7th CUBES mini-workshop:

A Z4 Symmetric Model for Self-Interacting Dark Matter and its Detection Probes

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K-Hotel, Sandong, Gurye, April 26th, 2025

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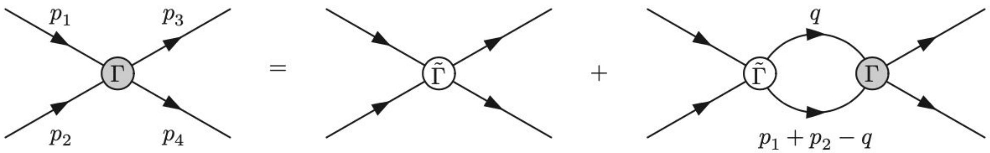
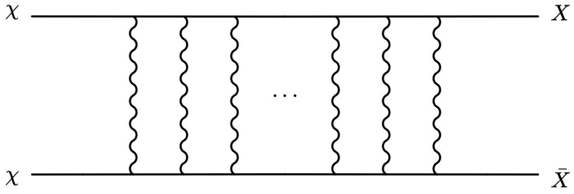
Introduction

In present Universe, the non-relativistic velocities of DM can lead to **non-perturbative** effects from quantum field theory. Particularly, the **DM (semi) annihilation and self-scattering can be enhanced by the Sommerfeld Effect.**



$$i\Gamma(p_1, p_2; p_3, p_4) \;=\; i\tilde{\Gamma}(p_1, p_2; p_3, p_4) + \int \frac{d^4q}{(2\pi)^4}\, \tilde{\Gamma}\; G(q)\; G(p_1 + p_2 - q)\; \Gamma$$

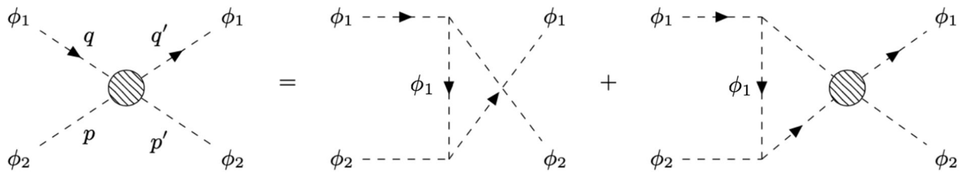
- SE requires suppression of (propagator)^-1. Standard scenario: **light mediator in the t-channel**.



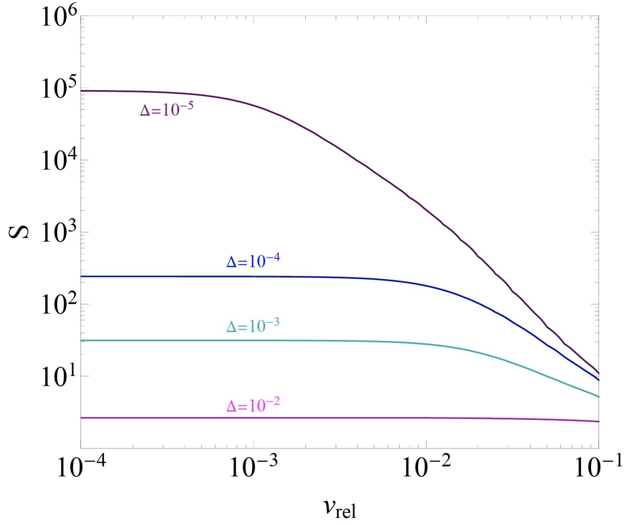
Problem: too restrictive.

Introduction

- Previous models by our group showed that it is possible to enhance DM cross sections without the need of a light mediator considering **two-component dark matter with a 2:1 mass ratio (self-resonant dark matter)**.



$$\left[-\frac{1}{2\mu}\left(\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}\right)+\frac{l(l+1)}{2\mu r^2}\right]R_l(r)-\frac{\alpha}{r}\,e^{-Mr}\,(-1)^l\,R_l(br)=E\,R_l(r)$$

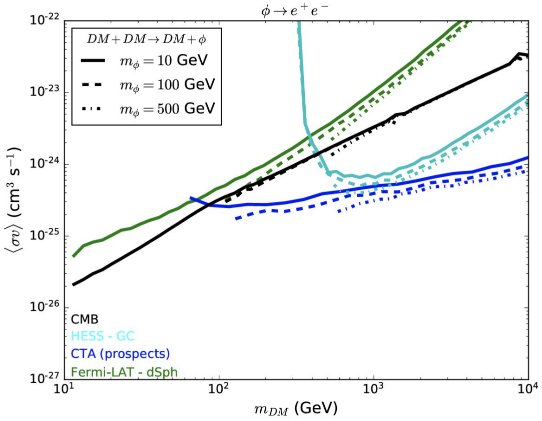


(work by HM Lee, SS Kim and B Zhu arXiv:2108.06278v2)

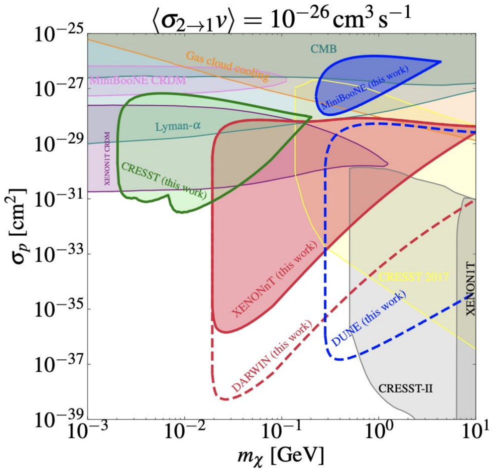
- This provides enhanced cross section for self- and semi- annihilation as well for self-interaction, allowing the fit of DM small scale data.

Introduction

- Multi-component DM models also allow **boosted dark matter**. With a sufficient mass-splitting, DM can be boosted and produce signals in both direct and indirect detection methods.



Queiroz et al., arXiv:1901.10494v2



Ibarra et al., arXiv:2501.12117v2

- Our proposal: **study a SRDM model stabilized by a Z4 symmetry and implement the new limits of DD for boosted DM scenario.**

Z4 symmetric model

- Setup: $$\mathcal{L} = |D_\mu \chi|^2 + |D_\mu \phi_1|^2 + |D_\mu \phi_2|^2 - \frac{1}{4} X_{\mu\nu} X^{\mu\nu} - \frac{1}{2} \sin\xi \, X_{\mu\nu} B^{\mu\nu} - V(\chi, \phi_1, \phi_2)$$

$$\begin{aligned} V(\chi,\phi_1,\phi_2) \;&=\; m_\chi^2|\chi|^2 + \lambda_\chi|\chi|^4 + m_1^2|\phi_1|^2 + \lambda_1|\phi_1|^4 + m_2^2|\phi_2|^2 + \lambda_2|\phi_2|^4 \\ &+ \sum_{i=1,2} \lambda_{\chi i}|\chi|^2|\phi_i|^2 + \lambda_{12}|\phi_1|^2|\phi_2|^2 + (g_1 m_1 \phi_2^\dagger \phi_1^2 + \text{h.c.}) \\ &+ \left(\kappa_1 \chi^\dagger \phi_2^2 + \kappa_2 \chi^\dagger \phi_2 \phi_1^2 + \text{h.c.}\right) \end{aligned}$$

	ϕ_1	ϕ_2	χ
$U(1)'$	$+1$	$+2$	$+4$
$U(1)' \supset Z_4$	i	-1	$+1$

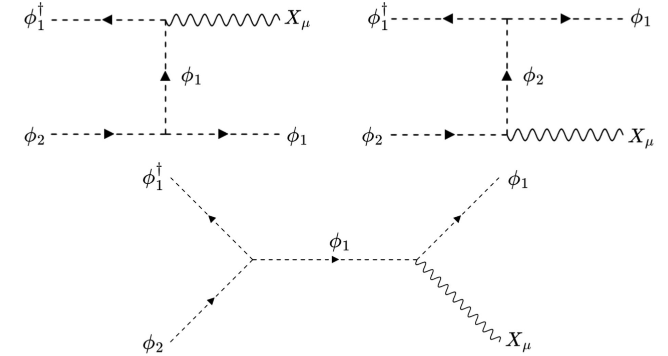
(U(1)' symmetry is broken into Z4 by the VEV of the dark Higgs)

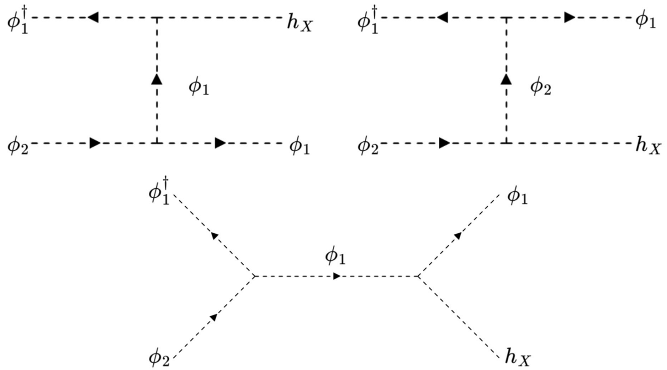
- The real and imaginary component of $\boldsymbol{\phi}2$ decoupled because of the induced mass splitting ($\boldsymbol{\phi}1$ and the lightest $\boldsymbol{\phi}2$ component will be the DM particle):

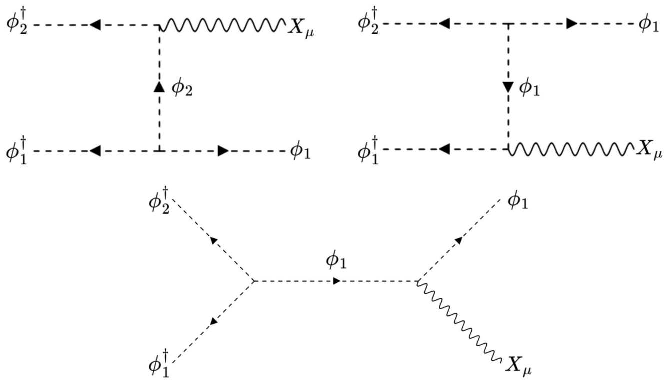
$$\begin{aligned} m_{1,\text{eff}}^2 &\equiv m_1^2 + \frac{1}{2}\lambda_{\chi 1} v_\chi^2, \\ m_{2,\text{eff}}^2 &\equiv m_2^2 + \frac{1}{2}\lambda_{\chi 2} v_\chi^2, \\ m_3^2 &\equiv \frac{1}{\sqrt{2}}\kappa_1 v_\chi, \\ g_2 &\equiv \frac{\kappa_2 v_\chi}{\sqrt{2} m_1}. \end{aligned}$$

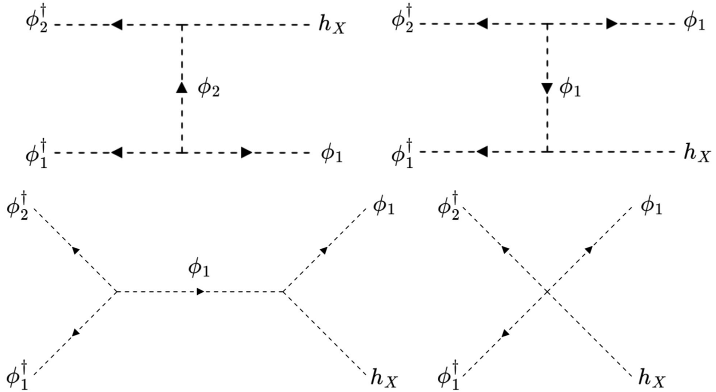
$$\begin{aligned} m_s^2 &= m_{2,\text{eff}}^2 + 2m_3^2, \\ m_a^2 &= m_{2,\text{eff}}^2 - 2m_3^2. \end{aligned}$$

Z4 Self-Resonant DM: Semi-annihilation diagrams



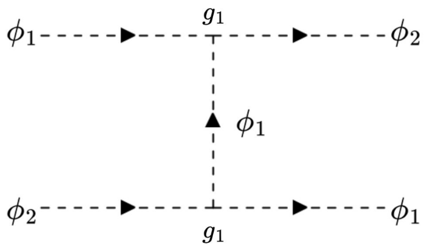


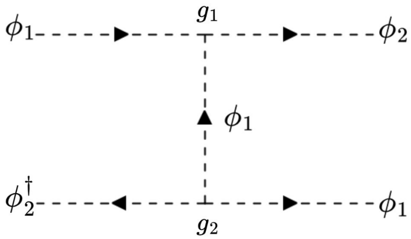




U-channel resonance

- For the Z4 model, two ladder diagrams contribute for the SE:





$$\widetilde{\Gamma}_{\phi_1\phi_2\to\phi_1\phi_2}(p,q;p',q') = -\frac{4g_1^2 m_1^2}{(p-q')^2 - m_1^2}$$

$$\widetilde{\Gamma}_{\phi_1\phi_2^\dagger\to\phi_1\phi_2}(p,q;p',q') = -\frac{4g_1 g_2 m_1^2}{(p-q')^2 - m_1^2}$$

- This defines two Bethe-Salpeter functions (**conversion between states**).

$$\begin{aligned} i\chi_A(p,q) &\simeq -G_2(p)G_1(q)\int\frac{d^4k}{(2\pi)^4}\Big[\widetilde{\Gamma}_{\phi_2\phi_1\to\phi_2\phi_1}(p,q;p+q-k,k)\chi_A(p+q-k,k) \\ &\quad +\widetilde{\Gamma}_{\phi_2\phi_1\to\phi_2^*\phi_1}(p,q;p+q-k,k)\chi_B(p+q-k,k)\Big], \\ i\chi_B(p,q) &\simeq -G_2(p)G_1(q)\int\frac{d^4k}{(2\pi)^4}\Big[\widetilde{\Gamma}_{\phi_2^*\phi_1\to\phi_2\phi_1}(p,q;p+q-k,k)\chi_A(p+q-k,k) \\ &\quad +\widetilde{\Gamma}_{\phi_2^*\phi_1\to\phi_2^*\phi_1}(p,q;p+q-k,k)\chi_B(p+q-k,k)\Big] \end{aligned}$$

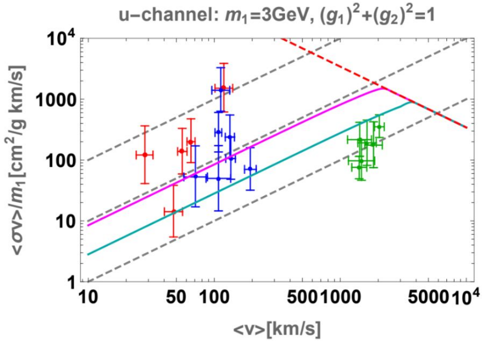
U-channel resonance

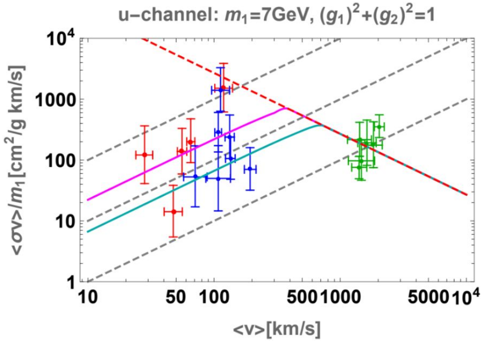
Defining the position-space wave-function from the Fourier transf., one obtains coupled Schrodinger-like equations:

$$-\frac{1}{2\mu}\nabla^2\begin{pmatrix}\psi_A(\vec{x})\\ \psi_B(\vec{x})\end{pmatrix}+V(\vec{x})\begin{pmatrix}1 & \frac{g_2}{g_1}\\ \frac{g_2}{g_1} & \left(\frac{g_2}{g_1}\right)^2\end{pmatrix}\begin{pmatrix}\psi_A\left(-\frac{m_2}{m_1}\vec{x}\right)\\ \psi_B\left(-\frac{m_2}{m_1}\vec{x}\right)\end{pmatrix}=\begin{pmatrix}E_A & 0\\ 0 & E_B\end{pmatrix}\begin{pmatrix}\psi_A(\vec{x})\\ \psi_B(\vec{x})\end{pmatrix}$$

$$V_{\text{eff}}(\vec{x})=-\left(1+\frac{g_2^2}{g_1^2}\right)\frac{\alpha}{r}\,e^{-Mr} \qquad M\equiv m_2\sqrt{2-\frac{m_2}{m_1}}.$$

- Fitting of small-scale data:



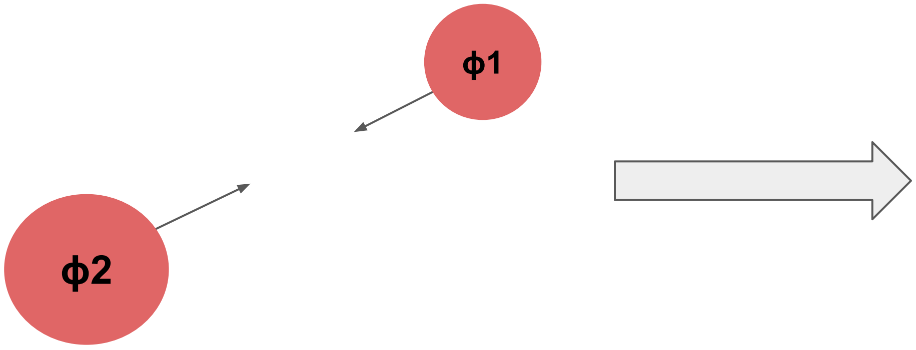


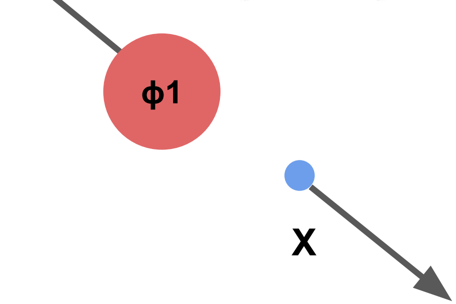
(Done by Hyun Min Lee and Seong-Sik Kim)

Boosted Dark Matter

Consider the semi-annihilation between DM into dark particle: relativistic velocities (boost) for the final states

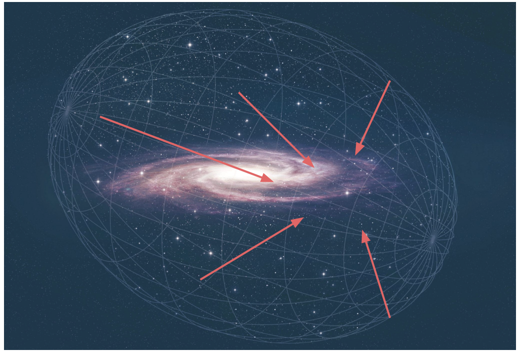
$$T_1' = \frac{m_2^2 - m_Y^2}{2(m_1 + m_2)}$$





$$\frac{d\Phi_{\phi_1}}{dE_1} = \frac{1}{8\pi m_1 m_2} \langle \sigma v \rangle_{\phi_1^\dagger \phi_2^{(\dagger)} \to \phi_1 Y} \cdot \frac{dN_{\phi_1}}{dE_1} \frac{1}{2} r_1 (1 - r_1) \int d\Omega \int_{\mathrm{l.o.s.}} ds\, \rho_{\mathrm{DM}}^2(r) = \frac{d\Phi_{\phi_1^\dagger}}{dE_1}$$

$$\Phi_{\mathrm{BSM}} = \frac{m_1}{2m_2} r_1(1 - r_1) \cdot \left(3.2 \times 10^{-3}\, \mathrm{cm}^{-2}\mathrm{s}^{-1}\right) \left(\frac{m_1}{100\, \mathrm{MeV}}\right)^{-2} \left(\frac{\langle \sigma v \rangle_{\phi_1^\dagger \phi_2^{(\dagger)} \to \phi_1 Y}}{10^{-26}\, \mathrm{cm}^3\, \mathrm{s}^{-1}}\right)$$



Boosted dark matter particles can travel through the galaxy and be detected by direct detection or neutrinos experiments!

Nucleus-boosted DM scattering

- Considering the **relativistic** nucleus-DM scattering, the DM wavelength can be of the size or smaller than the nucleus size -> **both coherent and incoherent scattering can be present!**

$$\left(\frac{d\sigma_{\rm SI}}{dT_A}\right)_{\rm coh} = \frac{\sigma_{\rm SI}^{\rm coh}}{T_{A,\rm max}}|F_{\rm SI}(q)|^2,$$
$$\left(\frac{d\sigma_{\rm SI}}{dT_A}\right)_{\rm inc} = \frac{\sigma_{\rm SI}^{\rm inc}}{T_{A,\rm max}}(1-|F_{\rm SI}(q)|^2)$$

$$\sigma_{\rm SI}^{\rm coh} \equiv \frac{2}{r_1}\sigma^{\rm coh}_{\phi_1,\phi_1^\dagger-A} = \frac{\mu_{A,1}^2}{2\pi m_1^2}\left[Z^2\left(c_p^{(1)}f_p\right)^2 + Z^2(g_p^{(1)})^2 + (A-Z)^2\left(c_n^{(1)}f_n\right)^2\right],$$
$$\sigma_{\rm SI}^{\rm inc} \equiv 2\,\sigma^{\rm inc}_{\phi_1,\phi_1^\dagger-A} = \frac{\mu_{A,1}^2}{2\pi m_1^2}\left[Z\left(c_p^{(1)}f_p\right)^2 + Z(g_p^{(1)})^2 + (A-Z)\left(c_n^{(1)}f_n\right)^2\right],$$

where $F_{\rm SI}(q) = \left(1+\frac{q^2}{\Lambda_A}\right)^{-2}$ is the nucleus form-factor.

Nucleus-boosted DM scattering

- The differential event rate will be given by

$$\frac{dR_A}{dT_A} = \frac{1}{m_A} \int_{T_{1,\mathrm{min}}}^{\infty} dT_1 \, \frac{d\sigma_{\mathrm{SI}}}{dT_A} \, \frac{d\Phi_{\phi_1}}{dT_1}$$

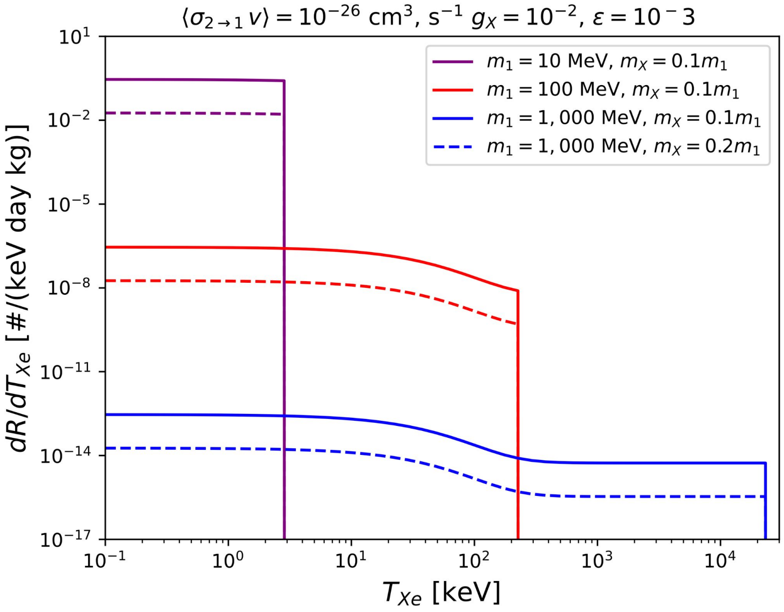
- We have to consider that a given recoil energy is produced minimum DM kinetic energy given by

$$T_{1,\mathrm{min}} = \left(\frac{T_A}{2} - m_1\right)\left[1 \pm \sqrt{1 + \frac{2T_A}{m_A}\frac{(m_1+m_A)^2}{(T_A-2m_1)^2}}\right]$$

- Reciprocally, a given DM energy can produce, at most, a given recoil energy:

$$T_{A,\mathrm{max}} = \frac{T_1(T_1+2m_\chi)}{(m_1+m_A)^2/(2m_A)+T_1}$$

DD for boosted DM: Recoil energy



Features:

- Suppression of rate for higher mX (DM-nucleon coupling):

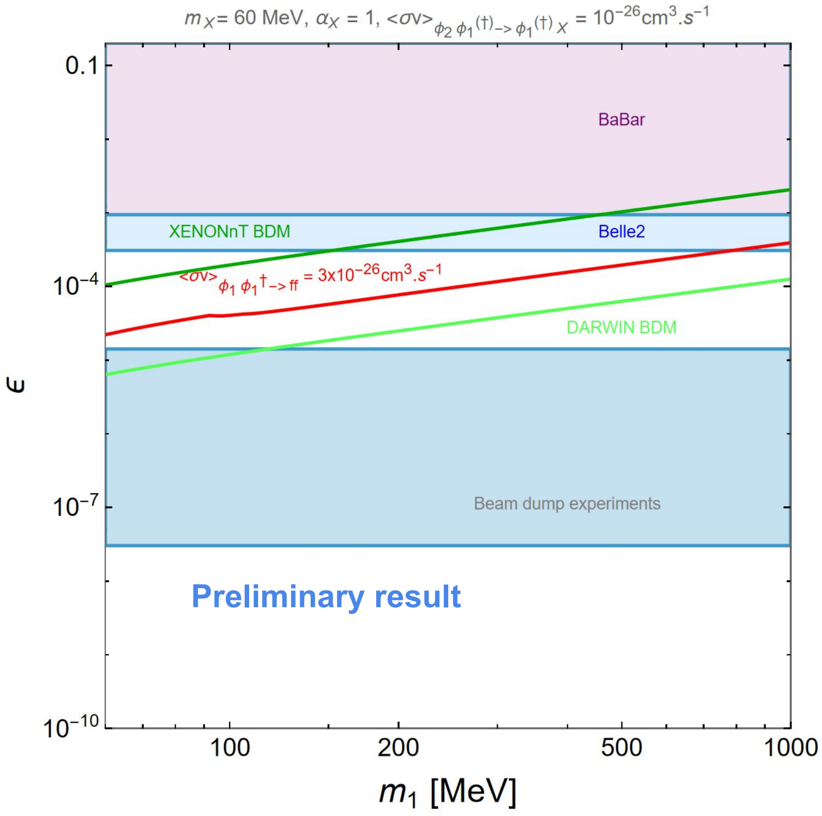
$$g_p^{(i)} = -\frac{2eq_i g_X \varepsilon m_i}{m_X^2}$$

- **Since the detectors have an energy threshold, the maximum recoil energy implies a minimum mass for detection:**

$$m_{1,\min} \simeq \frac{15m_A}{\frac{32m_A}{T_{\rm th}} - 9}\left(1 + \frac{4}{5}\sqrt{1 + \frac{2m_A}{T_{\rm th}}}\right)$$

Ex: Xenon (Tth = 3.3 KeV) → m > 10 MeV

DD for boosted DM: Limits



Exclusion limits for dark Higgs suppressed channel for mX = 60 MeV, αx=1 and $\langle\sigma_{\rm semi} v\rangle$= 10^(-26) cm3.s-1:

- Lower limits from attenuation above current collider constraints.
- XENONnT not able to probe abundance benchmark. **CMB bound strongly constraints BDM signal.**
- DARWIN (future experiment) excludes ϵ>10^(-4) for all masses in this benchmark.
- The limits are model-dependent and should be analyzed for different benchmarks.

Possible alternative scenario: **dark photon decay into hidden particles to increase signal** and preserve CMB constraint.

Conclusions and next steps

The Z4 symmetric model for self-resonant DM presents a non-perturbative enhancement that increase its detectability from different ways:

- **Galaxy scale data** fitting from **self-scattering DM**.
- **Self-annihilation** and **semi-annihilation** can be probed through in **indirect detection searches**.
- DD can probe the **nucleon-DM coupling** for both **halo DM** and **boosted DM** components. **DD for BDM can be relevant for sub-GeV masses in some scenarios.**

Next steps:

- Check **CMB limits** and **alternative scenario of invisible decays of dark photon**.
- **Fit small-scale data for sub-GeV DM**.
- Solve **Boltzmann equation** (possibly with help of softwares like micromegas, MadDM,etc)
- Determine **BDM limits for other experiments,** DD constraints from **electron-DM** and ID with **gamma-rays**.

Backup

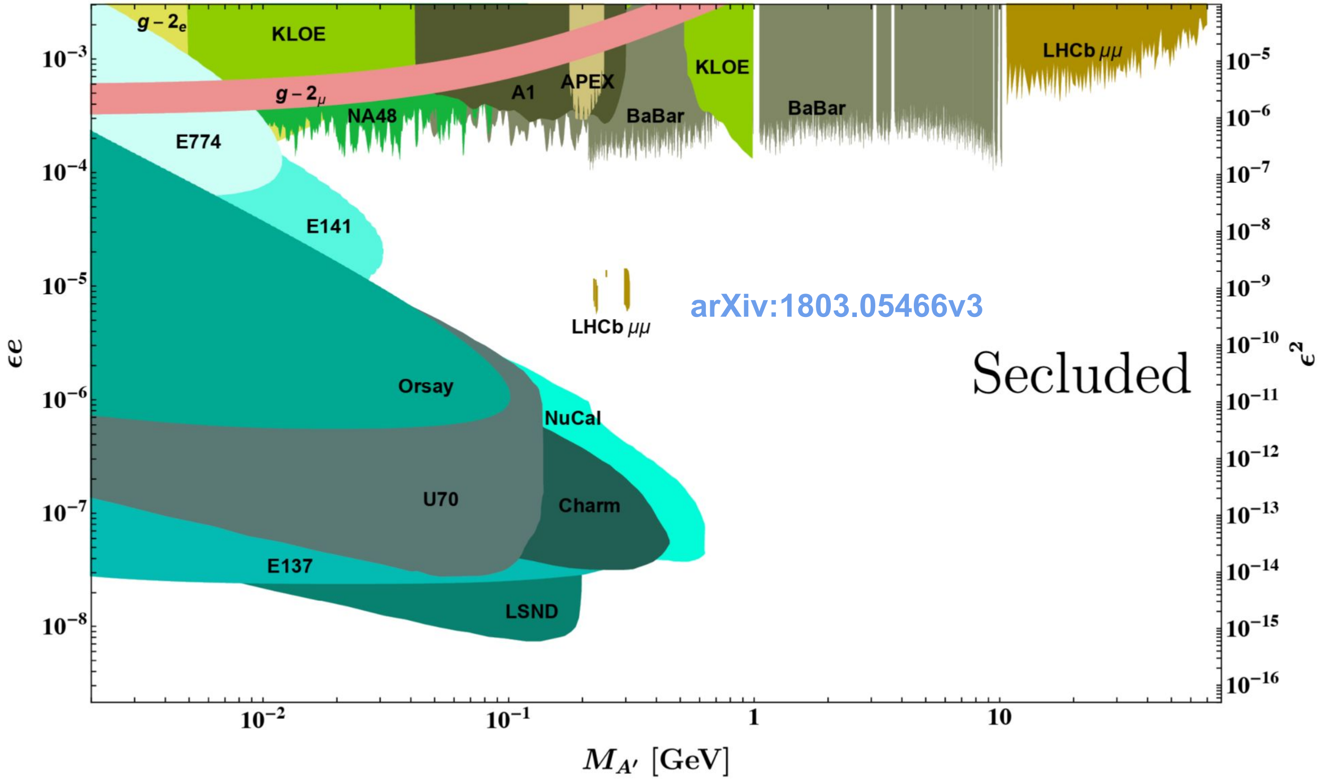
Boltzmann Equation for Relic Abundance

(Supposing CP symmetry in the dark sector)

$$\begin{aligned}
\dot{n}_1 + 3Hn_1 &= -\frac{1}{2}\langle\sigma v\rangle_{\phi_1\phi_1^\dagger\to f\bar{f},VV,h_Xh_X,XX}\left(n_1^2-(n_1^{\rm eq})^2\right) \\
&\quad +\frac{1}{2}\langle\sigma v\rangle_{\phi_2\phi_2^\dagger,\phi_2\phi_2,\phi_2^\dagger\phi_2^\dagger\to\phi_1\phi_1^\dagger}\left(n_2^2-n_1^2\right) \\
&\quad +\frac{1}{2}\sum_{i=X,h_X}\langle\sigma v\rangle_{\phi_2\phi_1^\dagger,\phi_2^\dagger\phi_1^\dagger\to\phi_1 i}\, n_1\left(n_2-n_2^{\rm eq}\right),
\end{aligned}$$

$$\begin{aligned}
\dot{n}_2 + 3Hn_2 &= -\frac{1}{2}\langle\sigma v\rangle_{\phi_2\phi_2^\dagger\to f\bar{f},VV,h_Xh_X,XX}\left(n_2^2-(n_2^{\rm eq})^2\right) \\
&\quad -\frac{1}{2}(\langle\sigma v\rangle_{\phi_2\phi_2^\dagger\to\phi_1\phi_1^\dagger}+2\langle\sigma v\rangle_{\phi_2\phi_2\to\phi_1\phi_1^\dagger})\left(n_2^2-n_1^2\right) \\
&\quad -\frac{1}{2}\sum_{i=X,h_X}\langle\sigma v\rangle_{\phi_2\phi_1^\dagger\to\phi_1 i}\, n_1\left(n_2-n_2^{\rm eq}\right).
\end{aligned}$$

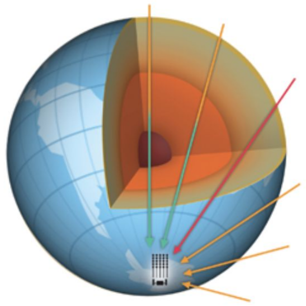
Beam dump Experiments



Lower limits: Earth's internal structure attenuation effect

If the proton-DM cross-section is too high, the boosted DM particles will scatter and lose kinetic energy below the detection threshold through their propagation inside Earth's internal structure, where the dense rock implies a high nuclear density.

$$\frac{dT_\chi}{dz} = -\sum_{\mathcal{T}} n_{\mathcal{T}} \int_0^{T_{\mathcal{T}}^{\max}} dT_{\mathcal{T}}\, T_{\mathcal{T}}\, \frac{d\sigma_{\chi\mathcal{T}}}{dT_{\mathcal{T}}}(T_\chi, T_{\mathcal{T}}),$$



Interaction with the dark sector

$$\mathcal{L}_{h_X-\phi_1,s,a} = -\lambda_\chi v_\chi h_\chi^3 - \frac{1}{4}\lambda_\chi h_\chi^4 - \frac{1}{2}\lambda_{\chi 1}(2v_\chi h_X + h_X^2)|\phi_1|^2 - \frac{1}{4}\lambda_{\chi 2}(2v_\chi h_X + h_X^2)(s^2 + a^2)$$
$$-\frac{1}{\sqrt{2}}\kappa_1 h_X(s^2 - a^2) - \frac{1}{2}\kappa_2 h_X s(\phi_1^2 + \phi_1^{\dagger 2}) - \frac{1}{2}i\kappa_2 h_X a(\phi_1^2 - \phi_1^{\dagger 2}). \quad (11)$$

$$\begin{aligned}
\mathcal{L}_X &= \frac{m_X^2}{v_\chi} h_X X_\mu X^\mu + 8g_X^2 h_X^2 X_\mu X^\mu \\
&\quad + ig_X X_\mu(\phi_1^\dagger \partial^\mu \phi_1 - \phi_1 \partial^\mu \phi_1^\dagger) + g_X^2 X_\mu X^\mu \phi_1^\dagger \phi_1 \\
&\quad + 2ig_X X_\mu(\phi_2^\dagger \partial^\mu \phi_2 - \phi_2 \partial^\mu \phi_2^\dagger) + 4g_X^2 X_\mu X^\mu \phi_2^\dagger \phi_2 \\
&= \frac{m_X^2}{v_\chi} h_X X_\mu X^\mu + 8g_X^2 h_X^2 X_\mu X^\mu \\
&\quad + ig_X X_\mu(\phi_1^\dagger \partial^\mu \phi_1 - \phi_1 \partial^\mu \phi_1^\dagger) + g_X^2 X_\mu X^\mu \phi_1^\dagger \phi_1 \\
&\quad - 2g_X X_\mu(s\partial^\mu a - a\partial^\mu s) + 2g_X^2 X_\mu X^\mu (s^2 + a^2).
\end{aligned}$$

Interaction with the SM sector

$$\mathcal{L}_{\rm NC} \simeq A_\mu J^\mu_{\rm EM} + Z_\mu \left(J^\mu_Z + \varepsilon\, t_W J^\mu_X \right) + X_\mu \left(-\varepsilon J^\mu_{\rm EM} + J^\mu_X \right)$$

$$\begin{aligned} J^\mu_{\rm EM} &= e \bar{f} \gamma^\mu Q_f f, \\ J^\mu_Z &= \frac{e}{2 s_W c_W} \bar{f} \gamma^\mu (\tau^3 - 2 s_W^2 Q_f) f, \\ J^\mu_X &= i g_X (\phi_1^\dagger \partial^\mu \phi_1 - \phi_1 \partial^\mu \phi_1^\dagger) - 2 g_X (s \partial^\mu a - a \partial^\mu s). \end{aligned}$$

$$\mathcal{L}_H \supset -\lambda_{\chi H} |H|^2 |\chi|^2 - \lambda_{H1} |H|^2 |\phi_1|^2 - \lambda_{H2} |H|^2 |\phi_2|^2 - \lambda_H |H|^4 - m_H^2 |H|^2$$

$$\begin{aligned} \mathcal{L}_{h_1,h_2} &= -y_{h_1 \phi_1^\dagger \phi_1} h_1 |\phi_1|^2 - y_{h_2 \phi_1^\dagger \phi_1} h_2 |\phi_1|^2 \\ &\quad -\frac{1}{2} y_{h_1 ss} h_1 (s^2 + a^2) - \frac{1}{2} y_{h_2 ss} h_2 (s^2 + a^2) - (\lambda_{h_1} h_1 + \lambda_{h_2} h_2) \bar{f} f \end{aligned}$$