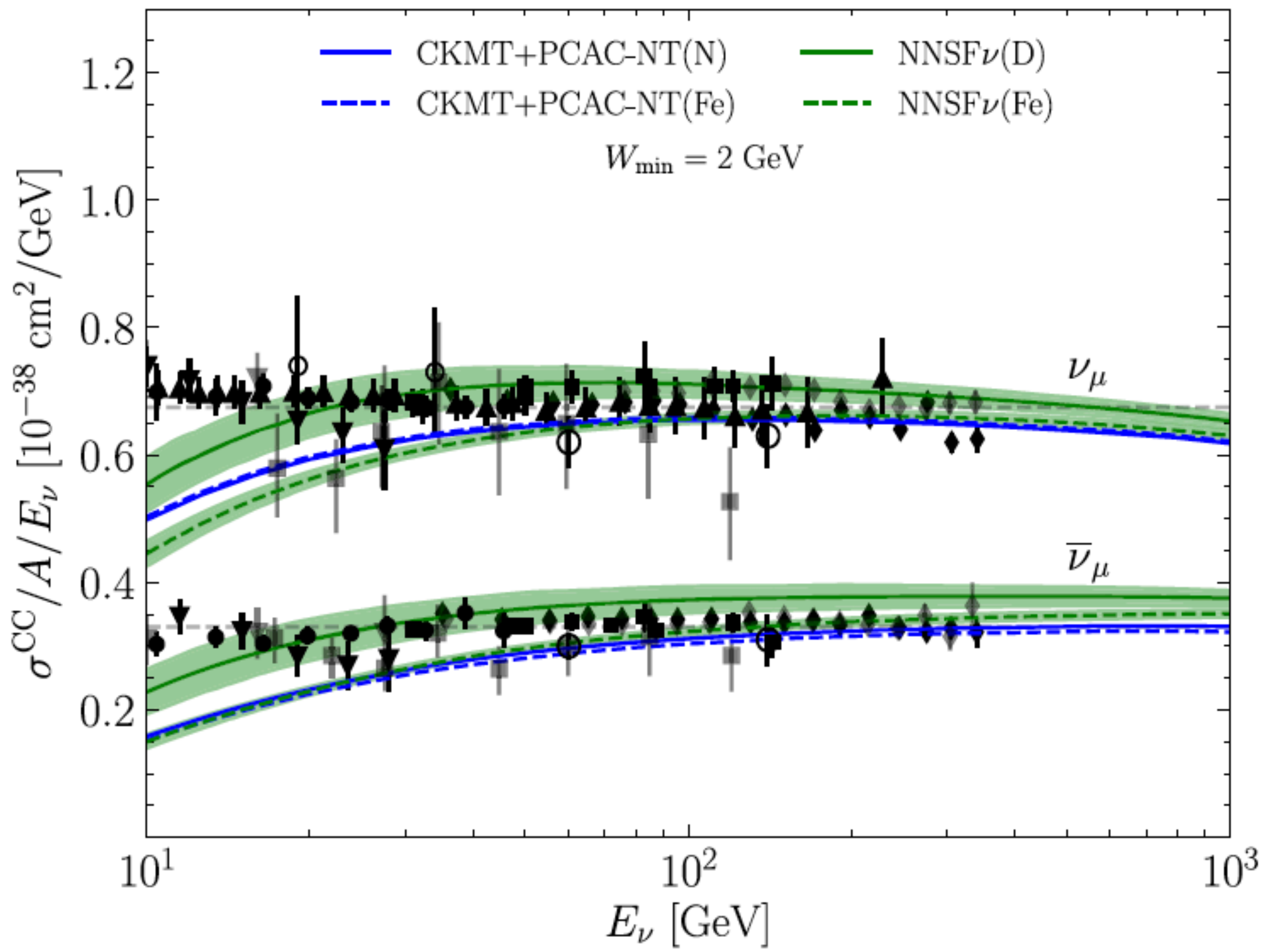

- For some heavier nuclear targets, disagreement between NNSFν and CKMT based result is relieved compared to the nucleon target.