# Reconciling cosmological tensions with inelastic dark matter and radiation in $U(1)_D$ framework

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# Motivation

## Cosmological Tensions

- Hubble tension
  local distance ladder measurements $73.0 \pm 1.0 \text{ km } s^{-1} \text{ Mpc}^{-1}$ vs CMB Planck $67.3 \pm 0.6 \text{ km } s^{-1} \text{ Mpc}^{-1}$

- S8 tension: Weak lensing survey prefer $S_8 = 0.77$, while Planck prefers $S_8 = 0.83$ (3$\sigma$)

- Lyman-$\alpha$ discrepancy: Small-scale matter power spectrum inferred from the eBOSS Lyα forest shows $4.9\sigma$ with ΛCDM fit to the CMB.

# DR

## Dark Radiation ($\Delta N_{\mathrm{eff}}$)

- Although, additional radiation species (DR) radiation can solve the Hubble tension, it cannot solve the large scale problem (S8 tension) and Lyman-alpha problem.

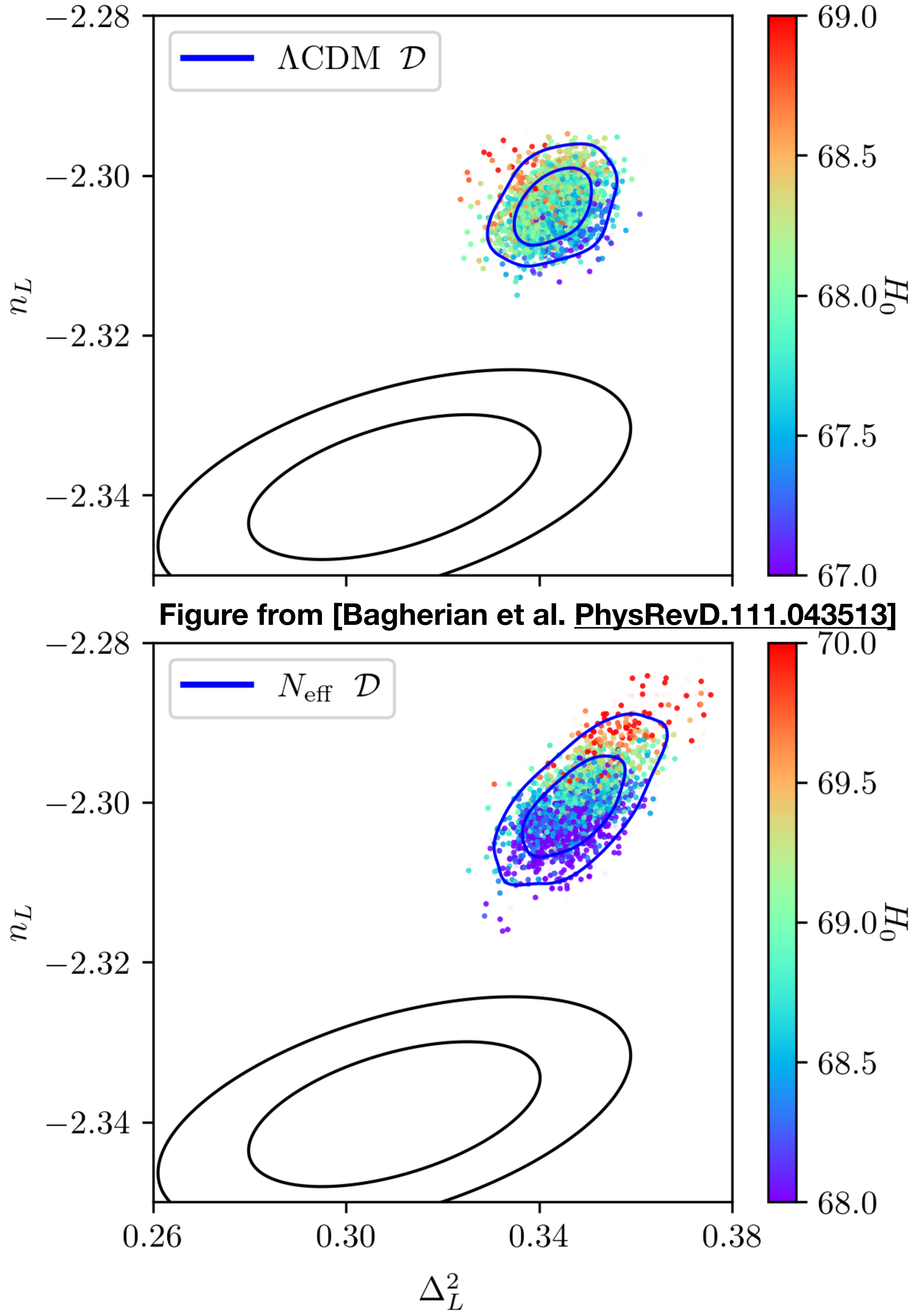



Figure from [Bagherian et al. PhysRevD.111.043513]

# SIDR

## Self Interacting Dark Radiation

- Self Interacting Dark Radiation (SIDR) model is for the explaining the large scale structure: S8 tension.

- However, they exacerbate the existing tension with Ly-α data.

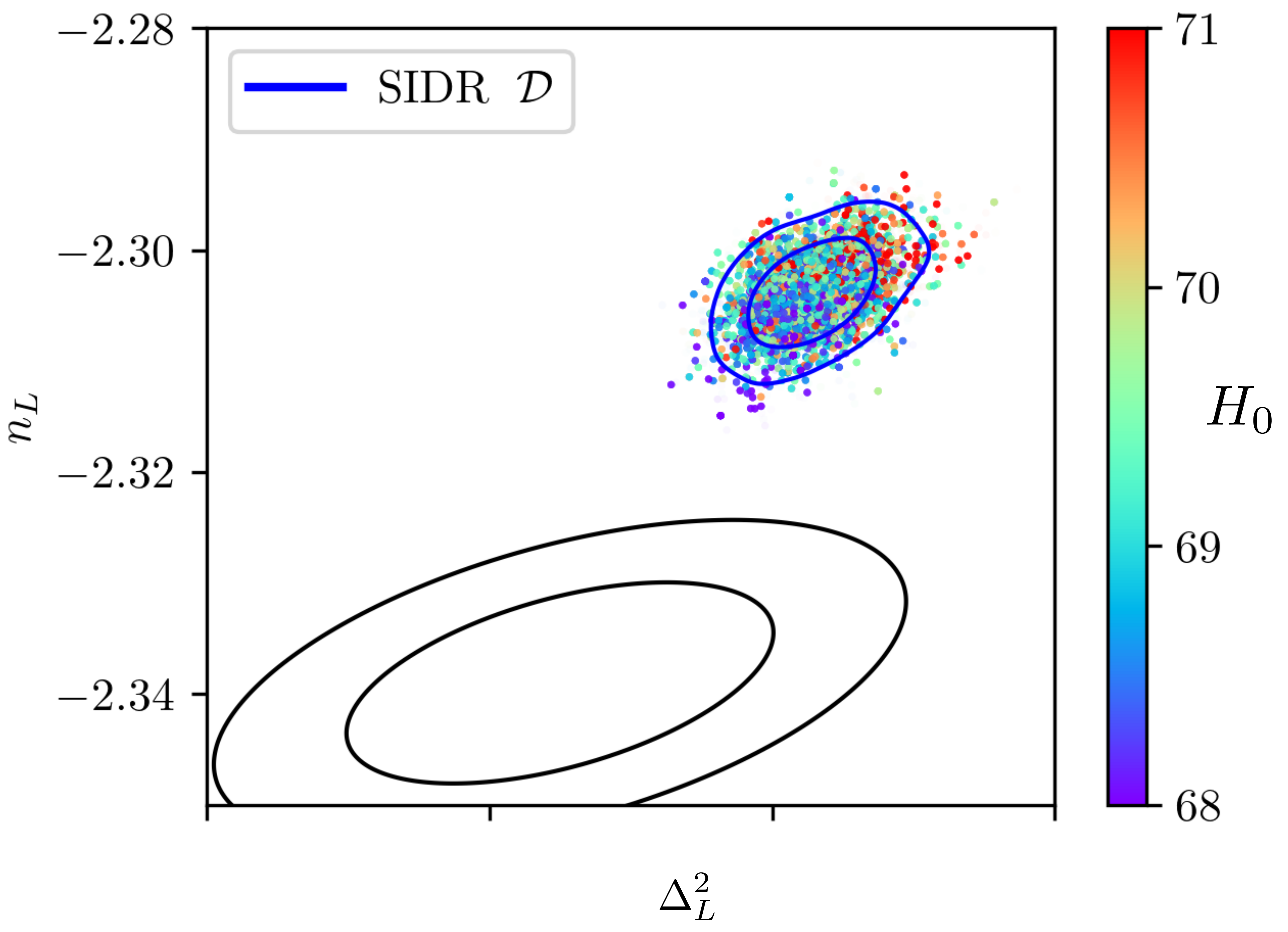



Figure from [Bagherian et al. PhysRevD.111.043513]

# WZDR

- Wess-Zumino Dark Radiation(WZDR) Model is two particle species before the CMB, one particle become non-relativistic and annihilated into the dark radiation.

- Call as a step that $\Delta N_{\rm eff}$ increases around the annihilation into dark radiation and call that time as $z_t$.

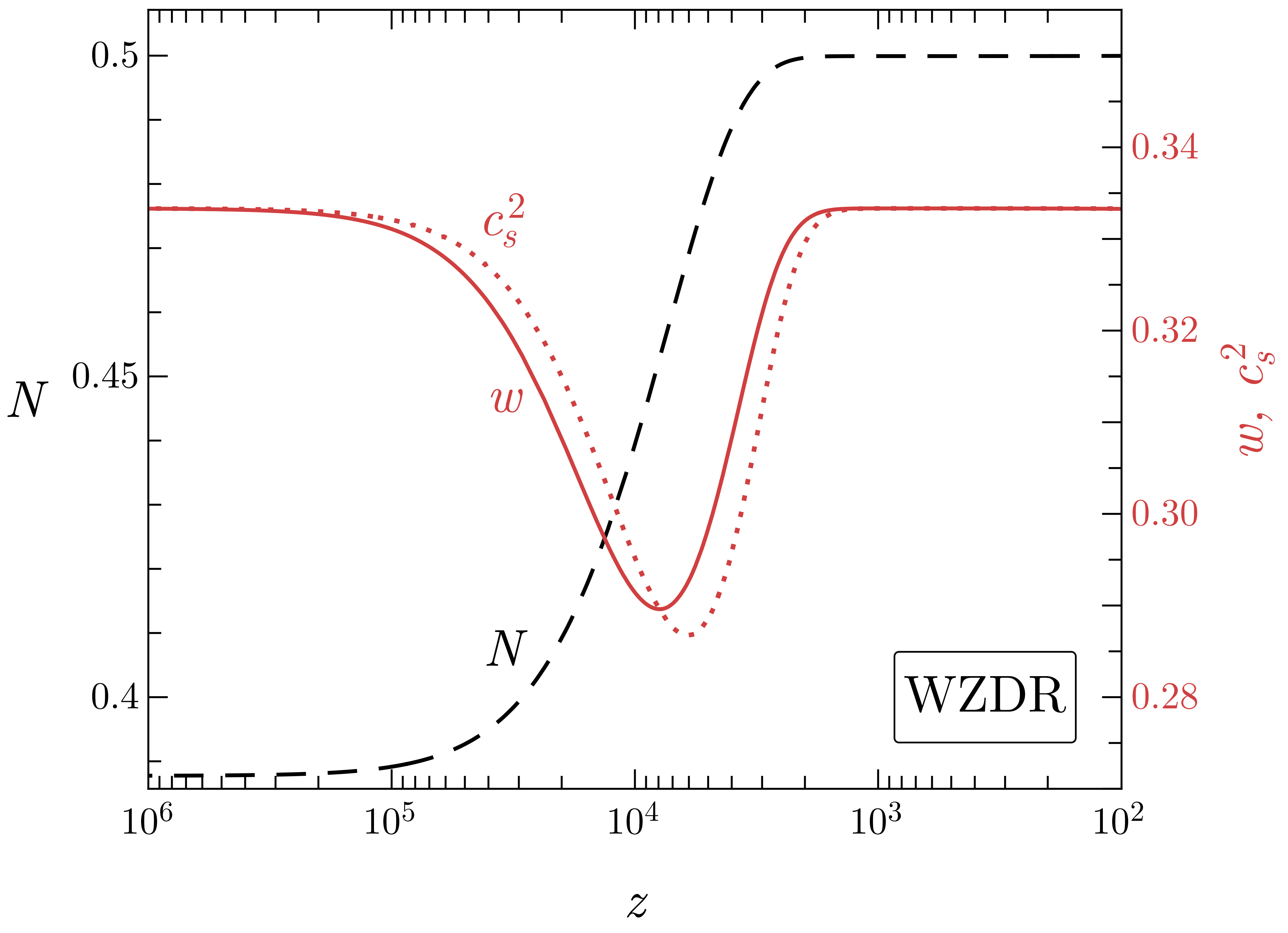



Figure from [Aloni et al. PhysRevD.105.123516]

# SIDR+/WZDR+

- The interaction between dark matter and dark radiation exist that the momentum transfer to the dark matter from DR, which can suppress the structure formation.
- The ratio of the momentum transfer rate to the Hubble expansion rate $R_\Gamma$ (0.07) and transition redshift $\log_{10} z_t$ (4.25) are new parameters to explain the tension.

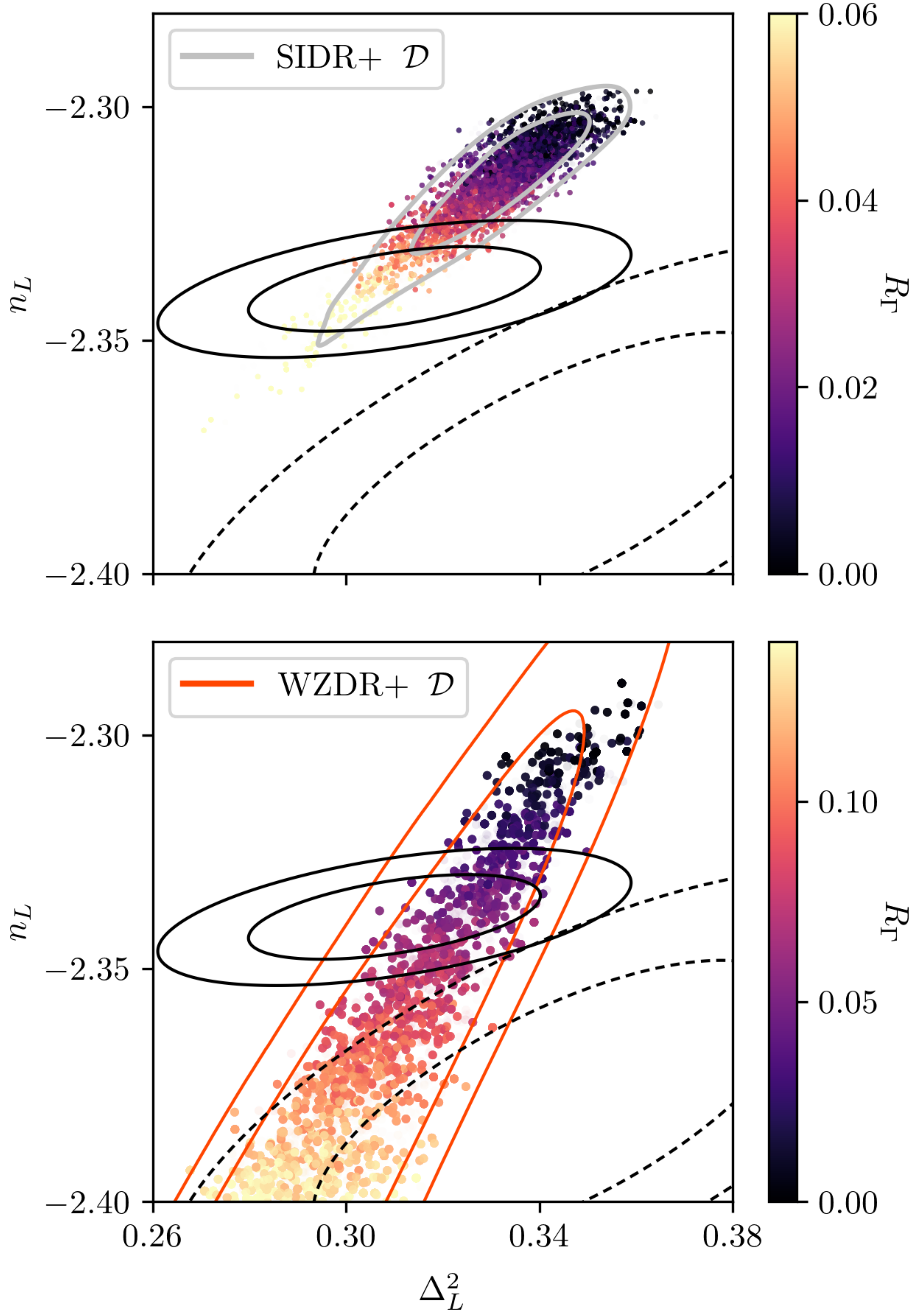



Figure from [Bagherian et al. PhysRevD.111.043513]

# Inelastic Dark Matter / Dark Gauge Boson

## Lagrangian

$$\mathcal{L} \supset i\overline{\psi}\gamma^\mu\partial_\mu\psi - M_\psi\overline{\psi}\psi + i\overline{\chi}\gamma^\mu D_\mu\chi - M_\chi\overline{\chi}\chi$$
$$+ (D_\mu\Phi)^\dagger(D^\mu\Phi) - Y\overline{\chi}\Phi\psi - \frac{1}{4}F_X^{\mu\nu}F^{X\,\mu\nu} - \frac{\epsilon}{2}F_X^{\mu\nu}B^{\mu\nu} + h.c.$$

$\epsilon$: Kinetic mixing with $U(1)_D$ gauge boson and $U(1)_Y$

$\phi$: Scalar particle that have charge under $U(1)_D$

$\chi, \psi$: vector like fermion, singlets under SM gauge group,  acts as dark matter

# Inelastic Dark Matter / Dark Gauge Boson

- After $U(1)_D$ breaking, the fermions $\psi$ and $\chi$ mix because of the Yukawa coupling among them as $\Phi$ acquires a VEV.

$$\begin{pmatrix} M_\chi & Yv_\phi/\sqrt{2} \\ Yv_\phi/\sqrt{2} & M_\psi \end{pmatrix}$$

- With the diagonalization of the above mass matrix we get the physical states:

$$\xi_1 = \cos\theta\ \chi - \sin\theta\ \psi \qquad \text{and} \qquad \xi_2 = \sin\theta\ \chi + \cos\theta\ \psi.$$

- From the inverse transformation, we can obtain:

$$M_{\xi_2} - M_{\xi_1} = \delta = \frac{\sqrt{2}Yv_\phi}{\sin 2\theta}$$

# Inelastic Dark Matter / Dark Gauge Boson

## Interaction between dark/hidden sector

$$\mathcal{L} \supset ig_X (Z_X)_\mu \left[c_\theta^2 \ \ \overline{\xi_1}\gamma^\mu \xi_1 + s_\theta^2 \ \ \overline{\xi_2}\gamma^\mu \xi_2 + c_\theta s_\theta \ (\overline{\xi_1}\gamma^\mu \xi_2 + \overline{\xi_2}\gamma^\mu \xi_1)\right]$$

$$- Y(\cos\beta \ h_2 + \sin\beta \ h_1)\left[c_{2\theta}(\overline{\xi_1}\xi_2 + \overline{\xi_2}\xi_1) + s_{2\theta} \ (\overline{\xi_2}\xi_2 - \overline{\xi_1}\xi_1)\right].$$

- Interaction in the hidden sector:

$$\xi_1\bar{\xi_1} \to Z_X Z_X, \ \xi_1\bar{\xi_2} \to Z_X Z_X, \ \xi_2\bar{\xi_1} \to Z_X Z_X, \ \xi_2\bar{\xi_2} \to Z_X Z_X, \ Z_X Z_X \to \phi\phi$$

# Production of hidden sector

- Interaction like makes hidden sector particles from heavy particle $S$ decay

$$\mathscr{L} \supset \kappa_1 S \bar{\xi}_1 \xi_1 + \kappa_2 S \bar{\xi}_2 \xi_2$$

- We can express energy injection from visible sector to hidden sector as $j_h$ and energy conservation,

$$\frac{d\rho_v}{dt} + 3H(1+\omega_v)\rho_v = -j_h,$$
$$\frac{d\rho_h}{dt} + 3H(1+\omega_h)\rho_h = j_h$$
$$j_h = n_S m_S \Gamma_S$$

# Thermal Equilibrium of hidden sector

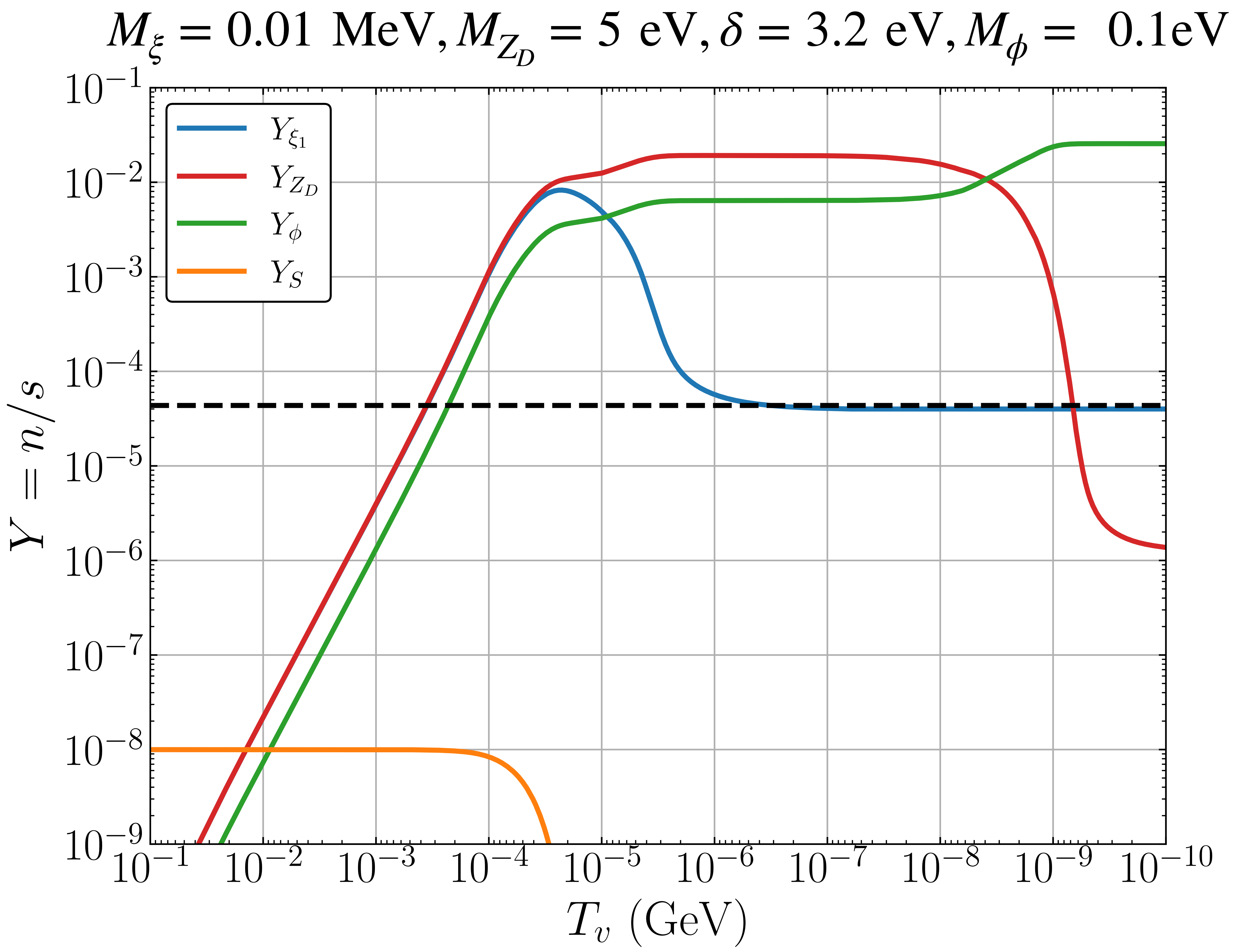
## Assumption

- Kinetic equilibrium is quickly achieved by strong gauge interaction between hidden sector particle, and the momentum distribution form Maxwell Boltzmann distribution [JCAP 08 (2023), 075].
- Evolution of Hidden sector temperature $T_h$ can be described as

$$\frac{d\xi}{dT_v} = -\frac{\xi}{T_v} + \frac{1}{T_v}\frac{3H(1+\omega_h)\rho_h - j_h}{3H(1+\omega)\rho_v + 3H(\omega-\omega_h)\rho_h + j_h}\frac{d\rho_v}{dT_v}/\frac{d\rho_h}{dT_h}.$$

# Evolution of dark sector

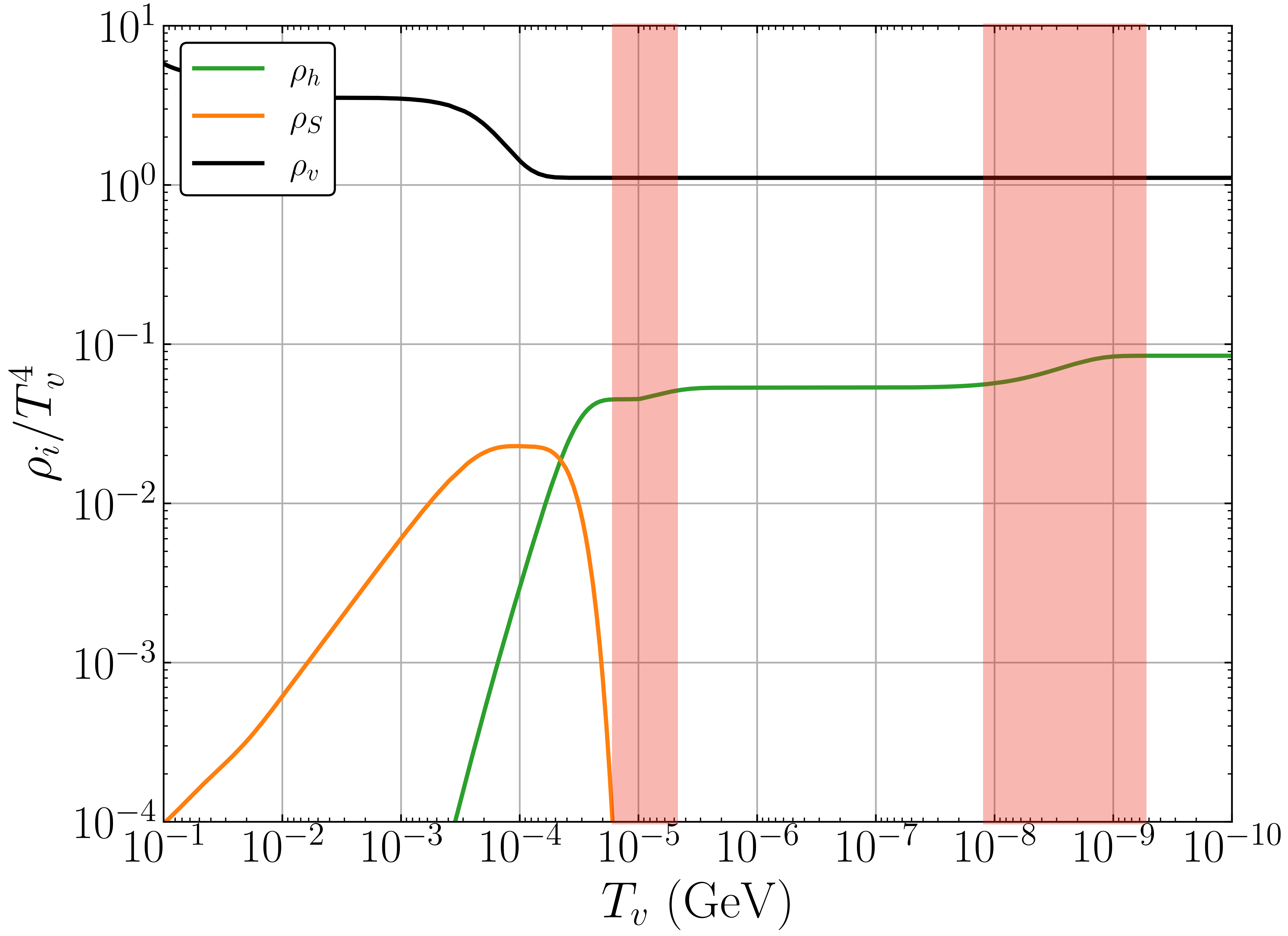
## Abundance




- S decays into dark sector, and make $\xi_1, \xi_2, Z_D, \phi$.
- Soon after dark matter($\xi_1$) becomes non-relativistic, it decays into dark gauge bosons.
- Similarly, when dark gauge boson becomes non-relativistic, it annihilated into dark radiations.

# Evolution of dark sector

## Density evolution




- S Decay: dumps the energy into hidden sector
- $\xi_1\xi_1 \to Z_D Z_D$ annihilation
- $Z_D Z_D \to \phi\phi$ annihilation at $T_v \sim M_{Z_D}$
- There is two stepping behaviors relative to the visible sector, which are annihilations of dark matter and gauge boson that makes hidden sector temperature increase.

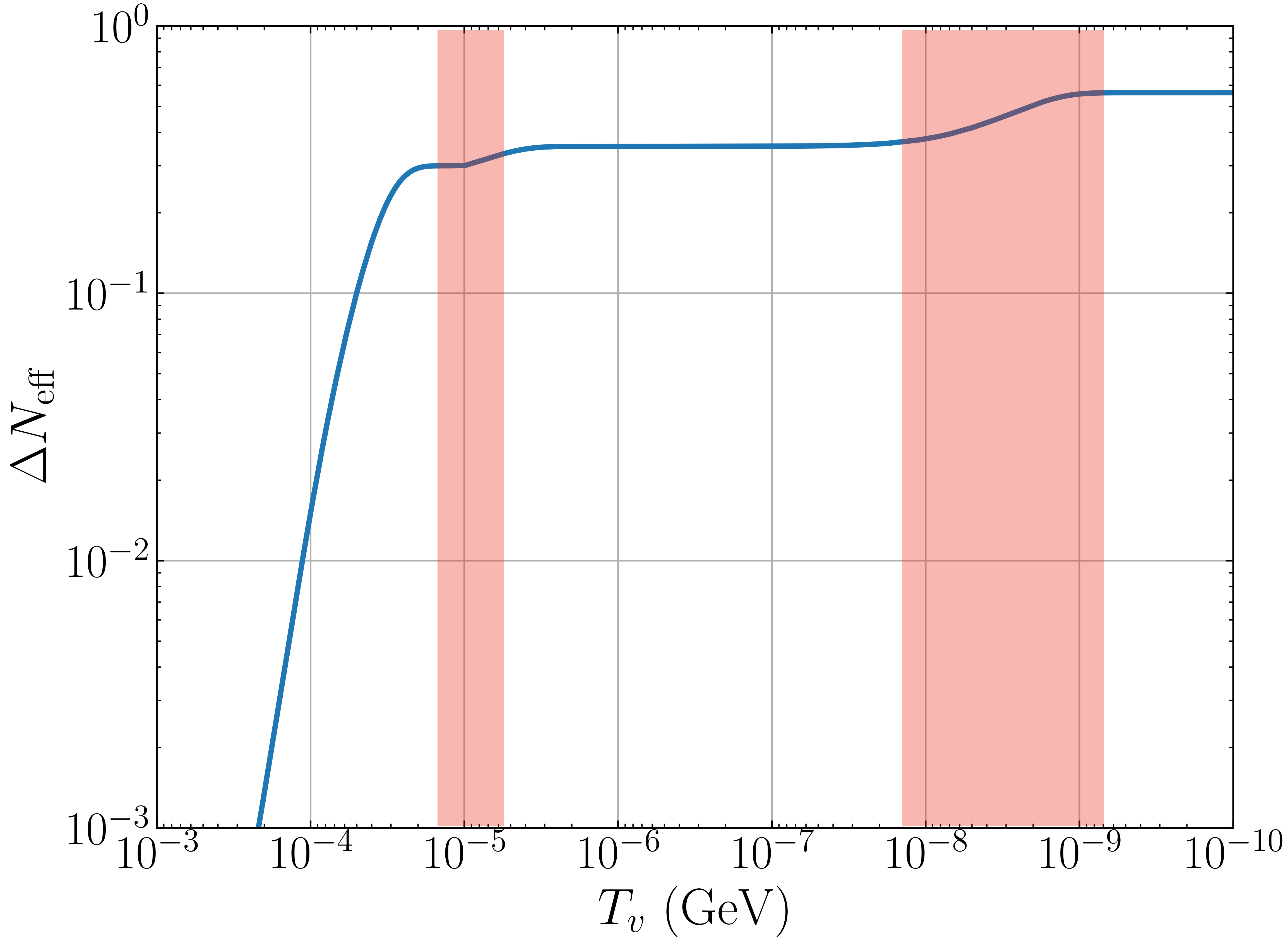
# Double stepping behavior of $\Delta N_{\text{eff}}$

- $\xi_1\xi_1 \to Z_D Z_D$ annihilation: boost $\Delta N_{\text{eff}}$ by a factor 1.23

$$\frac{\Delta N_{\text{eff}}|_{\text{after}}}{\Delta N_{\text{eff}}|_{\text{before}}} = \left( \frac{\frac{7}{8} g_*^{\xi_1} + g_*^{Z_D} + g_*^{\phi}}{g_*^{Z_D} + g_*^{\phi}} \right)^{1/3} \simeq 1.23.$$

- $Z_D Z_D \to \phi\phi$ annihilation: boost $\Delta N_{\text{eff}}$ by a factor of 1.58

$$\frac{\Delta N_{\text{eff}}|_{\text{after}}}{\Delta N_{\text{eff}}|_{\text{before}}} = \left( \frac{g_*^{Z_D} + g_*^{\phi}}{g_*^{\phi}} \right)^{1/3} \simeq 1.58.$$

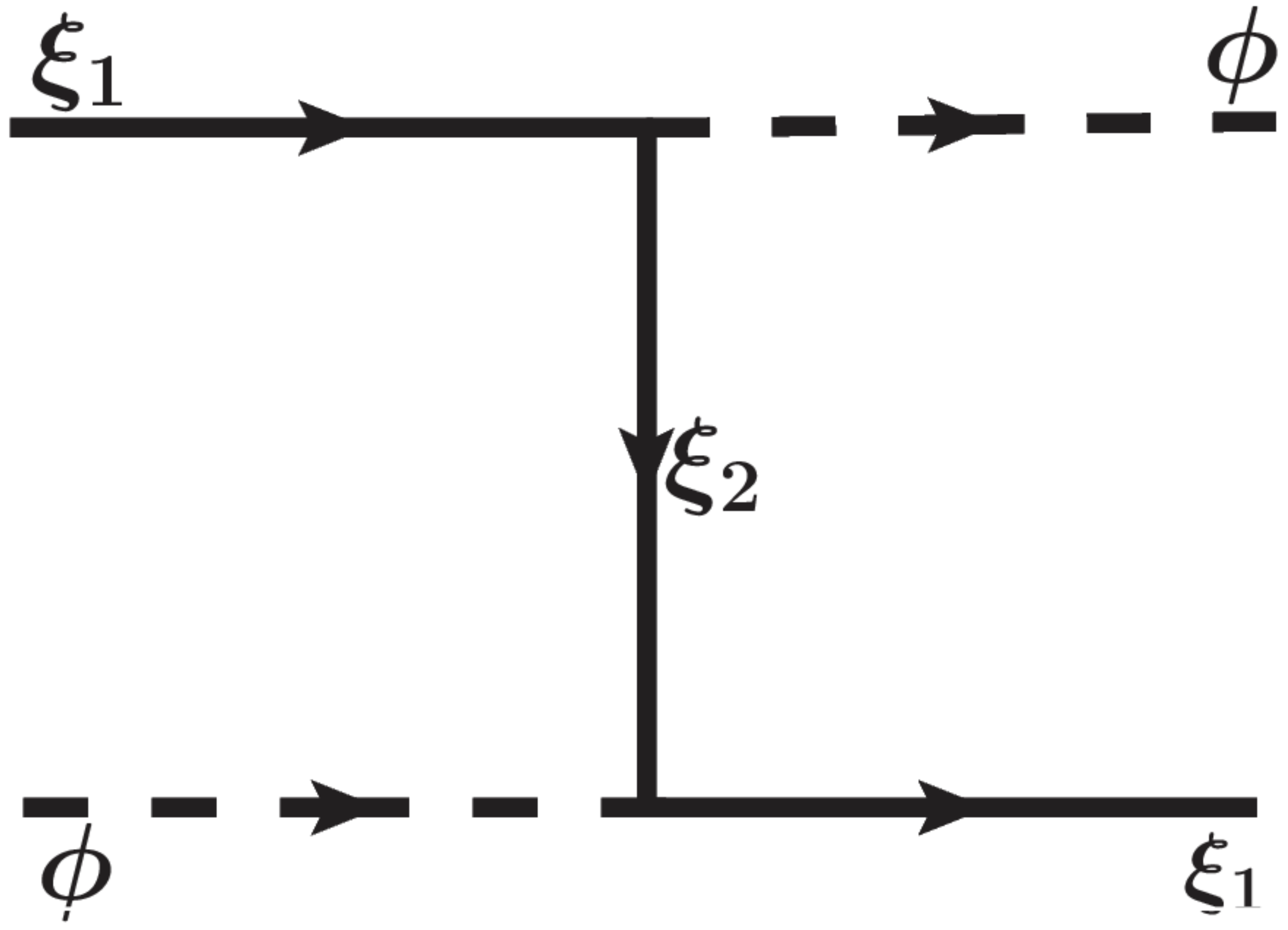


# DM-DR Interaction

- Dark matter and dark radiation elastic scattering dominated by u-channel process and Dark radiation transfer momentum to DM.

- A distinct temperature dependence for DM-DR interaction rate: a cut-off at the transition redshift  determined by the mass-splitting between inelastic dark fermions

$$R_\Gamma \equiv \frac{\Gamma}{H} \simeq 0.07 \left( \frac{y(1-2\sin^2\beta)}{4.1\times10^{-6}} \right)^4 \left( \frac{0.01\,\mathrm{MeV}}{M_{\xi_1}} \right), \quad \text{for} \quad T \gg \delta,$$




$$|\mathcal{M}|^2 \simeq \frac{y^4 \left(1-2\sin^2\beta\right)^4 M_{\xi_1}^2}{(k+\delta)^2}$$

# Conclusion

- Our SIDR+$z_t$ framework introduces a new approach to resolving cosmological tensions by leveraging self-interacting dark radiation and inelastic dark matter within a $U(1)_D$ gauge extension of the Standard Model.
- Temperature-dependent DM-DR interactions and flexible DR energy-density steps, allow it to address the Hubble tension, $S_8$ tension, and Lyman-$\alpha$ discrepancies while maintaining consistency with BBN constraints.

**Thank you for listening!**

# Backup slide

# Inelastic Dark Matter / Dark Gauge Boson

## Interaction between hidden sector and visible sector

$$\mathcal{L} \supset \epsilon g (Z_X)_\mu \overline{f}\gamma^\mu f + \epsilon g_X \frac{s_{\theta_W}}{c_{\theta_W}} Z_\mu \left[c_\theta^2 \; \overline{\xi_1}\gamma^\mu \xi_1 + s_\theta^2 \; \overline{\xi_2}\gamma^\mu \xi_2 + c_\theta s_\theta \; (\overline{\xi_1}\gamma^\mu \xi_2 + \overline{\xi_2}\gamma^\mu \xi_1)\right]$$

- Interaction like makes hidden sector particles from visible sector

$$f\bar{f} \to \xi_1\bar{\xi_1}, \; f\bar{f} \to \xi_1\bar{\xi_2}, \; f\bar{f} \to \bar{\xi_1}\xi_2, \; f\bar{f} \to \xi_2\bar{\xi_2}, \; f\bar{f} \to Z_X\gamma, \; f\gamma \to fZ_X$$

- Assuming interaction of dark/hidden sector with SM particles negligible

# Boltzmann equation

## DM

$$
\begin{aligned}
\frac{dY_{\xi_1}}{dT_v} = -\frac{s}{H}K_v \Bigg[ & \langle \sigma v \rangle_{f\bar{f}\to\xi_1\bar{\xi}_1}(T_v)\left(Y_f^{eq}(T_v)\right)^2 + \langle \sigma v \rangle_{f\bar{f}\to\xi_1\bar{\xi}_2}(T_v)\left(Y_f^{eq}(T_v)\right)^2 \\
& + \frac{1}{s}\langle \Gamma_{Z\to\xi_1\bar{\xi}_1}\rangle(T_v)Y_Z^{eq} + \frac{1}{s}\langle \Gamma_{Z\to\xi_1\bar{\xi}_2}\rangle(T_v)Y_Z^{eq} \\
& + \frac{1}{s}\langle \Gamma_{S\to\xi_1\bar{\xi}_1}\rangle(T_v)Y_S + \frac{1}{s}\langle \Gamma_{\xi_2\to\xi_1\phi}\rangle(T_h)Y_{\xi_2} \\
& + \langle \sigma v \rangle_{\bar{\xi}_1\xi_2\to\xi_1\bar{\xi}_1}(T_h)\left(Y_{\xi_1}Y_{\xi_2} - \frac{Y_{\xi_1}^{eq}(T_h)Y_{\xi_2}^{eq}(T_h)}{Y_{\xi_1}^{eq}(T_h)^2}Y_{\xi_1}^2\right) \\
& + \langle \sigma v \rangle_{\xi_2\bar{\xi}_2\to\xi_1\bar{\xi}_1}(T_h)\left(Y_{\xi_2}^2 - \left(\frac{Y_{\xi_2}^{eq}(T_h)}{Y_{\xi_1}^{eq}(T_h)}\right)^2 Y_{\xi_1}^2\right) \\
& - \langle \sigma v \rangle_{\xi_1\bar{\xi}_1\to Z_D Z_D}(T_h)\left(Y_{\xi_1}^2 - \left(\frac{Y_{\xi_1}^{eq}(T_h)}{Y_{Z_D}^{eq}(T_h)}\right)^2 Y_{Z_D}^2\right) \\
& - \langle \sigma v \rangle_{\xi_1\bar{\xi}_2\to Z_D Z_D}(T_h)\left(Y_{\xi_1}Y_{\xi_2} - \frac{Y_{\xi_1}^{eq}(T_h)Y_{\xi_2}^{eq}(T_h)}{Y_{Z_D}^{eq}(T_h)^2}Y_{Z_D}^2\right)\Bigg],
\end{aligned}
$$

# Boltzmann equation

## Gauge boson

$$\frac{dY_{Z_D}}{dT_v} = -\frac{s}{H}K_v \Bigg[\langle\sigma v\rangle_{f\bar{f}\to Z_D\gamma}(T_v)\left(Y_f^{eq}(T_v)\right)^2 + 2\langle\sigma v\rangle_{f\gamma\to fZ_D}(T_v)\left(Y_f^{eq}(T_v)\right)^2$$

$$+2\,\langle\sigma v\rangle_{\xi_1\bar{\xi_1}\to Z_D Z_D}(T_h)\left(Y_{\xi_1}^2 - \left(\frac{Y_{\xi_1}^{eq}(T_h)}{Y_{Z_D}^{eq}(T_h)}\right)^2 Y_{Z_D}^2\right)$$

$$+\,2\,\langle\sigma v\rangle_{\xi_2\bar{\xi_2}\to Z_D Z_D}(T_h)\left(Y_{\xi_2}^2 - \left(\frac{Y_{\xi_2}^{eq}(T_h)}{Y_{Z_D}^{eq}(T_h)}\right)^2 Y_{Z_D}^2\right)$$

$$+\,2\,\langle\sigma v\rangle_{\xi_1\bar{\xi_2}\to Z_D Z_D}(T_h)\left(Y_{\xi_1}Y_{\bar{\xi_2}} - \frac{Y_{\xi_1}^{eq}(T_h)Y_{\xi_2}^{eq}(T_h)}{Y_{Z_D}^{eq}(T_h)^2}Y_{Z_D}^2\right)$$

$$+\,2\,\langle\sigma v\rangle_{\xi_2\bar{\xi_1}\to Z_D Z_D}(T_h)\left(Y_{\bar{\xi_1}}Y_{\xi_2} - \frac{Y_{\xi_1}^{eq}(T_h)Y_{\xi_2}^{eq}(T_h)}{Y_{Z_D}^{eq}(T_h)^2}Y_{Z_D}^2\right)$$

$$-2\langle\sigma v\rangle_{Z_D Z_D\to\phi\phi}\left(Y_{Z_D}^2 - \left(\frac{Y_{Z_D}^{eq}(T_h)}{Y_{\phi}^{eq}(T_h)}\right)^2 Y_{\phi}^2\right)\Bigg],$$

# Boltzmann equation

## DR

$$\frac{dY_\phi}{dT_v} = -\frac{s}{H} K_v \left[ 2\langle \sigma v \rangle_{Z_D Z_D \to \phi\phi} \left( Y_{Z_D}^2 - \left( \frac{Y_{Z_D}^{eq}(T_h)}{Y_\phi^{eq}(T_h)} \right)^2 Y_\phi^2 \right) \right]$$

# Momentum Transfer rate

## Momentum transfer from DR to DM

- The momentum transfer equation can be expressed as

$$\dot{\vec{p}}_{\rm DM} = \frac{a}{2E_p} \int \frac{d^3k}{(2\pi)^3 2E_k} f(k;T) \int \frac{d^3k'}{(2\pi)^3 2E_{k'}} \frac{d^3p'_{\rm DM}}{(2\pi)^3 2E_{p'_{\rm DM}}} (2\pi)^4 \delta^{(4)}\left(p_{\rm DM} + k - p'_{\rm DM} - k'\right)$$
$$\times |\mathcal{M}|^2 \left(\vec{p'}_{\rm DM} - \vec{p}_{\rm DM}\right)$$

- Momentum transfer rate $\Gamma$ can be calculated as

$$\Gamma \simeq \frac{1}{8(2\pi)^3 M_{\rm DM}^3} \int k^3 f(k;T) dk \int d\cos\theta |\mathcal{M}|^2 (1 - \cos\theta)$$

# Momentum Transfer

## Momentum transfer from DR to DM

- Momentum transfer rate is proportional to $T^2$ at relatively high temperatures but suppressed to $T^4$ when $T_h \ll \delta$.

$$
\begin{aligned}
\Gamma &\simeq \frac{y^4\left(1-2\sin^2\beta\right)^4 T_h^2}{32\pi^3 M_{\xi_1}} f(x) \\
&\simeq 2.43\times 10^{-34}\mathrm{GeV}\left(\frac{T_h}{100\mathrm{eV}}\right)^2\left(\frac{y\left(1-2\sin^2\beta\right)}{4.1\times 10^{-6}}\right)^4\left(\frac{0.01\mathrm{MeV}}{M_{\xi_1}}\right), \ \text{for } T_h \gg \delta, \\
&\simeq 1.46\times 10^{-43}\mathrm{GeV}\left(\frac{T_h}{0.1\mathrm{eV}}\right)^4\left(\frac{y\left(1-2\sin^2\beta\right)}{4.1\times 10^{-6}}\right)^4\left(\frac{0.01\mathrm{MeV}}{M_{\xi_1}}\right)\left(\frac{10\mathrm{eV}}{\delta}\right)^2, \ \text{for } T_h \ll \delta,
\end{aligned}
$$

# MPS of the SIDR+/WZDR+ Model