PQ inflation for axion kinetic misalignment and baryogenesis

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1. [Inflation models with Peccei-Quinn symmetry and axion kinetic misalignment, H.M Lee, A.G. Menkara, M.J Seong, J.H Song August 30, 2024]

2. [Peccei-Quinn Inflation at the Pole and Axion Kinetic Misalignment, H.M Lee, A.G. Menkara, M.J Seong, J.H Song JHEP 05 Oct 26, 2023]

2025 CUBES-7 Workshop

Why KSVZ or DFSZ?

1. We need an **additional CP phase** to solve Strong CP problem

2. We can make **complex VEV** by considering the dynamics of $U(1)_{PQ}$ scalar field: Φ

[CP Conservation in the Presence of Pseudoparticles, R. D. Peccei and Helen R. Quinn (1977)]

3. If there is a quark that charged under $U(1)_{PQ}$, we can transit the **phase of VEV** to QCD phase by chiral rotation ($U(1)_{PQ}$ transformation).

-KSVZ-
new vector-like quarks charged under $U(1)_{PQ}$

[Weak-Interaction Singlet and Strong CP Invariance, Kim (1979)]

Not ruled out models

-DFSZ-
SM quarks charged under $U(1)_{PQ}$ (requires 2HDM)

[A Simple solution to the strong CP problem with a harmless axion, DINE, FISCHLER, SREDNICKI (1981)

$\Phi_{U(1)_{PQ}}$ mediating inflation

1. PQ field is conformally coupled to gravity ⇒ we can construct **PQ pole inflation**

[Peccei-Quinn Inflation at the Pole and Axion Kinetic Misalignment, H.M Lee, A.G. Menkara, M.J Seong, J.H Song Oct 26, 2023]

2. In this set-up, We can discuss the **Axion quality problem** for KSVZ, and DFSZ models

[Inflation models with Peccei-Quinn symmetry and axion kinetic misalignment, H.M Lee, A.G. Menkara, M.J Seong, J.H Song August 30, 2024]

3. We can determine the **reheating temperature** depending on free parameter in each model

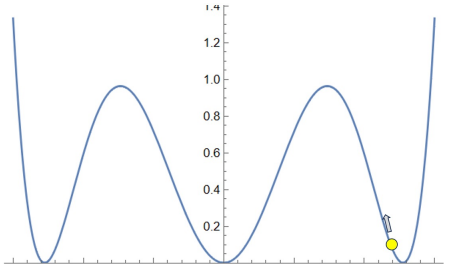
4. We think of axial mode of PQ field as axion (DM), and we can find the **axion decay constant** to get the correct **DM relic density**

5. We also look for whether we can produce the current **baryon asymmetry** for each model by Spontaneous baryogenesis

PQ pole inflation $U(1)_{PQ}$

$$\frac{\mathcal{L}_J}{\sqrt{-g_J}} = -\frac{1}{2}M_P^2\,\Omega R(g_J) + \mathcal{L}_{\mathrm{kin},J} - \Omega^2 V_E$$

Jordan frame



KSVZ model

$$\mathcal{L}_{\mathrm{kin},J} = |\partial_\mu \Phi|^2$$

$$\Omega(\Phi) = 1 - \frac{1}{3M_P^2}|\Phi|^2$$

Kinetic term

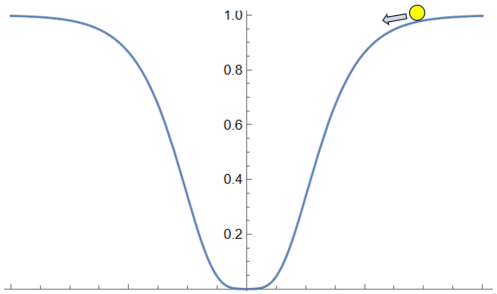
DFSZ model

$$\mathcal{L}_{\mathrm{kin},J} = |D_\mu H_1|^2 + |D_\mu H_2|^2 + |\partial_\mu \Phi|^2$$

$$\Omega = 1 - \frac{1}{3M_P^2}|H_1|^2 - \frac{1}{3M_P^2}|H_2|^2 - \frac{1}{3M_P^2}|\Phi|^2$$

PQ pole inflation $U(1)_{PQ}$

$$\frac{\mathcal{L}_E}{\sqrt{-g_E}} = -\frac{1}{2}M_P^2 R + \frac{1}{\Omega}(\mathcal{L}_{kin,E}) + \frac{3}{4}M_P^2\frac{(\partial_\mu\Omega)^2}{\Omega^2} - V_E$$ Einstein frame



KSVZ model

$$\frac{\mathcal{L}_E}{\sqrt{-g_E}} = -\frac{1}{2}M_P^2 R + \frac{1}{2}(\partial_\mu\phi)^2 + 3M_P^2\sinh^2\left(\frac{\phi}{\sqrt{6}M_P}\right)(\partial_\mu\theta)^2 - V_E(\phi,\theta)$$

DFSZ model

$$\frac{1}{\Omega}\frac{\mathcal{L}_{\mathrm{kin},E}}{\sqrt{-g_E}} = -\frac{1}{2}R + \frac{(\partial_\mu h_1)^2 + (\partial_\mu h_2)^2 + h_1^2(\partial_\mu\eta_1)^2 + h_2^2(\partial_\mu\eta_2)^2 + (\partial_\mu\rho)^2 + \rho^2(\partial_\mu\theta)^2}{2(1-\frac{1}{6}h_1^2-\frac{1}{6}h_2^2-\frac{1}{6}\rho^2)} + \frac{3}{4}\frac{\left(\partial_\mu\left(\frac{1}{6}h_1^2+\frac{1}{6}h_2^2+\frac{1}{6}\rho^2\right)\right)^2}{(1-\frac{1}{6}h_1^2-\frac{1}{6}h_2^2-\frac{1}{6}\rho^2)^2}.$$

$$H_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ h_1 e^{i\eta_1}\end{pmatrix}, \quad H_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ h_2 e^{i\eta_2}\end{pmatrix}, \quad \Phi = \frac{1}{\sqrt{2}}\rho\, e^{i\theta}$$

-Charge neutral-

Peccei-Quinn pole inflation $U(1)_{PQ}$

$$\frac{\mathcal{L}_J}{\sqrt{-g_J}} = -\frac{1}{2}M_P^2\,\Omega R(g_J) + \mathcal{L}_{\mathrm{kin},J} - \Omega^2 V_E$$

$$V_E(\phi,\theta) \;=\; V_{\mathrm{PQ}}(\phi) + V_{\mathrm{PQV}}(\rho,\theta)$$

KSVZ model

$$V_{\mathrm{PQ}} \;= V_0 + \frac{1}{4}\lambda_\Phi(\rho^2 - f_a^2)^2$$

$$V_{\mathrm{PQV}} = \frac{1}{2^{n/2}}\left(\frac{\phi}{M_P}\right)^n M_P^4 \sum_{k=0}^{[n/2]} |c_k| \cos\Big((n-2k)\theta + A_k\Big)$$

For $N = 60$

$n_s = 0.966 \quad r = 0.0033$

CMB normalization

$\lambda_\Phi = 1.1 \times 10^{-11}$

DFSZ model

$$\begin{aligned} V_{\mathrm{PQ}} \;=\; & \lambda_\Phi|\Phi|^4 + \lambda_1|H_1|^4 + \lambda_2|H_2|^4 + \lambda_3|H_1|^2|H_2|^2 + \lambda_4\left(H_1^\dagger H_2\right)\left(H_2^\dagger H_1\right) \\ & + \lambda_{1\Phi}|H_1|^2|\Phi|^2 + \lambda_{2\Phi}|H_2|^2|\Phi|^2 + \left(2^{p/2-1}\kappa_p H_1^\dagger H_2 \Phi^p + \mathrm{h.c.}\right) \\ & + \mu_\Phi^2|\Phi|^2 + m_1^2|H_1|^2 + m_2^2|H_2|^2 + V_0. \end{aligned}$$

$$V_{\mathrm{PQV}} = \sum_{n,l}\sum_{k=0}^{[l/2]} \frac{c_{n,l,k}}{2M^{2n+l-4}}\,(H_1^\dagger H_2)^n |\Phi|^{2k}\Phi^{l-2k} + \mathrm{h.c.}$$

Stability of the non-inflation direction

In DFSZ, It is Multi-field inflation

$$\Phi = \frac{1}{\sqrt{2}}\rho\, e^{i\theta} \qquad H_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ h_1 e^{i\eta_1} \end{pmatrix}, \qquad H_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ h_2 e^{i\eta_2} \end{pmatrix}$$

We have to stabilize the

$$\tau_1 \equiv \frac{h_1}{\rho} \quad \Rightarrow \quad \langle \tau_1 \rangle$$

$$\tau_2 \equiv \frac{h_2}{\rho} \quad \Rightarrow \quad \langle \tau_2 \rangle$$

(False vacuum)

Perturbation around VEV

$$\tau_1 \equiv \langle \tau_1 \rangle + \bar{\tau}_1$$

$$\tau_2 \equiv \langle \tau_2 \rangle + \bar{\tau}_2$$

Stabilizing Condition: The Hessian of the squared mass matrix > 0

$$\frac{\mathcal{L}_E}{\sqrt{-g_E}} \supset \begin{pmatrix} \bar{\tau}_1 & \bar{\tau}_2 \end{pmatrix} \begin{pmatrix} m_{11}^2 & m_{12}^2 \\ m_{21}^2 & m_{22}^2 \end{pmatrix} \begin{pmatrix} \bar{\tau}_1 \\ \bar{\tau}_2 \end{pmatrix} \qquad \det \begin{vmatrix} m_{11}^2 & m_{12}^2 \\ m_{21}^2 & m_{22}^2 \end{vmatrix} > 0$$

Ex) the case of $\langle \tau_1 \rangle = \langle \tau_2 \rangle = 0$ (pure PQ inflation),

Diagonalized mass: $m_{\pm}^2 = \frac{1}{4}\left[\lambda_{1\Phi} + \lambda_{2\Phi} - 4\lambda_\Phi \pm \sqrt{(\lambda_{1\Phi} - \lambda_{2\Phi})^2 + 4\rho^{2p-4}\kappa_p^2}\right]\rho^4$

$$\det \begin{vmatrix} m_+^2 & 0 \\ 0 & m_-^2 \end{vmatrix} > 0 \quad \Rightarrow \quad (\lambda_{1\Phi} - 2\lambda_\Phi)(\lambda_{2\Phi} - 2\lambda_\Phi) > \rho^{2p-4}\kappa_p^2$$

Higgs directions are

decoupled if $m_{\pm,\mathrm{eff}}^2 \gg H_I^2 \simeq 3\lambda_\Phi$

Axion Quality Problem

$$\mathcal{L}_{\text{gluons}} = \frac{g_s^2}{32\pi^2}\left(\bar{\theta} + \xi\frac{a}{f_a}\right)G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$$

KSVZ model

$$\xi = 1$$

Due to a new heavy quark

$$\mathcal{L}_{Q,\text{int}} = -y_Q \Phi \bar{Q}_R Q_L + \text{h.c.}$$

$$v_\Phi \gg v_1, v_2 \quad \Rightarrow \quad f_a \simeq q_\Phi v_\Phi \qquad a_{QCD} \simeq a_\Phi$$

DFSZ model

$$\xi = 3pq_\Phi$$

Due to PQ conserving potential and Yukawa term

$$\begin{aligned}\mathcal{L} \supset & (2^{p/2-1}\kappa_p H_1^\dagger H_2 \Phi^p + h.c.) \\ & + y^d_{ij}\bar{f}_L H_1 f^d_R + y'^d_{ij} H_2 f^d_R \\ & + y^u_{ij}\bar{f}_L \tilde{H}_1 f^u_R + y'^u_{ij}\tilde{H}_2 f^u_R\end{aligned}$$

Axion Quality Problem

$$\mathcal{L}_{\text{gluons}} = \frac{g_s^2}{32\pi^2}\left(\bar{\theta} + \xi\frac{a}{f_a}\right)G^a_{\mu\nu}\tilde{G}^{a\mu\nu} \qquad |\theta_{\text{eff}}| = \left|\bar{\theta} + \xi\frac{\langle a\rangle}{f_a}\right| < 10^{-10}$$ (EDM bound)

KSVZ model $\xi = 1$

$$\left(\frac{f_a}{M_P}\right)^n \lesssim \frac{2^{n/2}\xi}{(n-2k)|c_k|}\left(\frac{\Lambda_{\text{QCD}}}{M_P}\right)^4 \times 10^{-10}$$

The order of boundary is the same

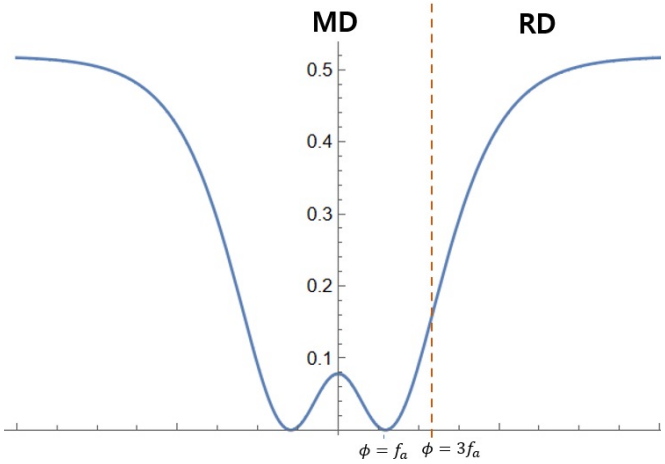
DFSZ model $\xi = 3pq_\Phi$

$$\left(\frac{f_a}{M_P}\right)^l \lesssim \frac{2^{l/2}\xi\, q_\Phi^{l-1}}{(l-2k)|c_{0,l,k}|}\left(\frac{\Lambda_{\text{QCD}}}{M_P}\right)^4 \times 10^{-10}$$

For type II, Y

Inflaton condensation

Right after the end of inflation $V_{PQ} = \frac{\lambda_\phi}{4}(\phi^2 - f_a^2)^2$



$$\phi = \phi_0(t)\mathcal{P}(t)$$

$\phi_0(t)$ The amplitude of the inflaton oscillation

$\mathcal{P}(t)$ Periodic function

$$\langle w_\phi \rangle = \frac{p_\phi}{\rho_\phi} = \frac{m-1}{m+1}$$

$$m_\phi^2 = V_E''(\phi_0) = 2\alpha_m m(2m-1)\phi_0^{2m-2}$$

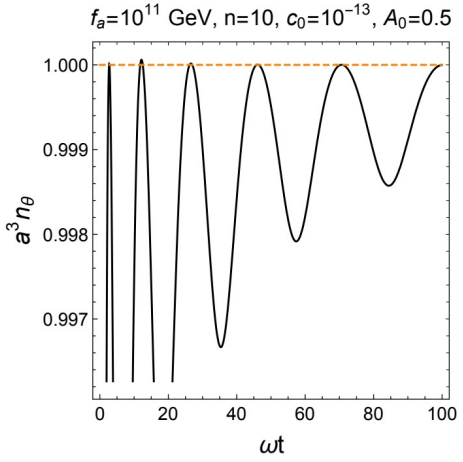
m=1 => quadratic potential $V(\phi) \simeq \lambda_\phi f_a^2 \phi'^2$

m=2 => quartic potential $V(\phi) \simeq \frac{1}{4}\lambda_\phi \phi_0^4(t)$

Evolution of Axion velocity after inflation

$$\epsilon_{\theta,\mathrm{end}} = \frac{1}{2V_E^2}\left(\frac{\partial V_E}{\partial\theta}\right)^2 \lesssim 1$$

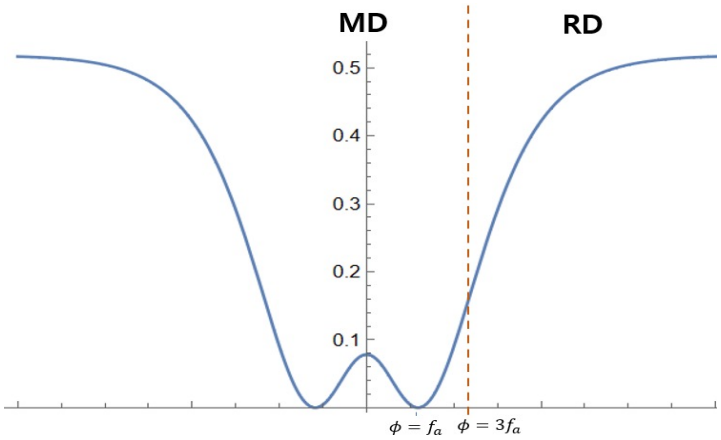
Initial axial velocity : $\dot{\theta} \simeq -\frac{1}{3H}\frac{\frac{\partial V_E}{\partial\theta}}{6M_P^2\sinh^2\left(\frac{\phi}{\sqrt{6}M_P}\right)} = -\frac{\sqrt{2\epsilon_\theta}\,H}{6\sinh^2\left(\frac{\phi}{\sqrt{6}M_P}\right)}$



$$a^3 n_\theta = a^3\phi^2\dot{\theta} \simeq \text{const}$$

After end of inflation, PQ violation terms are suppressed by $\phi \ll M_p$

, so PQ noether charge will be conserved



Redshift

1. Radiation-like $\phi(a_{\mathrm{RH}}) > 3f_a$

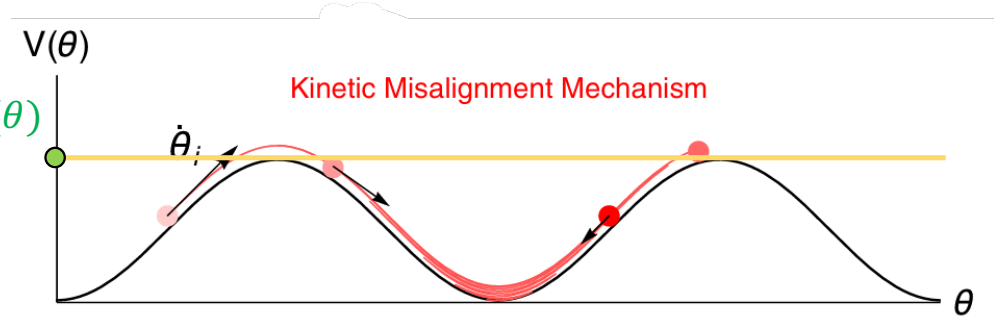
$$n_\theta(T_{\mathrm{RH}}) = n_{\theta,\mathrm{end}}\left(\frac{\pi^2 g_*(T_{\mathrm{RH}})T_{\mathrm{RH}}^4}{45V_E(\phi_{\mathrm{end}})}\right)^{3/4}$$

2. Matter-like $\phi(a_{RH}) < 3f_a$

$$n_\theta(T_{\mathrm{RH}}) = n_{\theta,\mathrm{end}}\left(\frac{\pi^2 g_*(T_{\mathrm{RH}})T_{\mathrm{RH}}^4}{45V_E(\phi_{\mathrm{end}})}\right)^{3/4}\left(\frac{T_{\mathrm{RH}}}{T_{\mathrm{RH}}^c}\right)$$

Kinetic Misalignment mechanism

$K = V_{max}(\theta)$



$K = V_{max}(\theta)$ ⇨ $\frac{1}{2} f_a^2 \dot{\theta}^2(T_*) = 2 m_a^2(T_*) f_a^2$

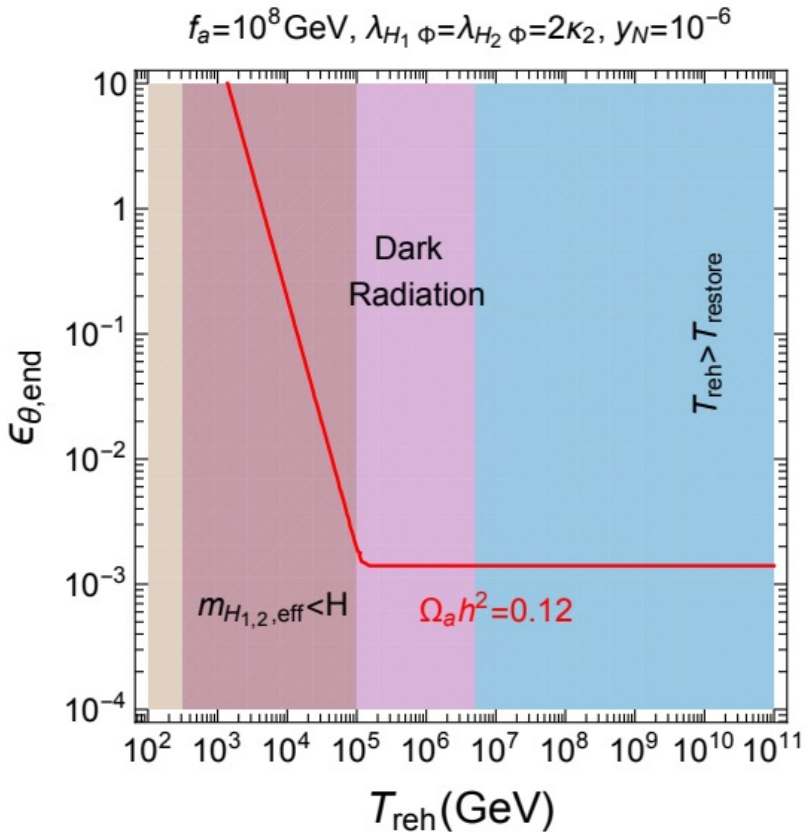
(starts oscillation) $m_a(T_*) \geq 3H(T_*)$

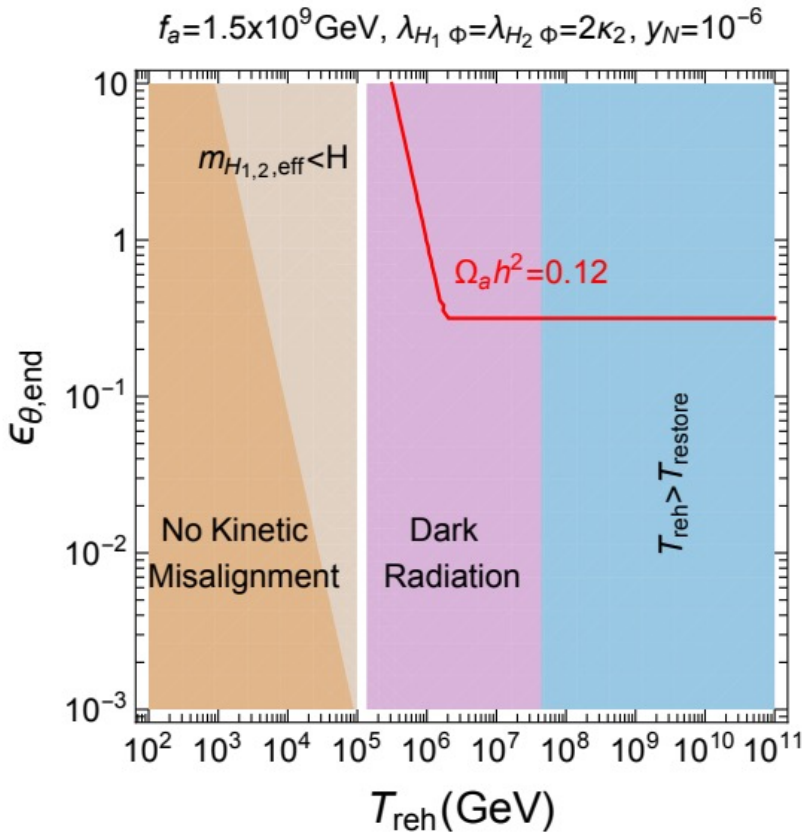
$$\Omega_a h^2 = 0.12 \left(\frac{10^9\,\mathrm{GeV}}{f_a} \right) \left(\frac{Y_\theta}{40} \right) \qquad Y_\theta = \frac{n_\theta(T_{\mathrm{RH}})}{s(T_{\mathrm{RH}})}$$

Axion relic density

$$\Omega_a h^2 = 0.12\left(\frac{10^9\,\mathrm{GeV}}{f_a}\right)\left(\frac{Y_\theta}{40}\right)$$

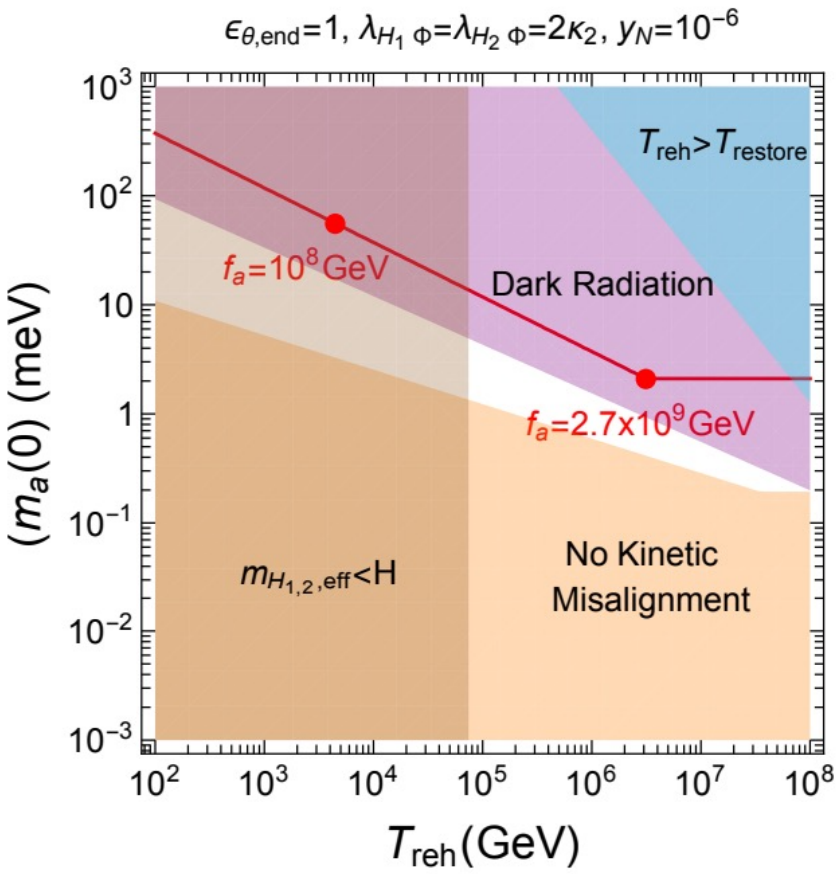
For the DFSZ model,

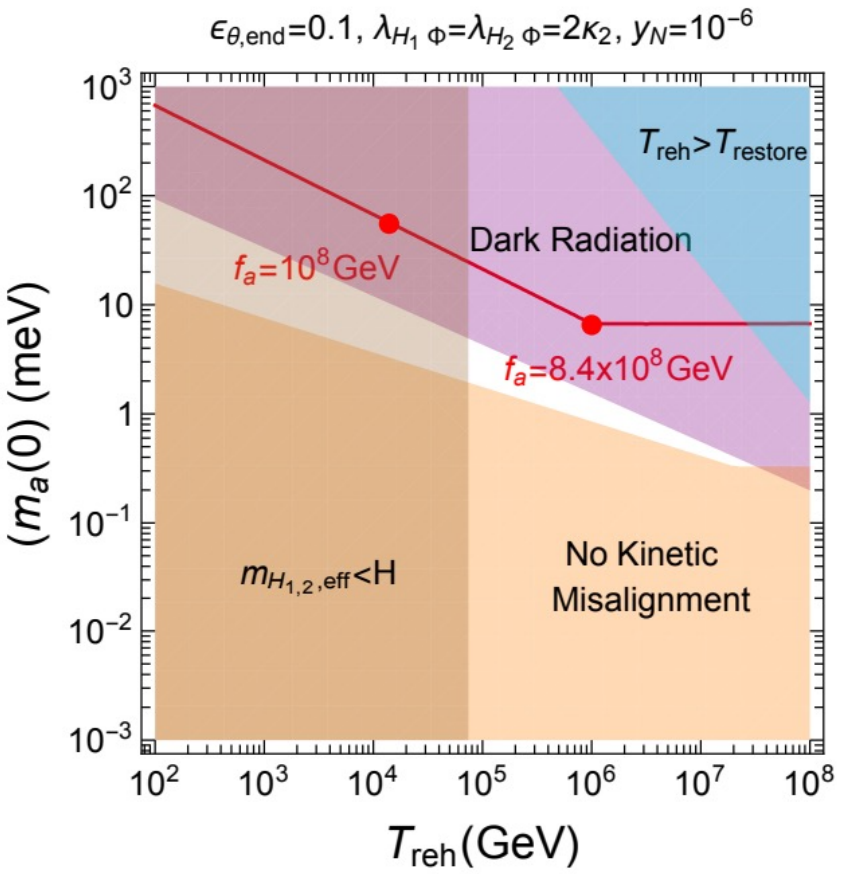




The range of axion decay constant

For the DFSZ model,





Spontaneous baryogenesis with seesaw model from PQ pole inflation

Set up

mass hierarchy: $M_{N_1} \ll M_{N_2}, M_{N_3}$

$$\mathcal{L} = \mathcal{L}_{\rm DFSZ} + \mathcal{L}_{\rm Yukawa} + \mathcal{L}_{\rm anomaly}$$

$$-\mathcal{L}_{\rm Yukawa} = y_u q u^c H_u + y_d q d^c H_d + y_e l e^c H_d + y_\nu l N H_u + h.c.$$

$$-\mathcal{L}_{\rm DFSZ} = y_H \Phi^2 H_u H_d + \frac{1}{2} y_N \Phi \overline{N}_R^c N_R + \text{h.c.}$$

$$-\mathcal{L}_{\rm anomaly} = \frac{c_s}{32\pi^2}\frac{a}{v_a} G\tilde{G} + \frac{c_w}{32\pi^2}\frac{a}{v_a} W\tilde{W}$$

Imposing PQ charge for DFSZ

PQ charge	Φ	H_u	H_d	N	u^c	d^c	e^c	l
	1	-1	-1	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{3}{2}$

$$-\mathcal{L}_{\text{Yukawa}} = y_u q u^c H_u + y_d q d^c H_d + y_e l e^c H_d + y_\nu l N H_u + h.c.$$

$$-\mathcal{L}_{\text{DFSZ}} = y_H \Phi^2 H_u H_d + \text{h.c.}$$

Shifting chemical potential

By taking a local $U(1)_{PQ}$ transformation,

we get the $(\partial_\mu \theta \cdot J^\mu_{PQ})$ term and anomaly term

$$J^\mu_{\rm PQ} = -\frac{3}{2}\bar{l}\gamma^\mu l - \frac{1}{2}\bar{e}\gamma^\mu e + \frac{1}{2}\bar{d}\gamma^\mu d + \frac{1}{2}\bar{u}\gamma^\mu u + \frac{1}{2}\overline{N}_R\gamma^\mu N_R - \frac{1}{2}\overline{N}^c_R\gamma^\mu N^c_R + \left(iH^\dagger_{u,d}\partial^\mu H_{u,d} - iH_{u,d}\partial^\mu H^\dagger_{u,d}\right)\ldots$$

$$-\mathcal{L}_{\rm anomaly} = \frac{c_s}{32\pi^2}\frac{a}{v_a}G\tilde{G} + \frac{c_w}{32\pi^2}\frac{a}{v_a}W\tilde{W}$$

Energy shifting

$$E_{\psi\pm} = \sqrt{\mathbf{p}^2 + m^2_\psi} \pm (\text{PQ charge})\cdot\dot{\theta}$$

$$f \simeq e^{-\frac{E+\mu}{T}} = e^{-\frac{E_0 \pm (\text{PQ charge})\cdot\dot{\theta}+\mu}{T}}$$

Effective chemical potential

$$\mu'_{\rm eff} = \mu \pm (\text{PQ charge})\cdot\dot{\theta}$$

Chemical equilibrium around M_N

Wash-out (in) effects appear when $\frac{M_N}{z_{out}} < T < \frac{M_N}{z_{in}}$ (ex. For K=50, $z_{out} \cong 7.8$)

Quarks and higgs couplings

$$\mu_q + \mu_{u^c} + \mu_{x_{H_u}} = 0, \quad \mu_q + \mu_{d^c} + \mu_{x_{H_d}} = 0,$$
$$\mu_l + \mu_{e^c} + \mu_{x_{H_d}} = 0, \quad \mu_l + \mu_{x_{H_u}} = -x_N\dot{\theta}, \qquad \mu_N \cong 0$$

$H_u H_d \to WW$

$$\mu_{H_u} + \mu_{H_d} = (x_{H_u} + x_{H_d})\dot{\theta},$$

Strong/Weak sphaleron

$$N_f(3\mu_q + \mu_l) = c_w\dot{\theta}, \quad N_f(2\mu_q + \mu_{u^c} + \mu_{u^c}) = c_s\dot{\theta}$$

Charge neutrality

$$N_f(\mu_q - 2\mu_{u^c} + \mu_{d^c} - \mu_l + \mu_{e^c}) + 2\mu_{H_u} - 2\mu_{H_d} = 0$$

$$\Rightarrow \quad \mu_{B-L} = \frac{1}{9}\left(8c_w - 16c_s + 69x_N\right)\dot{\theta}$$

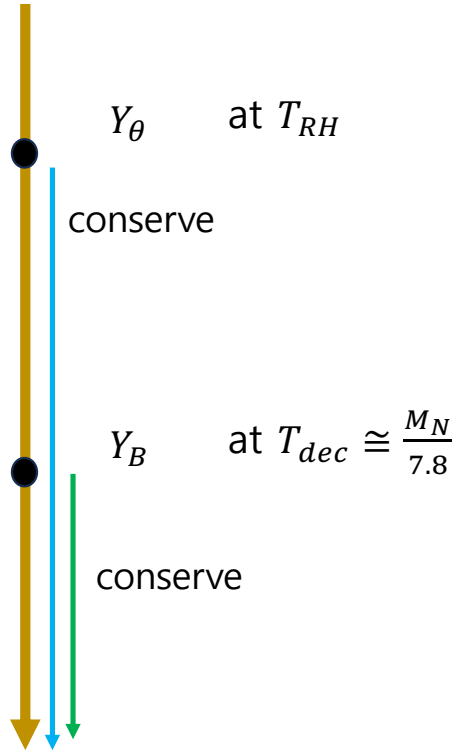
Baryon asymmetry

At the reheating temperature T_{RH}, $\quad Y_\theta \equiv \frac{n_\theta}{s} = \frac{v_a^2 \dot{\theta}}{\frac{2\pi^2}{45} g_* T^3}$

$$Y_{B-L} \equiv \frac{n_{B-L}}{s} \xrightarrow{n_{B-L} = \frac{1}{6}\mu_{B-L} T^2} Y_B = c_B Y_\theta \left(\frac{T_{B-L}}{v_a}\right)^2$$

-Successful cogenesis-
DM abundance & baryon asymmetry

$$1.13 \frac{N_{DW}^2}{c_B} \left(\frac{f_a}{10^9 \mathrm{GeV}}\right) \left(\frac{10^3 \mathrm{GeV}}{T_{B-L}}\right)^2 = 1$$

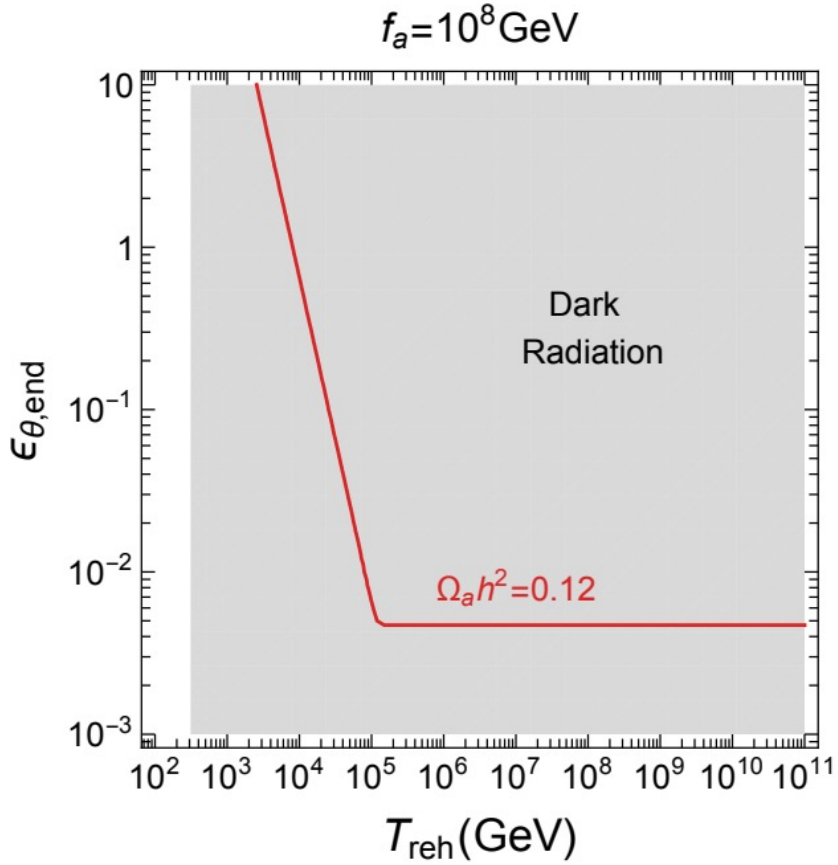


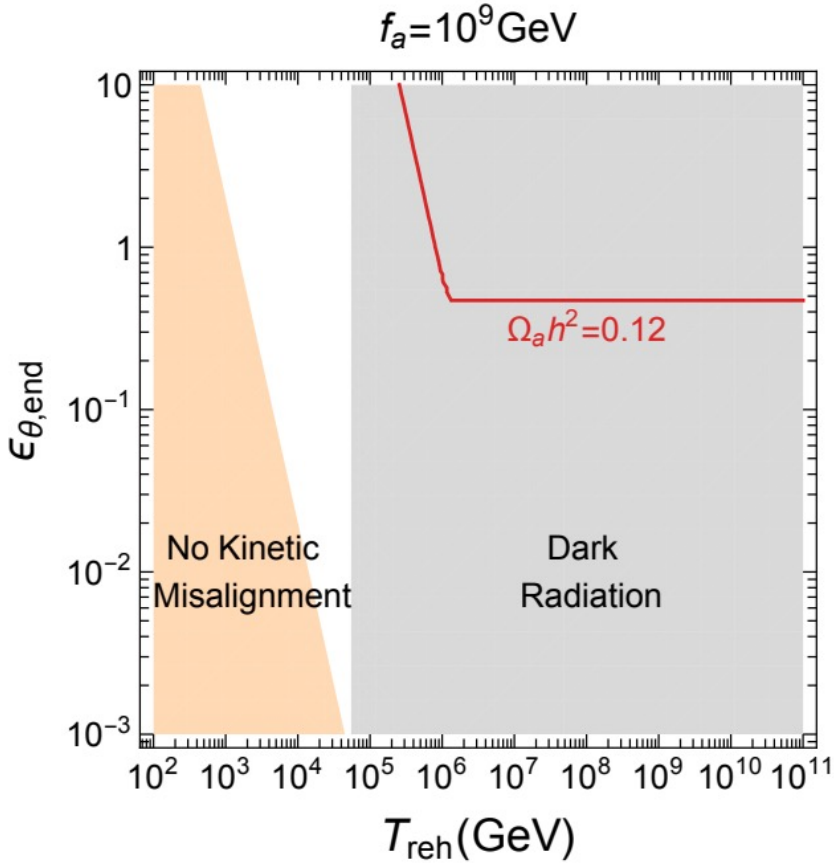
Axion relic density

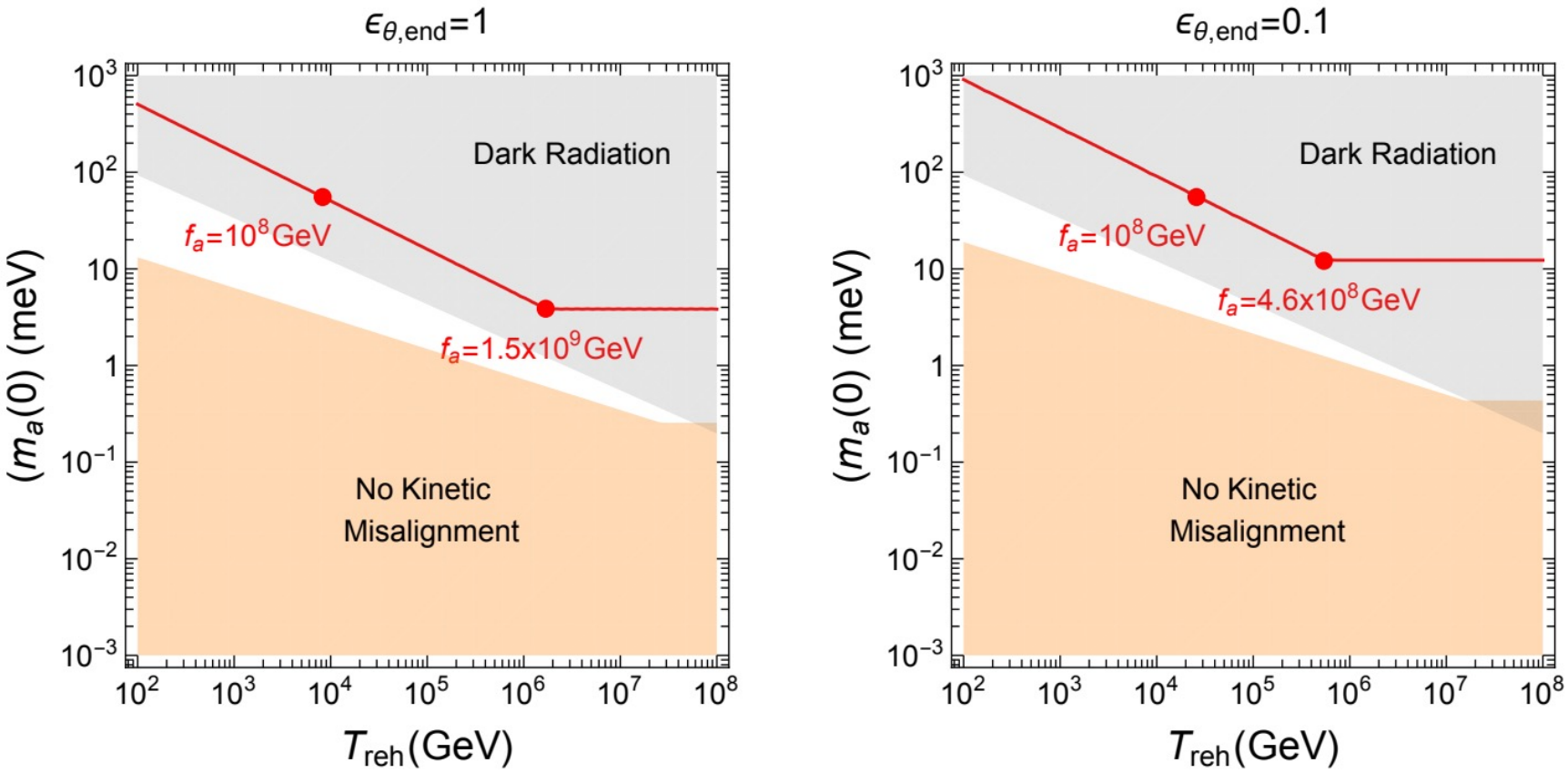
$$Y_{a,\mathrm{mis}} = 0.11\left(\frac{f_a}{10^9\,\mathrm{GeV}}\right)^{13/6}$$

$$\Omega_a h^2 = 0.12\left(\frac{10^9\,\mathrm{GeV}}{f_a}\right)\left(\frac{Y_\theta}{40}\right)$$

For the KSVZ model,





The range of axion decay constant

For the KSVZ model,