# String Bundles in Multi-Axion Models and the QCD Axion Domain-Wall Problem

October 27, 2025

@IBS Conference on Dark World 2025

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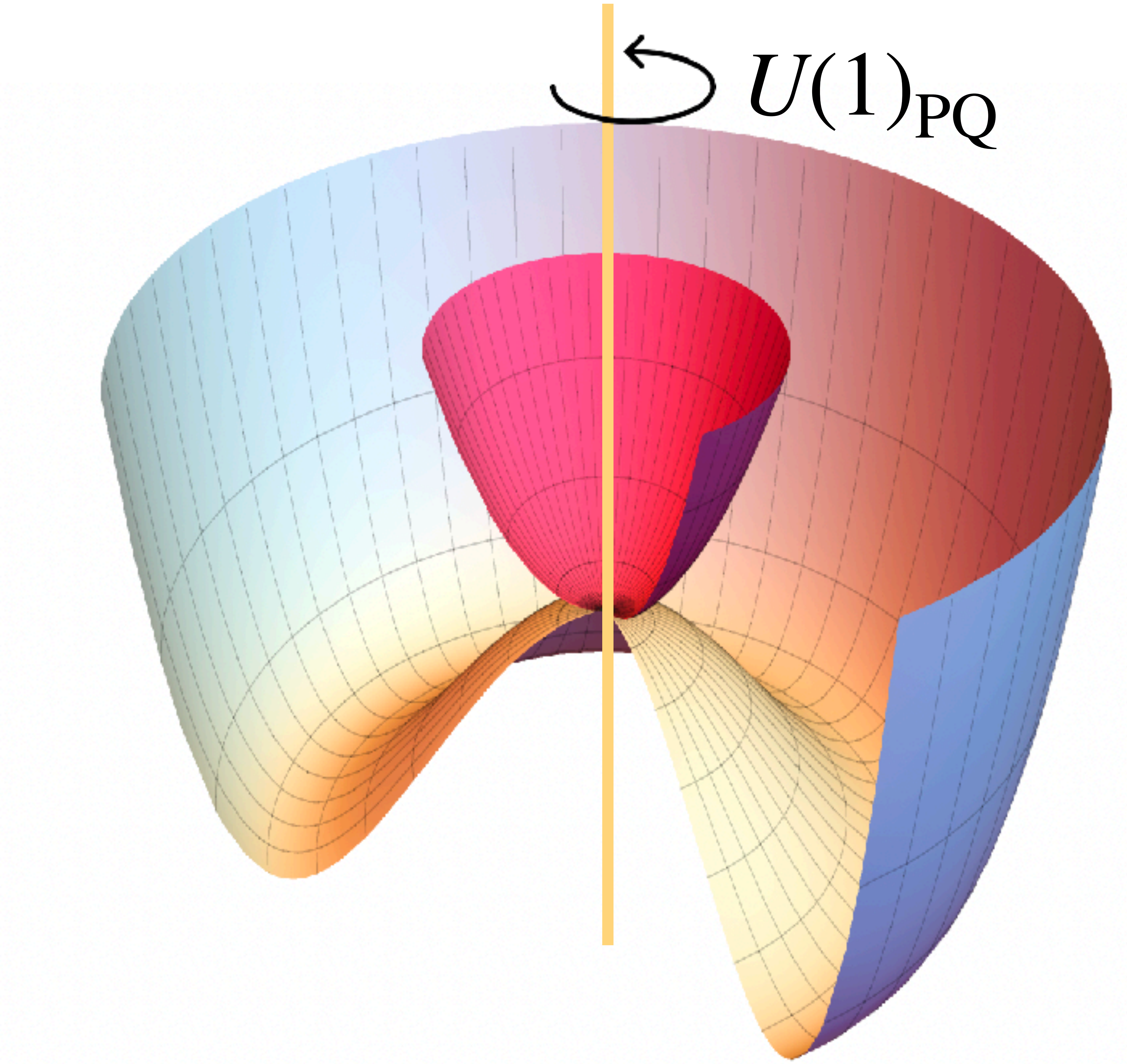
Based on Lee, Murai, FT and Yin 2409.09749, 2407.09478, 2507.07075

# QCD axion

The QCD axion is a pseudo Nambu-Goldstone boson associated with SSB of U(1) Peccei-Quinn symmetry.




Peccei, Quinn `77, Weinberg `78, Wilczek `78

See talks by Rybka and Ahn

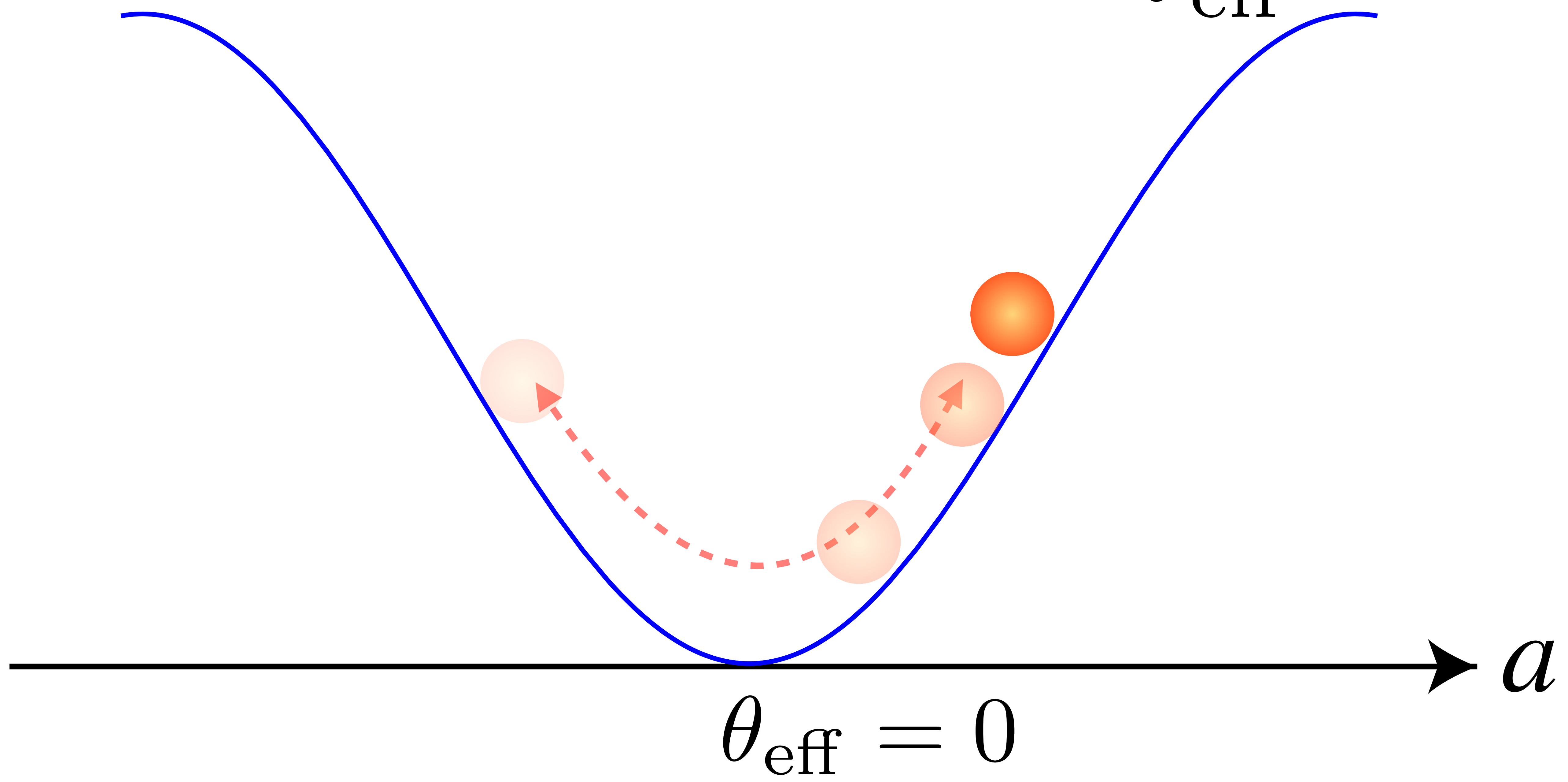
# QCD axion

Dynamically solves the strong CP problem.

$$\mathcal{L}_\theta = \boxed{\theta} \frac{g_s^2}{32\pi^2} G^{a\mu\nu} \tilde{G}^a_{\mu\nu} \quad \Rightarrow \quad \mathcal{L}_\theta = \underbrace{\left( \theta + \frac{a}{f_a} \right)}_{\theta_{\rm eff}} \frac{g_s^2}{32\pi^2} G^{a\mu\nu} \tilde{G}^a_{\mu\nu}$$

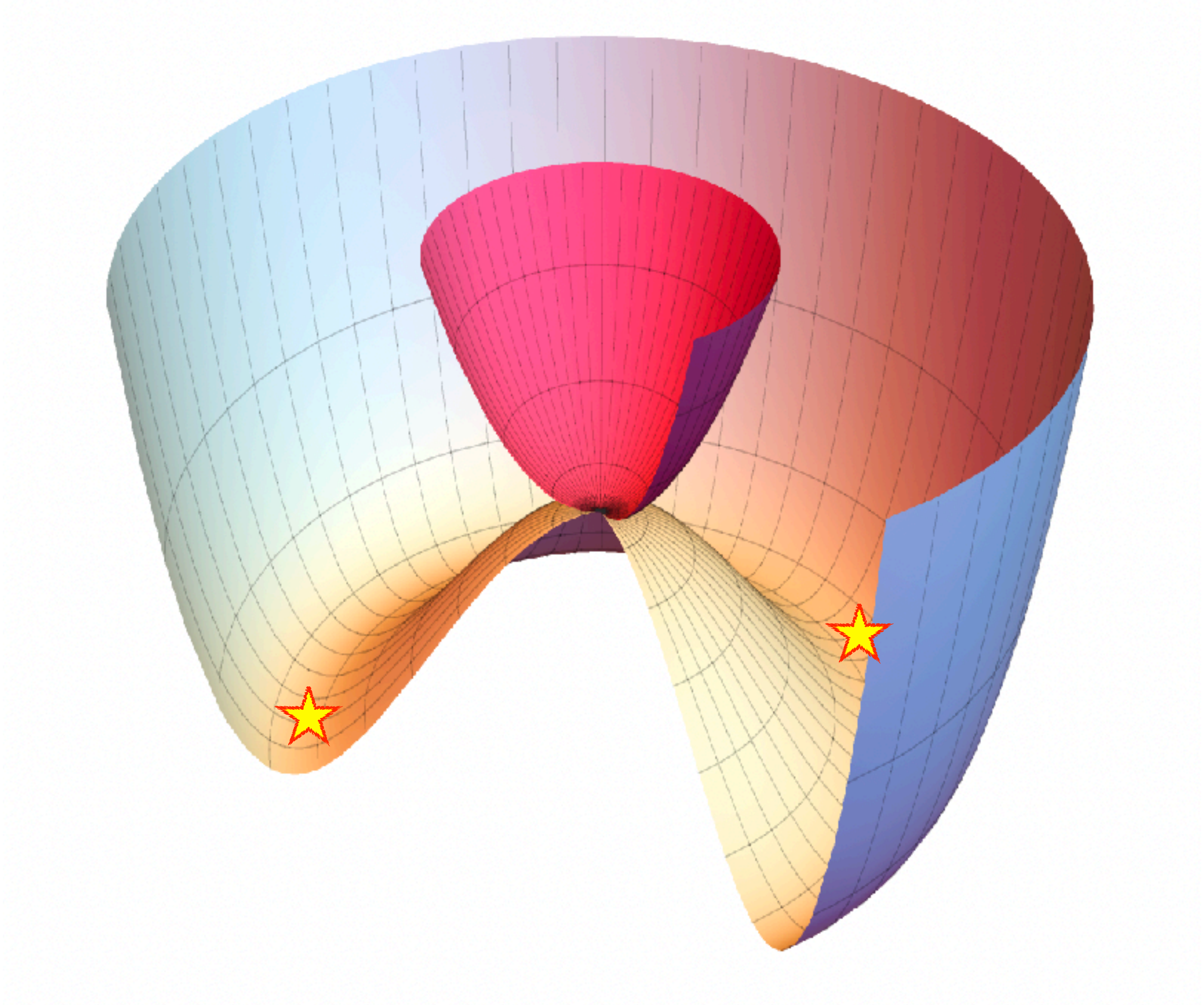
$|\bar{\theta}| \lesssim \mathcal{O}(10^{-10})$ : strong CP problem

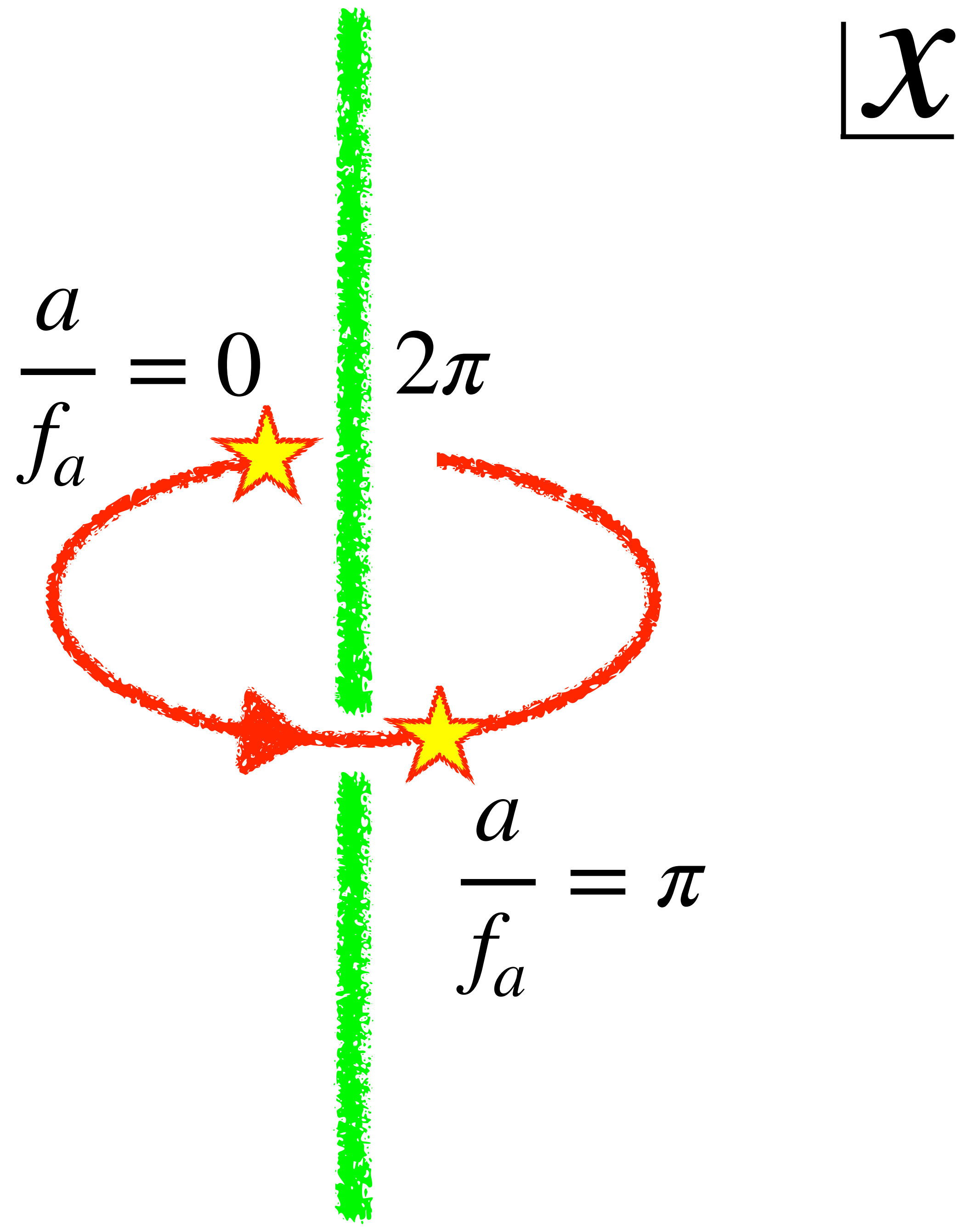
Neutron EDM,
Abel et al, 2001.11966



# Axion production from strings/walls

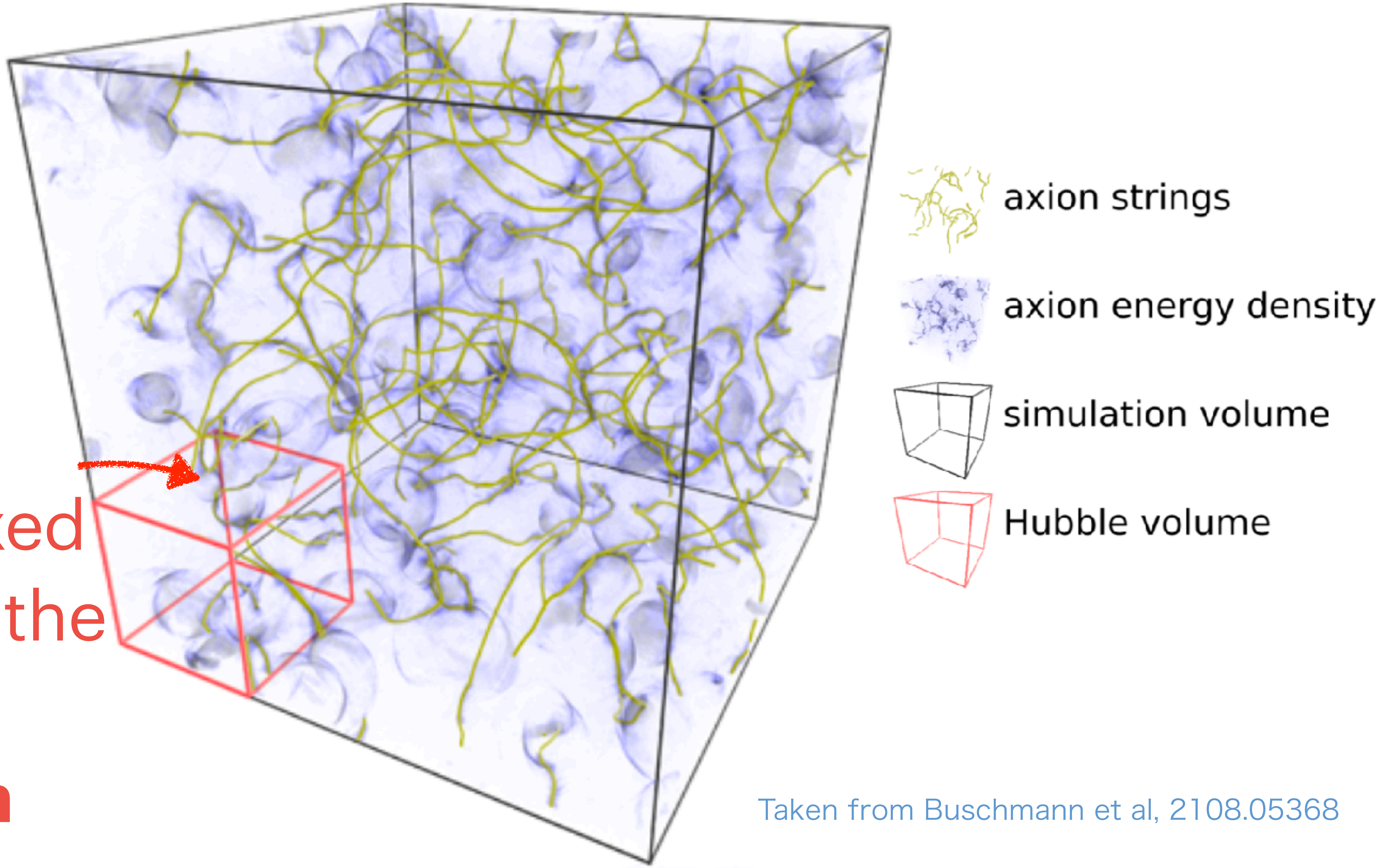
Cosmic strings and domain walls, formed in post-inflationary scenarios, produce axion dark matter.






Cosmic strings

# Axion production from strings/walls




(Approximately) fixed numer of strings in the Hubble volume: **Scaling solution**

Taken from Buschmann et al, 2108.05368

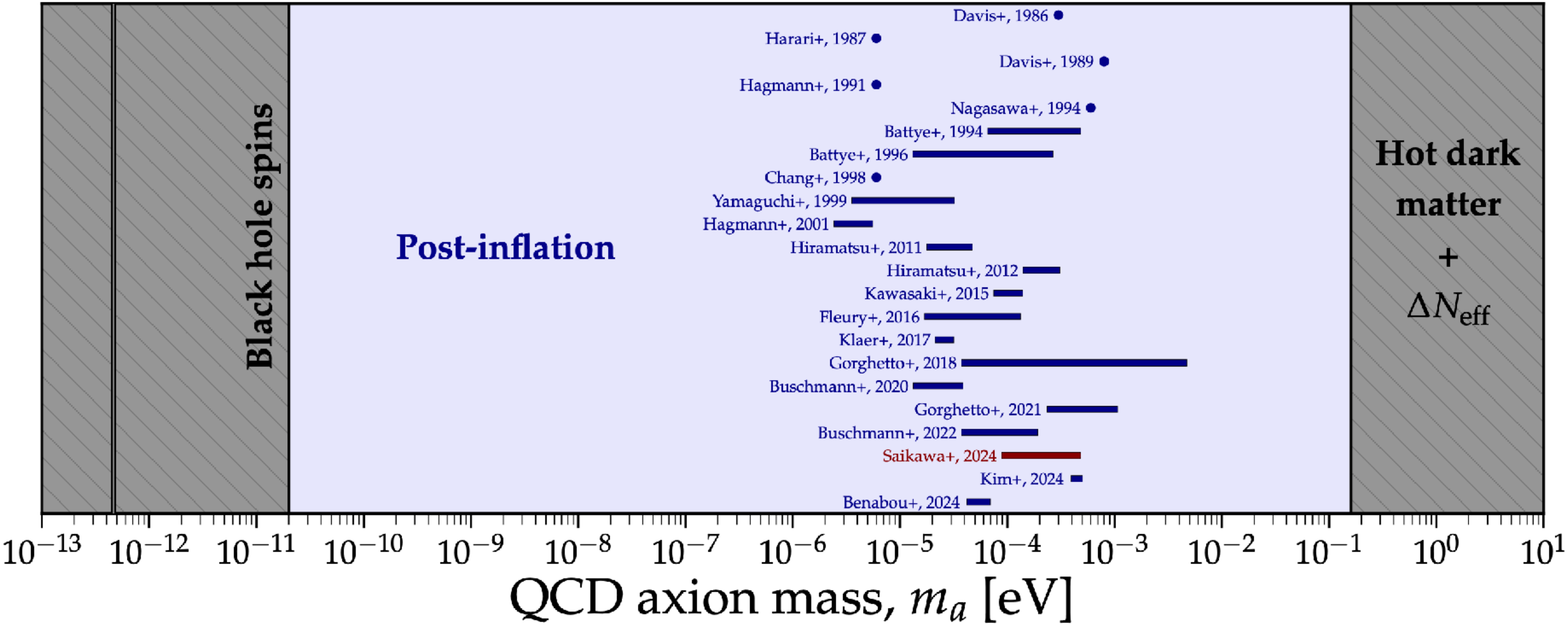
# Axion production from strings/walls

$$N_{\mathrm{DW}} = 1$$


Hiramatsu, Kawasaki, Saikawa, Sekiguchi,, 1202.5851

Studying the evolution of strings and domain walls is crucial for predicting the mass of axion dark matter.

# Prediction of post-inflationary scenario

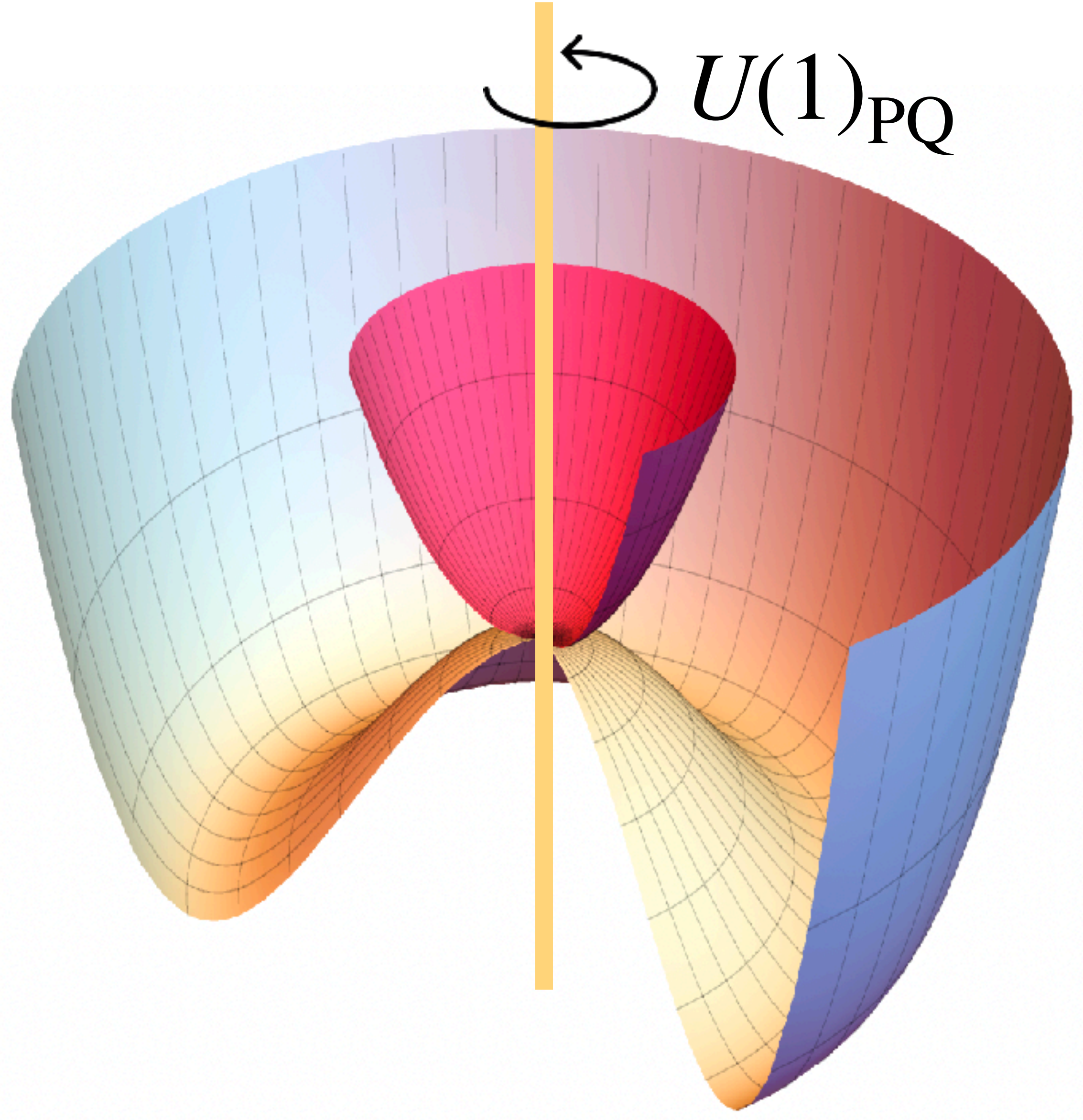



Kaltschmidt, Redondo, Saikawa and Vaquero, arXiv:2502.02398

# Key assumption :

Single PQ scalar model

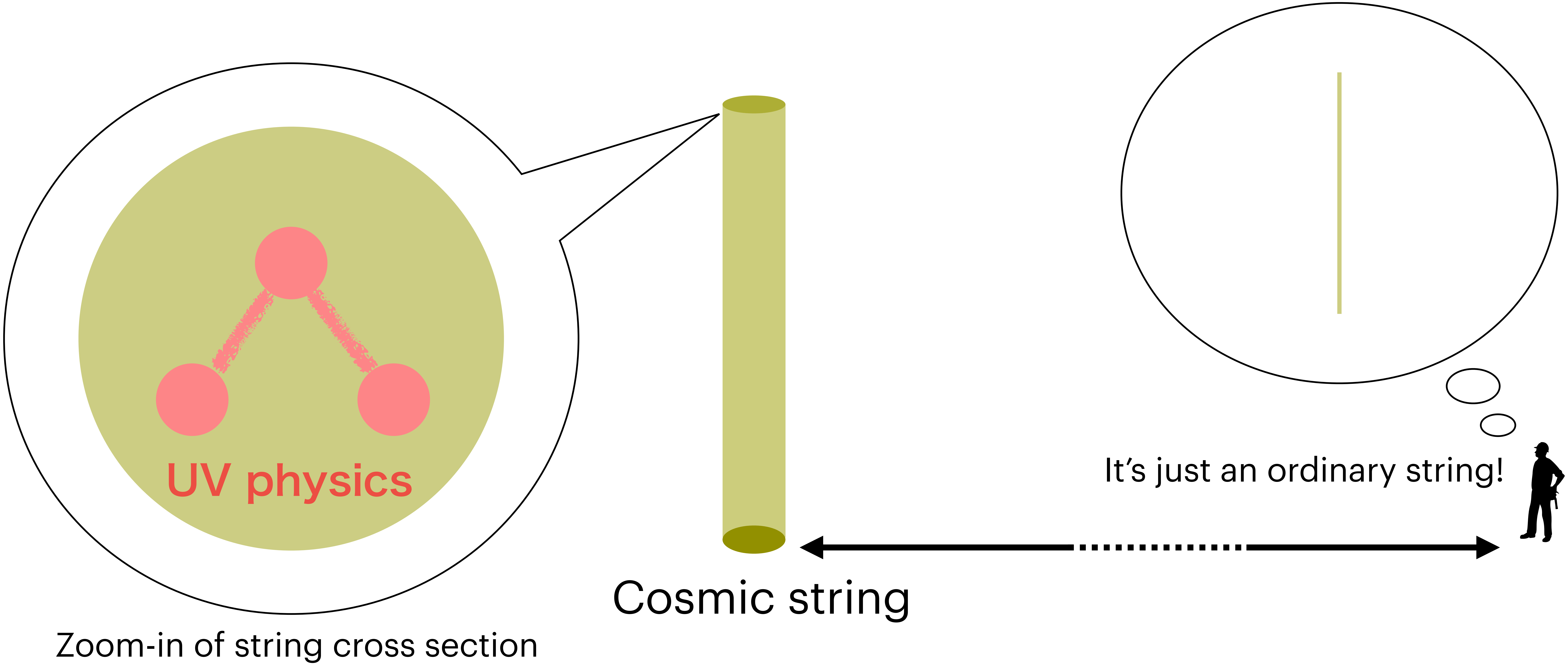
$$\Phi = \frac{f_a + s}{\sqrt{2}} e^{ia/f_a}$$




- Prevalent in high-precision lattice calculations.
- Simplifying assumption or crucial factor?

Origin and breaking of U(1) PQ are unknown!

# Is UV physics always confined in the string core?



# Two axion system

We introduce two PQ scalars

$$\Phi_1 = \frac{f_1}{\sqrt{2}} e^{i\frac{\phi_1}{f_1}} \quad \text{and} \quad \Phi_2 = \frac{f_2}{\sqrt{2}} e^{i\frac{\phi_2}{f_2}}$$

$\theta_1 \equiv \phi_1/f_1$ and $\theta_2 \equiv \phi_2/f_2$

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and the potential for axions

$$V_1(\phi_1, \phi_2) = \Lambda^4 \left[1 - \cos\left(n_1 \frac{\phi_1}{f_1} + n_2 \frac{\phi_2}{f_2}\right)\right]$$

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$$V_2(\phi_1, \phi_2) = \Lambda'^4 \left[1 - \cos\left(n_1' \frac{\phi_1}{f_1} + n_2' \frac{\phi_2}{f_2} + \alpha\right)\right]$$

$$\Lambda \gg \Lambda'$$

$n_1, n_2, n_1', n_2' \in \mathbf{Z}$

with the post-inflationary initial condition.

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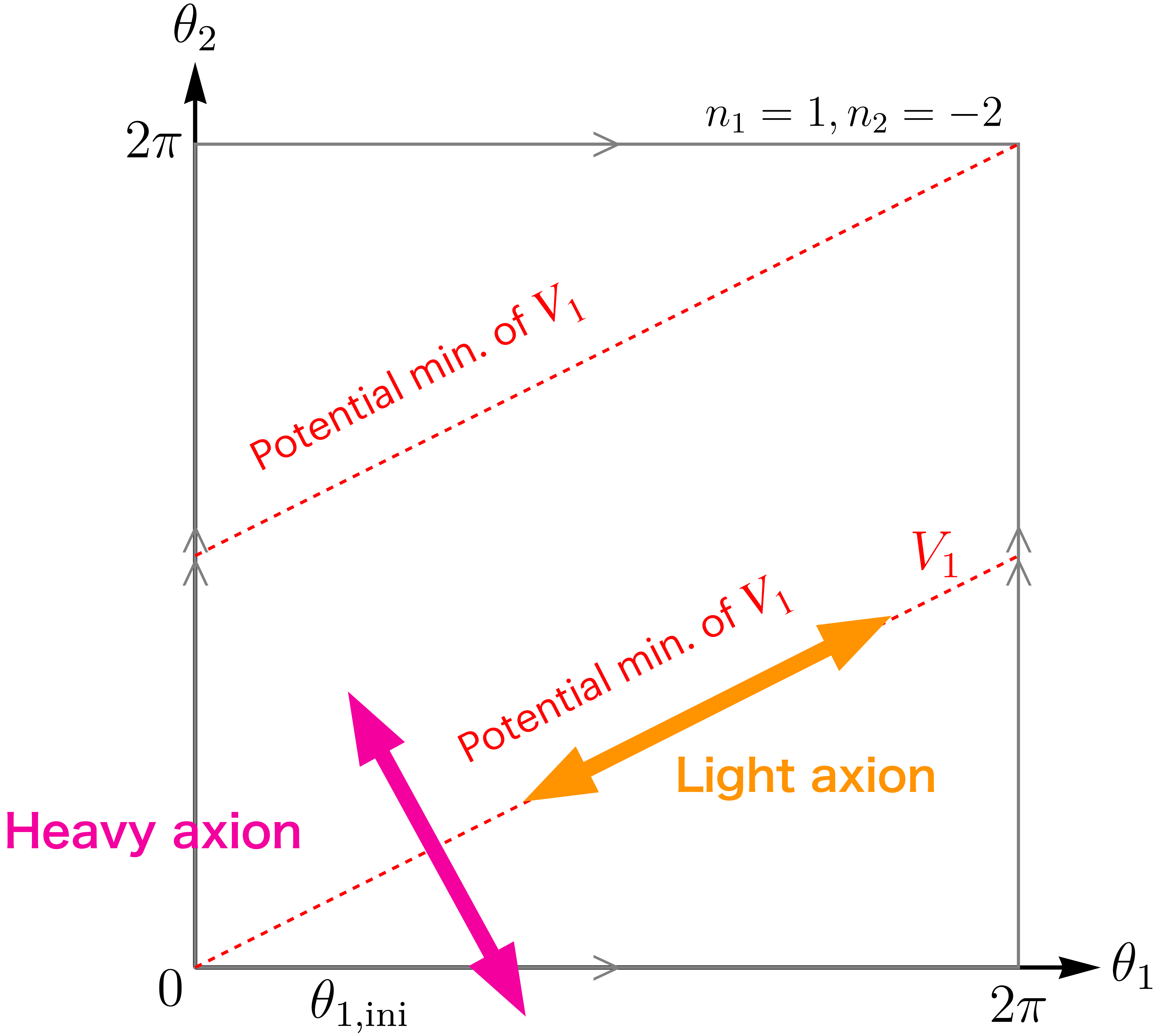
with the post-inflationary initial condition.

# Two axion system

$$V_1(\phi_1,\phi_2) = \Lambda^4\left[1-\cos\left(n_1\frac{\phi_1}{f_1}+n_2\frac{\phi_2}{f_2}\right)\right]$$

One linear combination of two axions becomes heavy, leaving the orthogonal one (nearly) massless.

$$\phi_{\text{heavy}} \propto n_1\frac{\phi_1}{f_1}+n_2\frac{\phi_2}{f_2}$$



# Two types of strings and multiple DWs

Both $\phi_1, \phi_2$-strings quickly reach the scaling solution, and when $V_1$ becomes relevant, $n_1$ ($n_2$) domain walls appear, attached to the $\phi_1(\phi_2)$-string.

1

2

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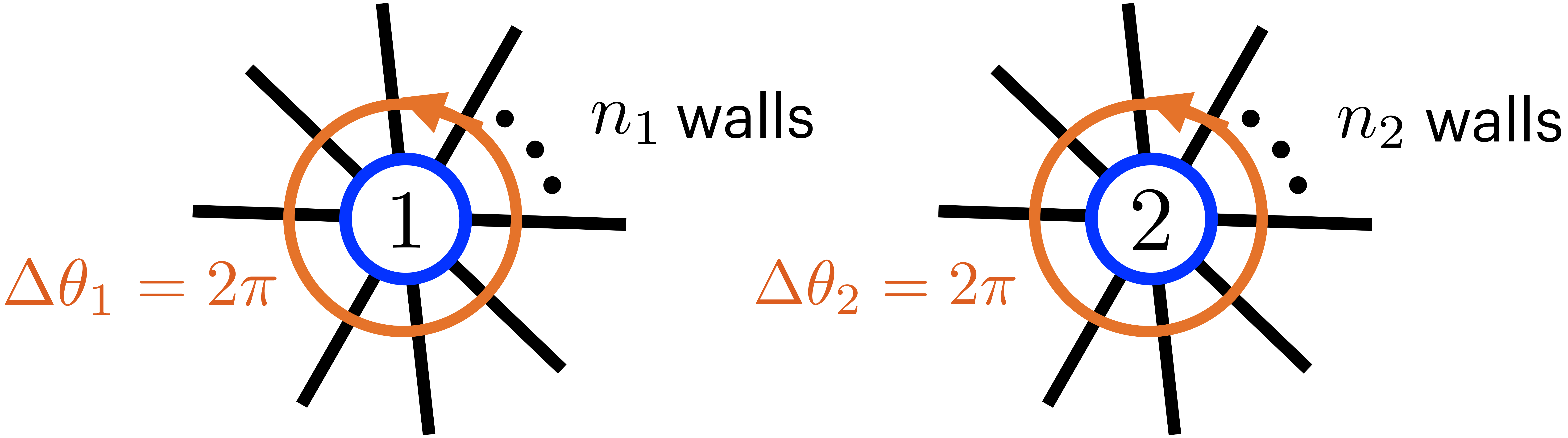
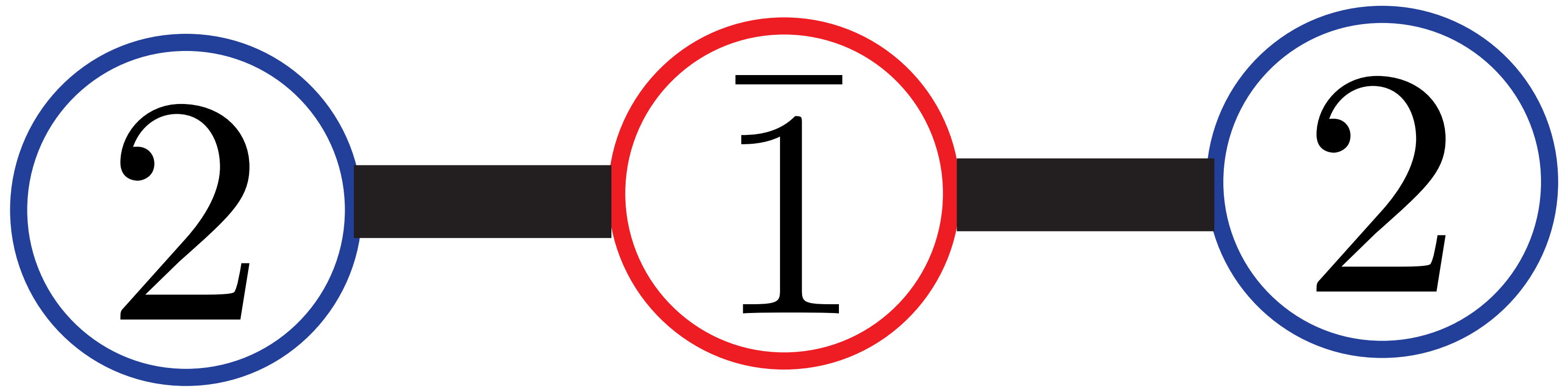



$$V_1(\phi_1, \phi_2) = \Lambda^4 \left[1 - \cos\left(n_1 \frac{\phi_1}{f_1} + n_2 \frac{\phi_2}{f_2}\right)\right] \qquad n_1, n_2 \in \mathbf{Z}$$

# Case of $(n_1, n_2) = (2, 1)$

$$V_1(\phi_1, \phi_2) = \Lambda^4 \left[1 - \cos\left(n_1 \frac{\phi_1}{f_1} + n_2 \frac{\phi_2}{f_2}\right)\right]$$




$\phi_2$-string $\phi_1$-anti-string $\phi_2$-string

**String bundle**
(= ordinary cosmic string)

cf. Higaki, Jeong, Kitajima, Sekiguchi and FT, 1606.05552,
See also Eto, Hiramatsu, Saito and Sakakihara, 2309.04248

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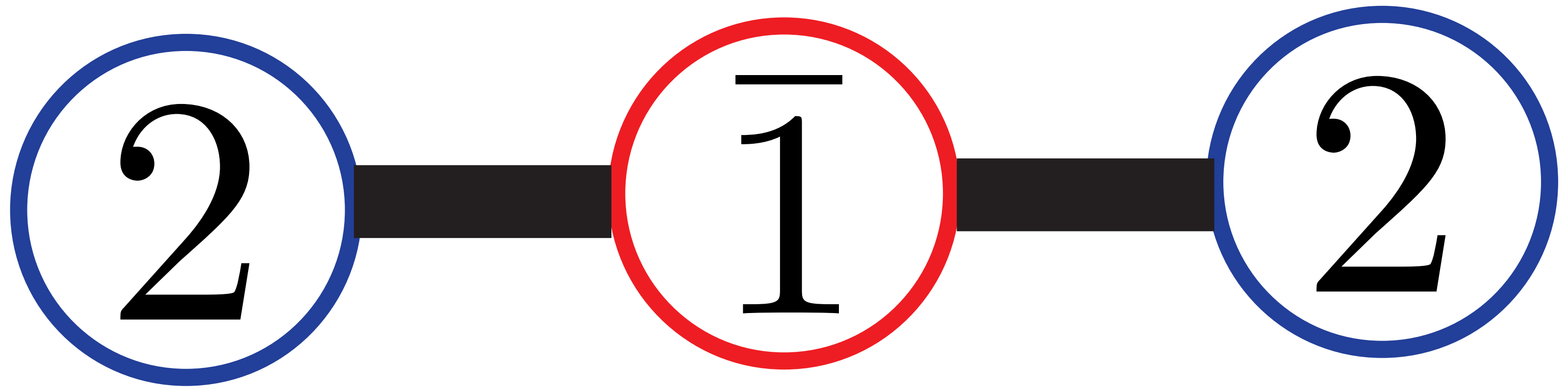


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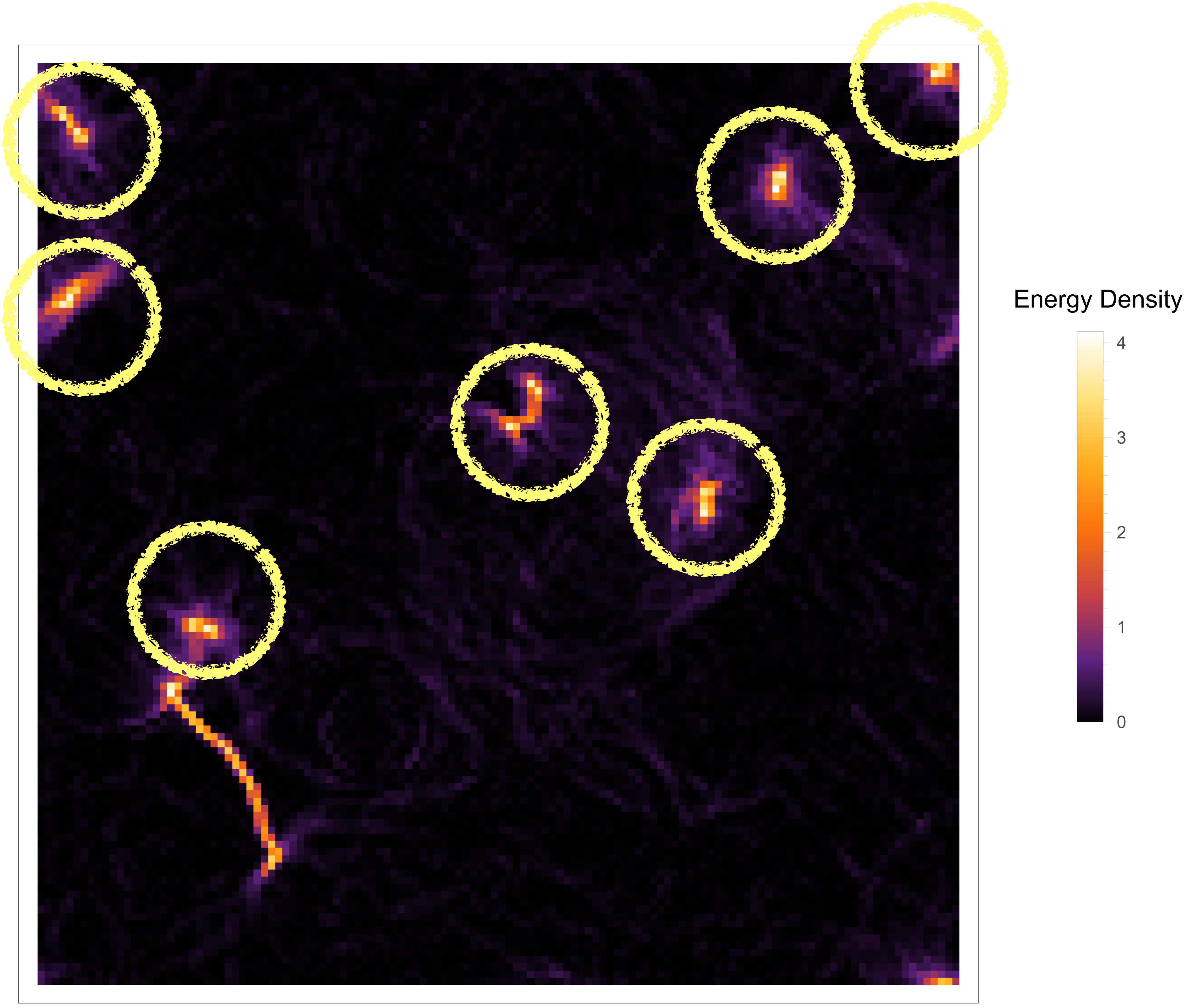


**Numerical results (2D)**

Lee, Murai, FT and Yin 2409.09749

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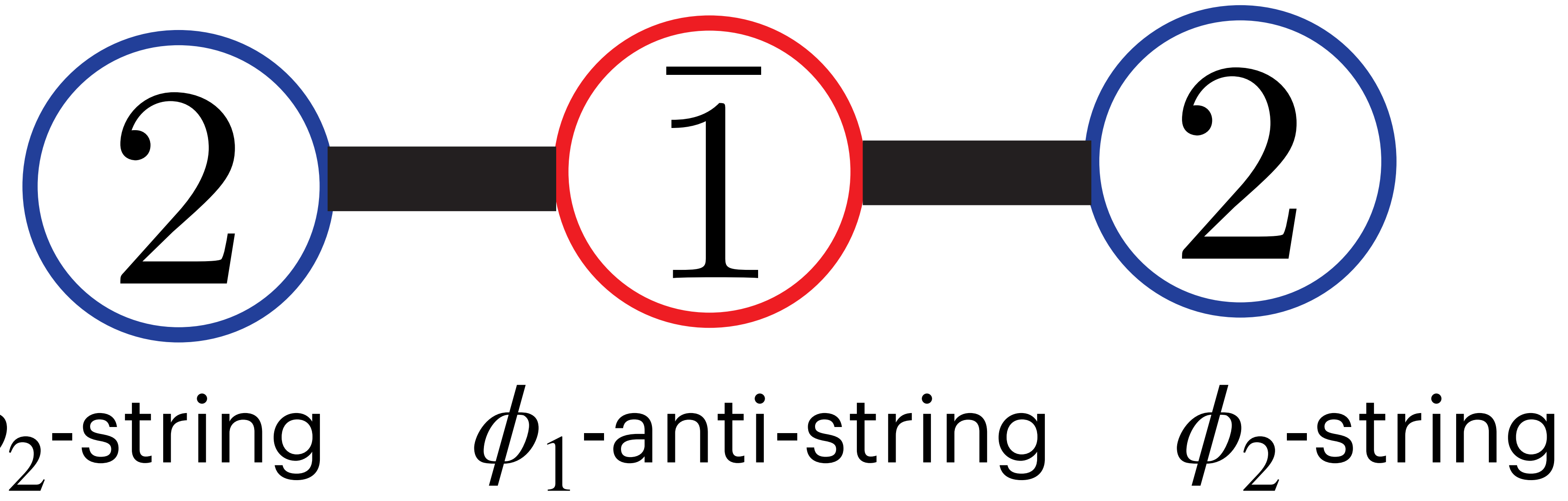


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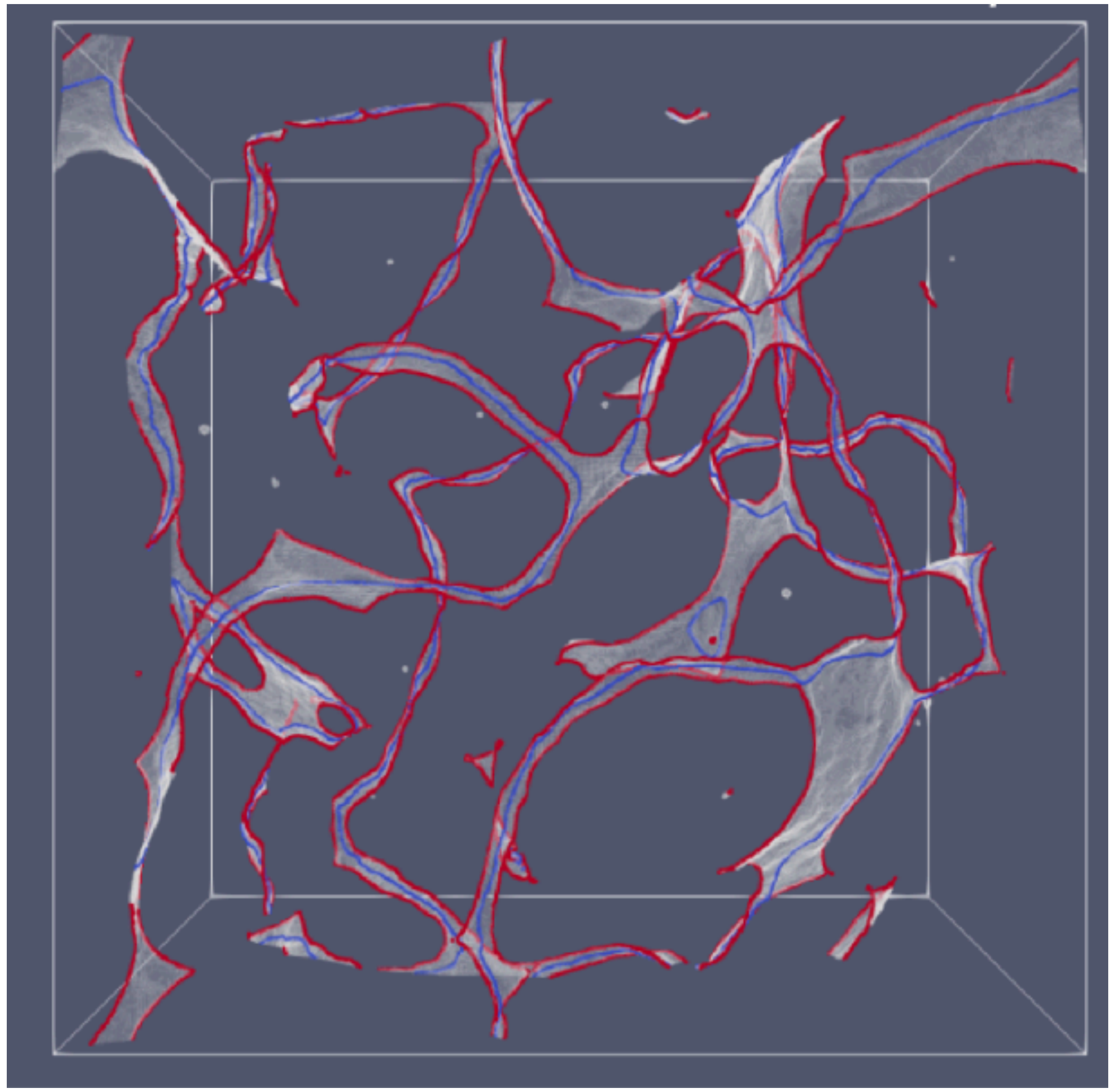

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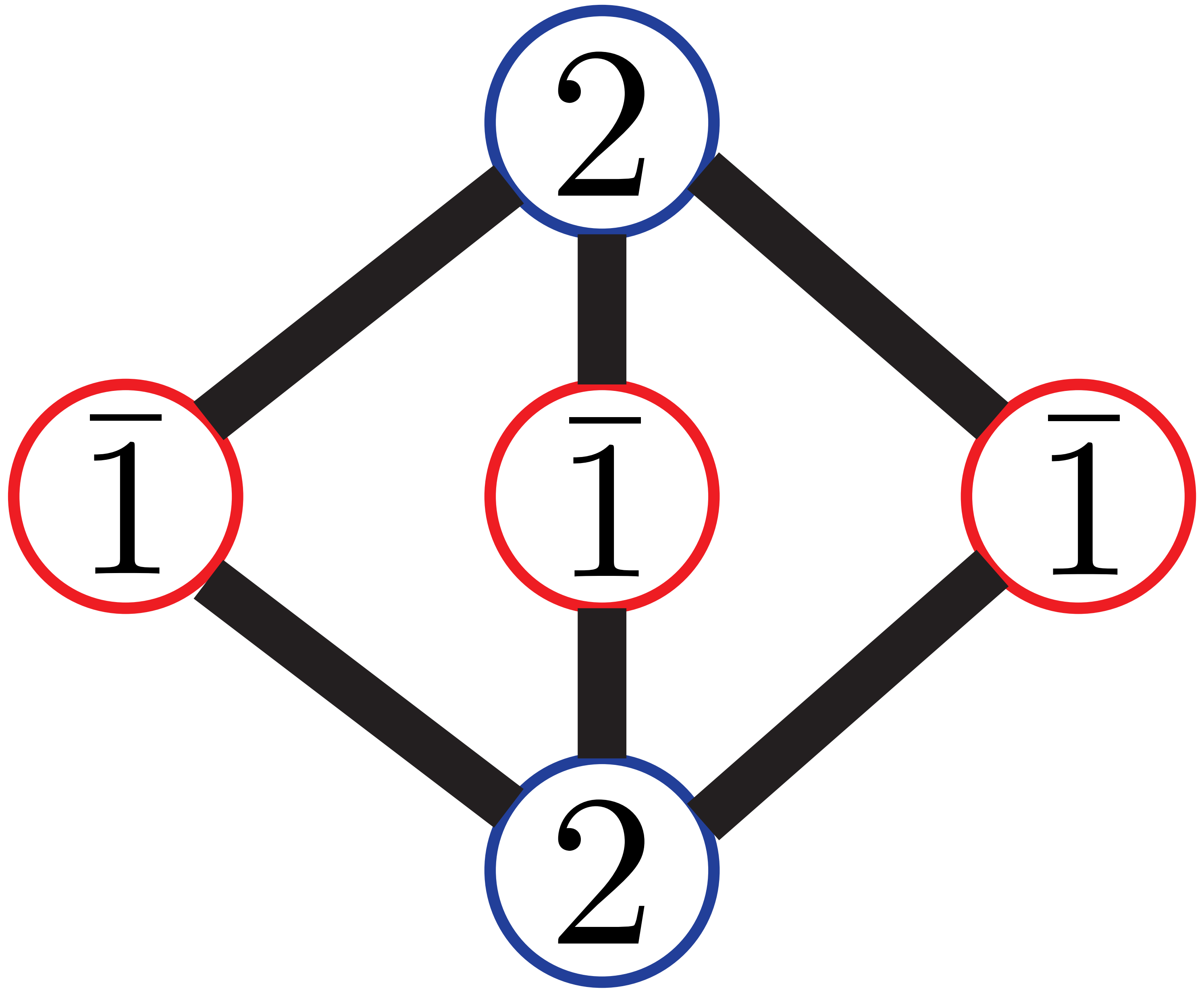


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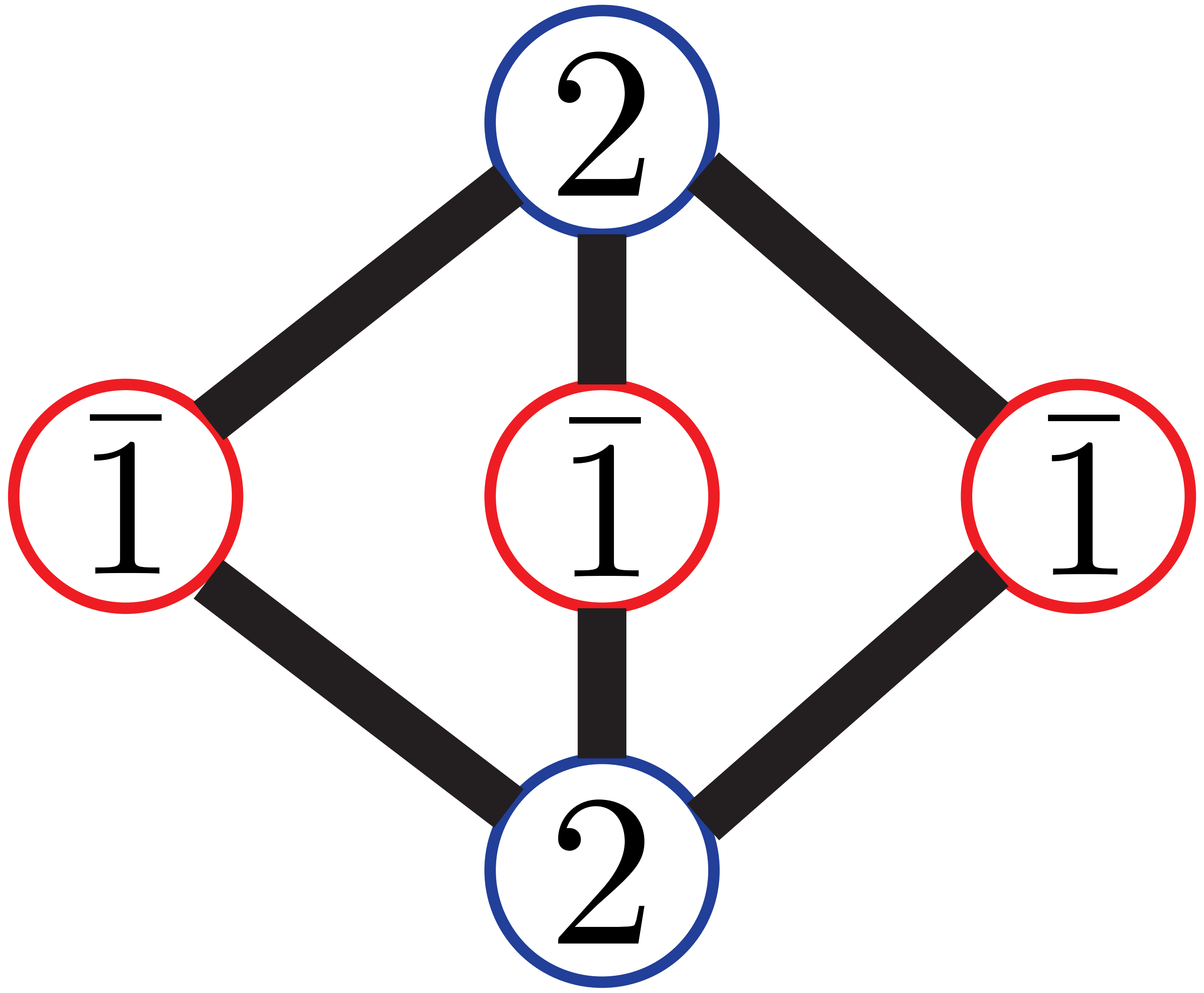


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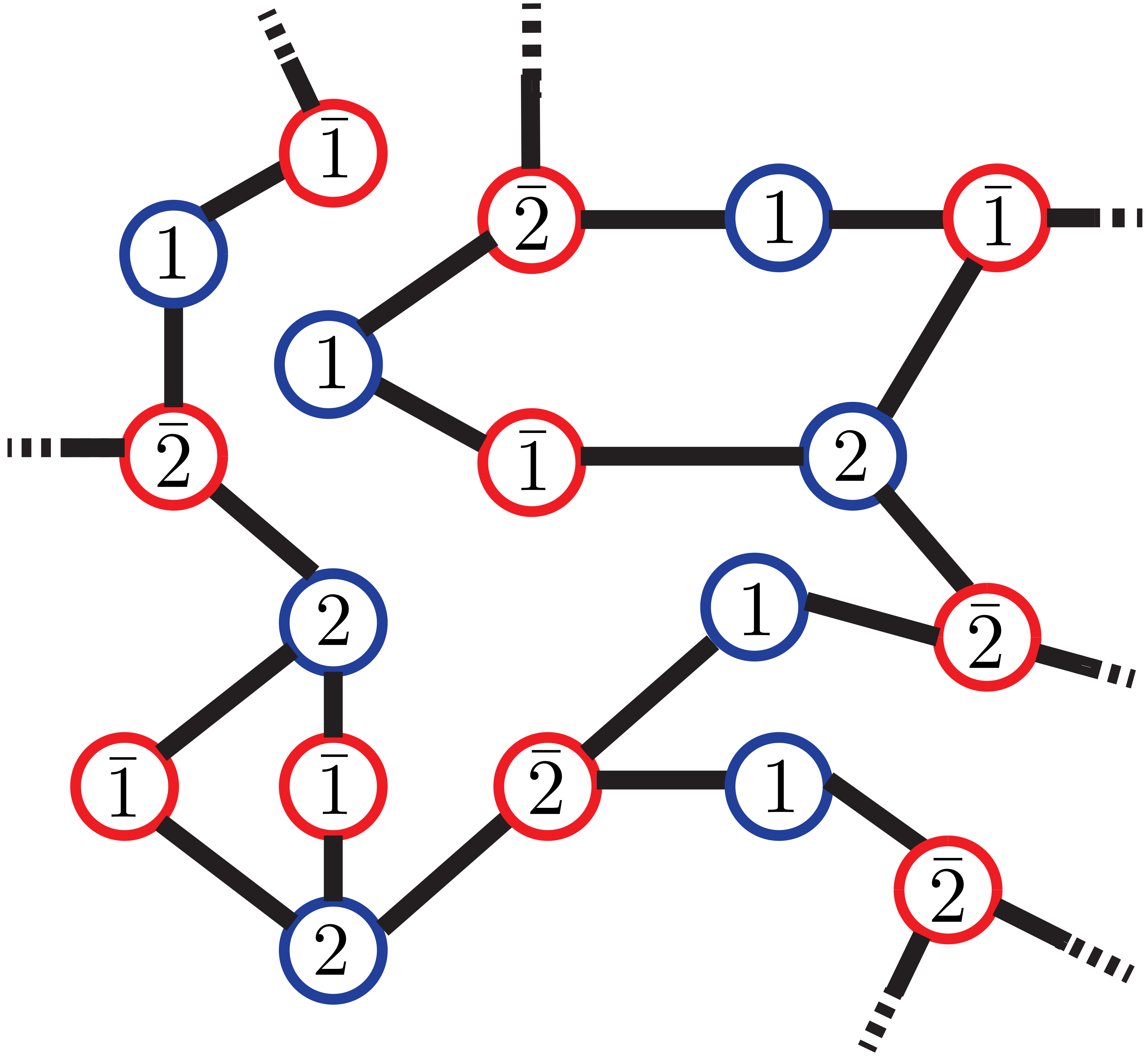
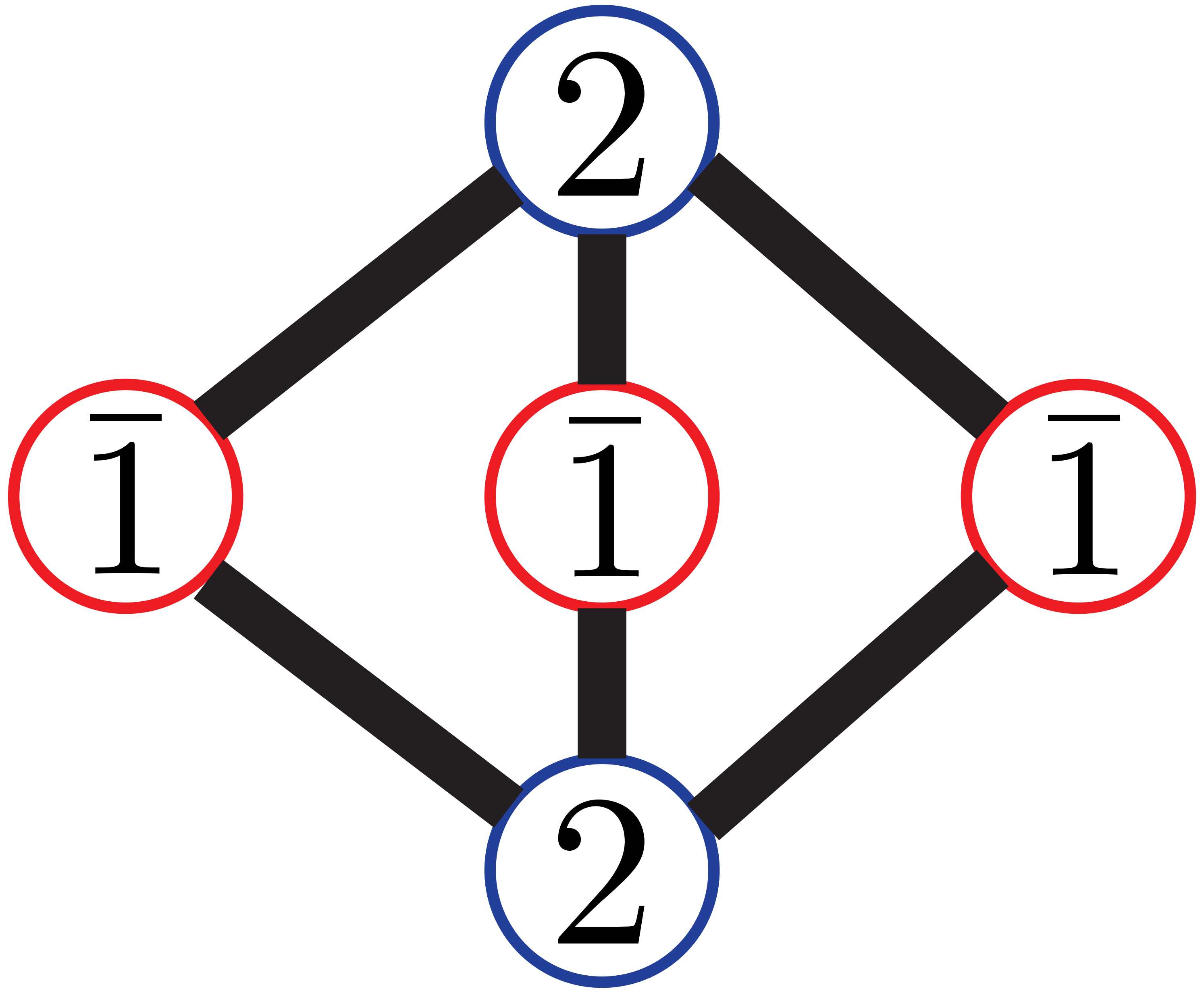



**String-wall network**

# Case of $(n_1, n_2) = (2, 3)$

**String-wall network forms in stead of string bundles.**

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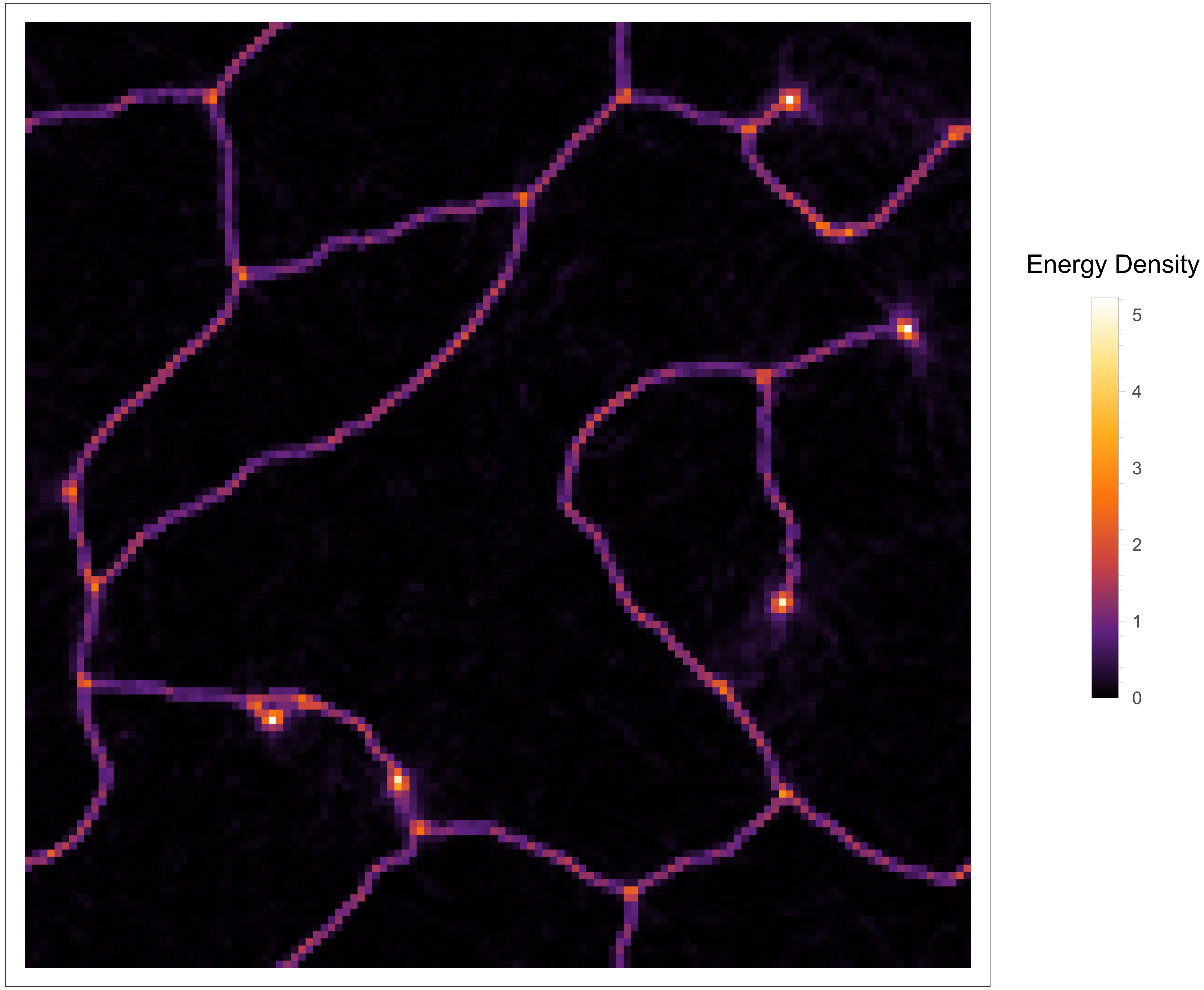


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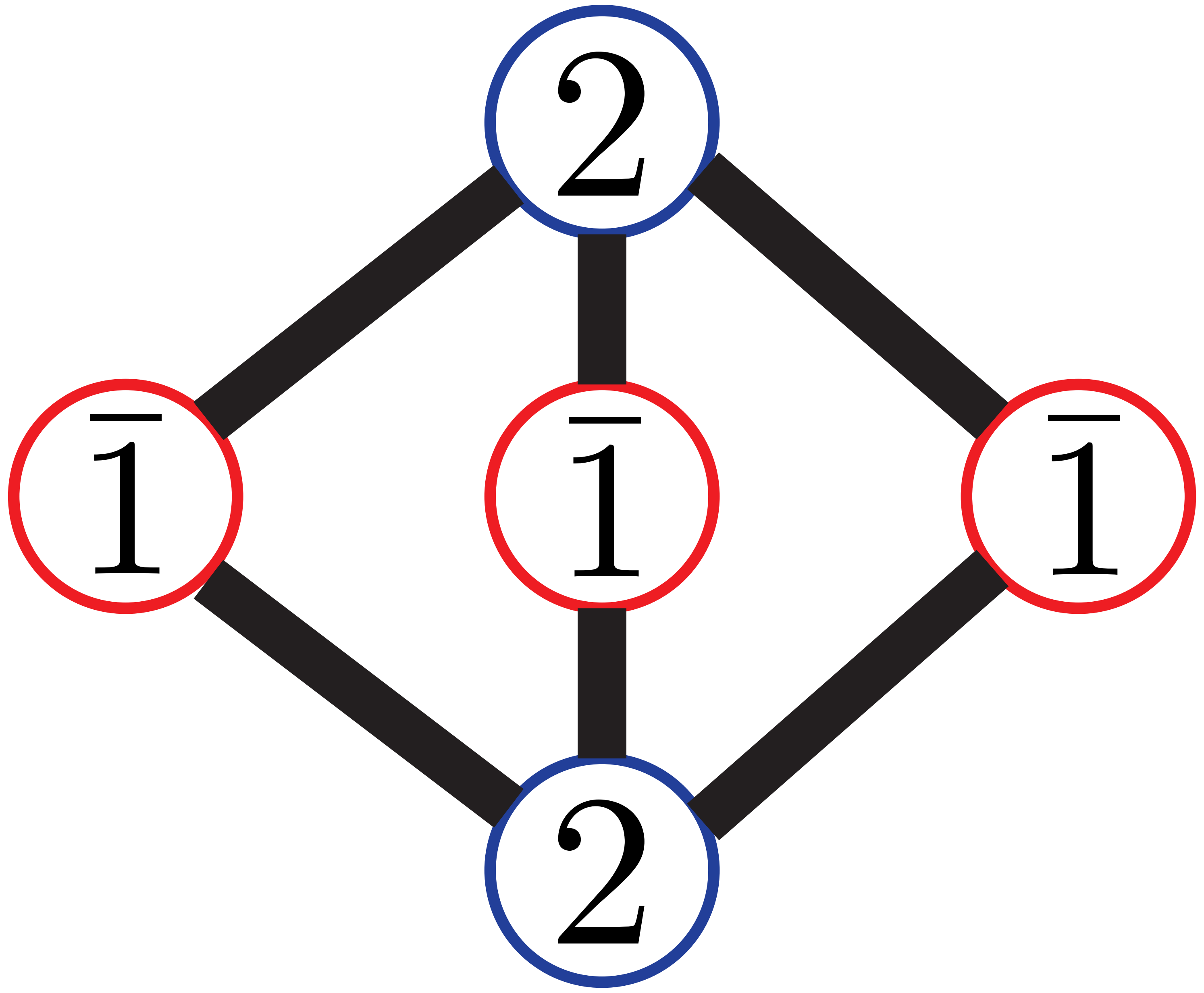


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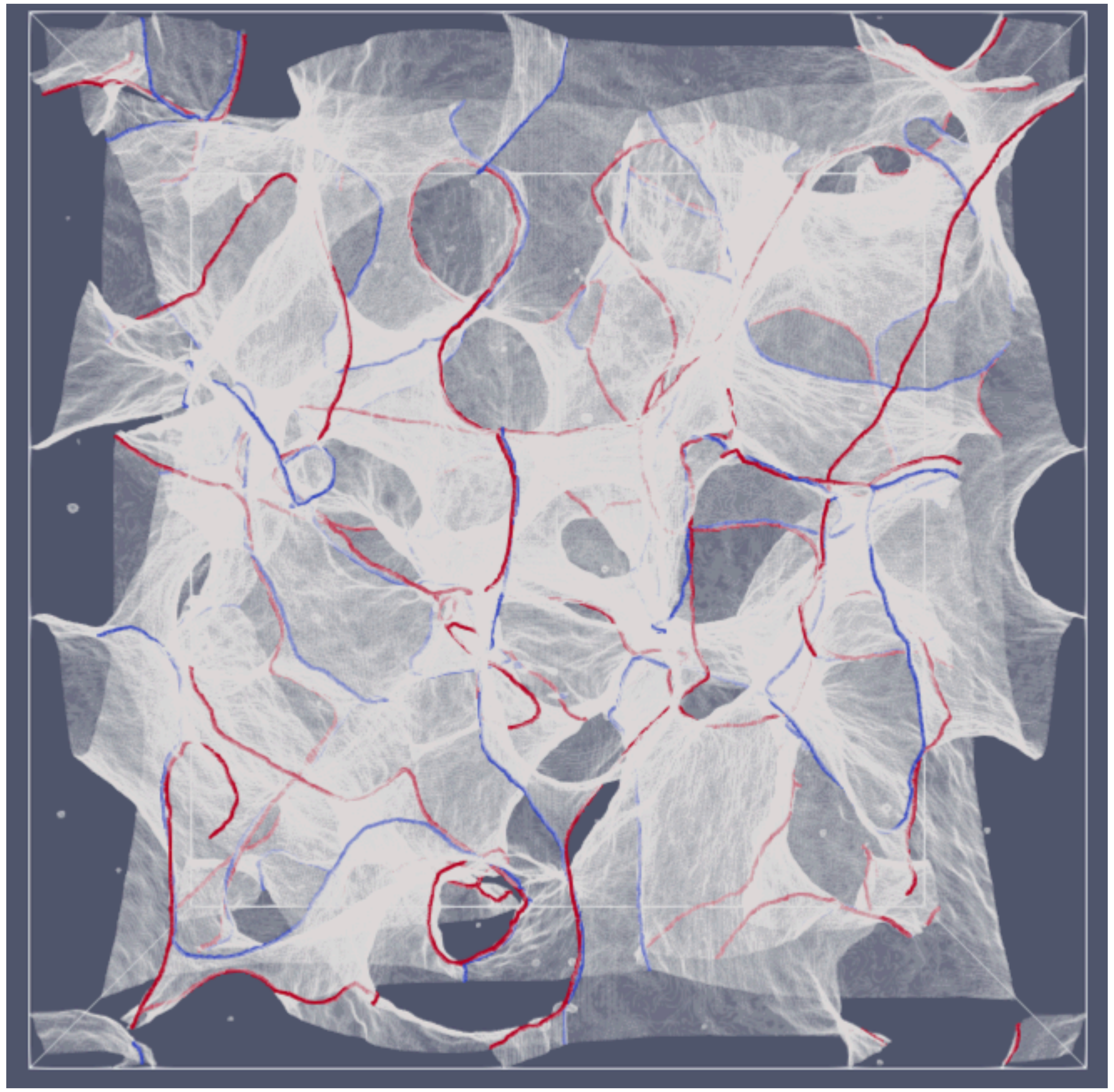
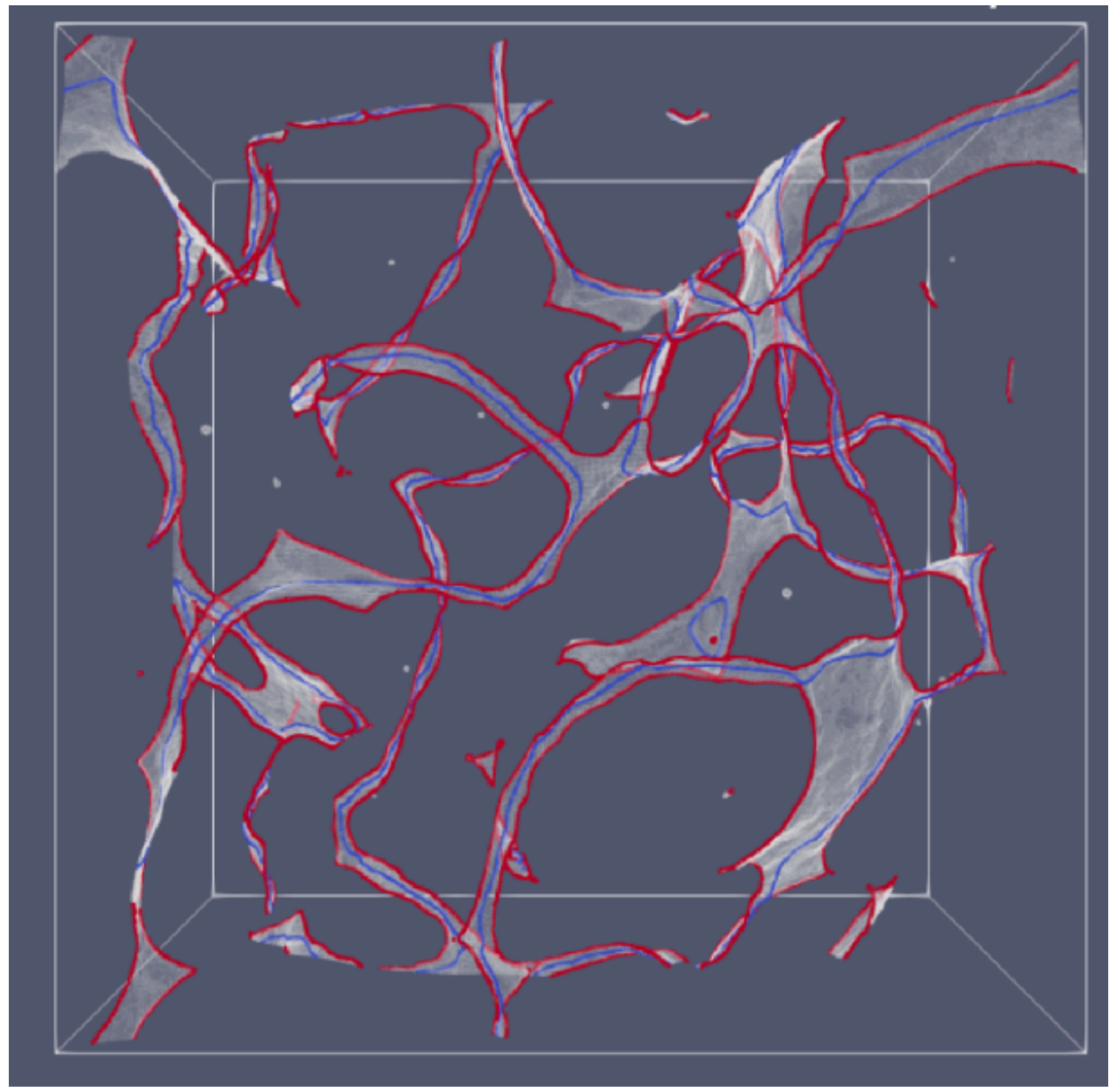


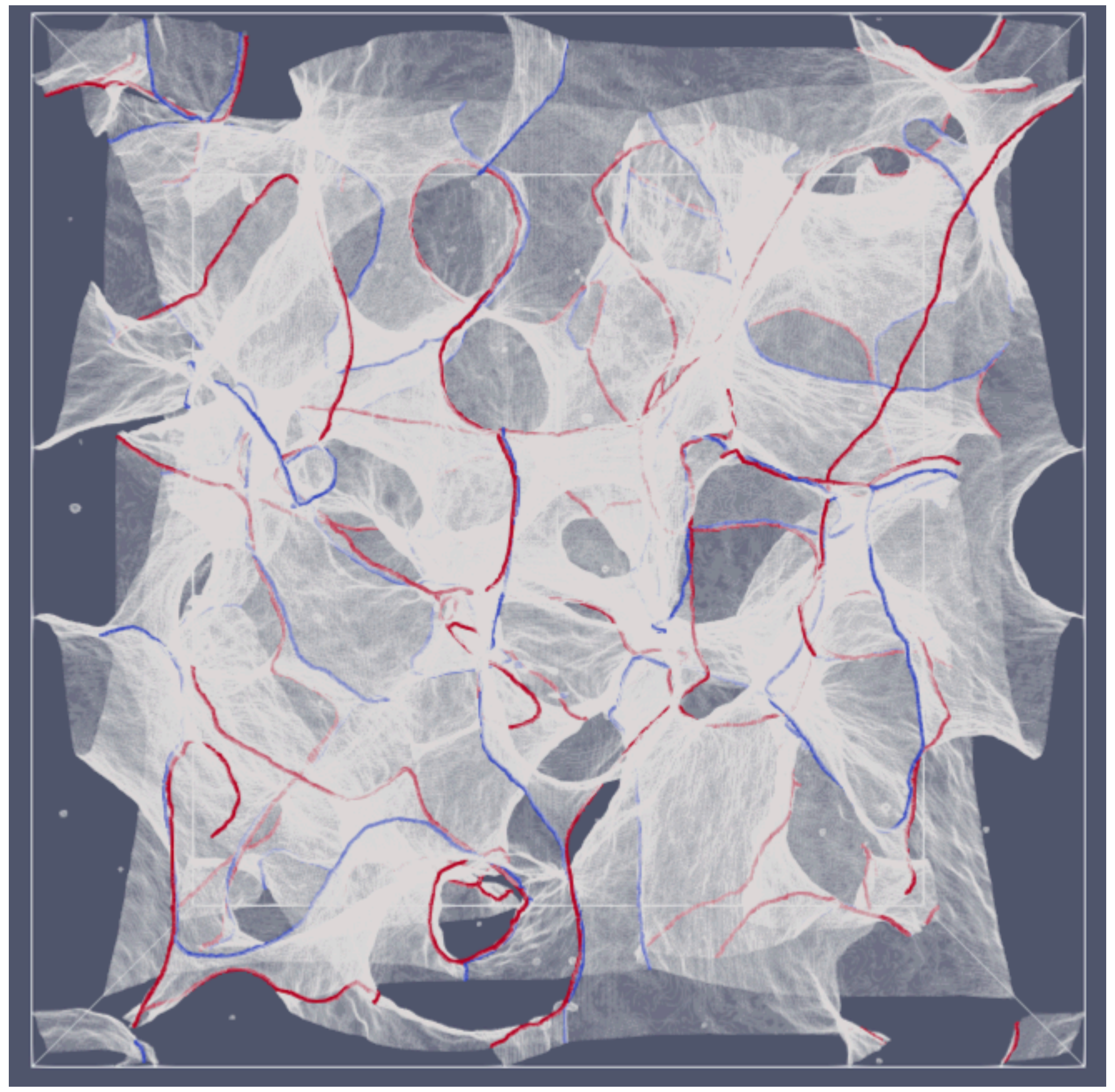
**Numerical results (3D)**

Coutesy of Junseok Lee

In the post-inflationary scenario, a string-wall network forms instead of ordinary cosmic strings if $|n_1|, |n_2| \geq 2$.



$|n_1| = 1$ or $|n_2| = 1$



$|n_1|, |n_2| \geq 2.$

These heavy axion DWs also produce a large amount of GWs.

Higaki, Jeong, Kitajima, Sekiguchi and FT, 1606.05552

# Mixed initial conditions

We may impose a mixed "pre-post" initial condition, i.e.,

pre-inflationary initial condition for $\phi_1$

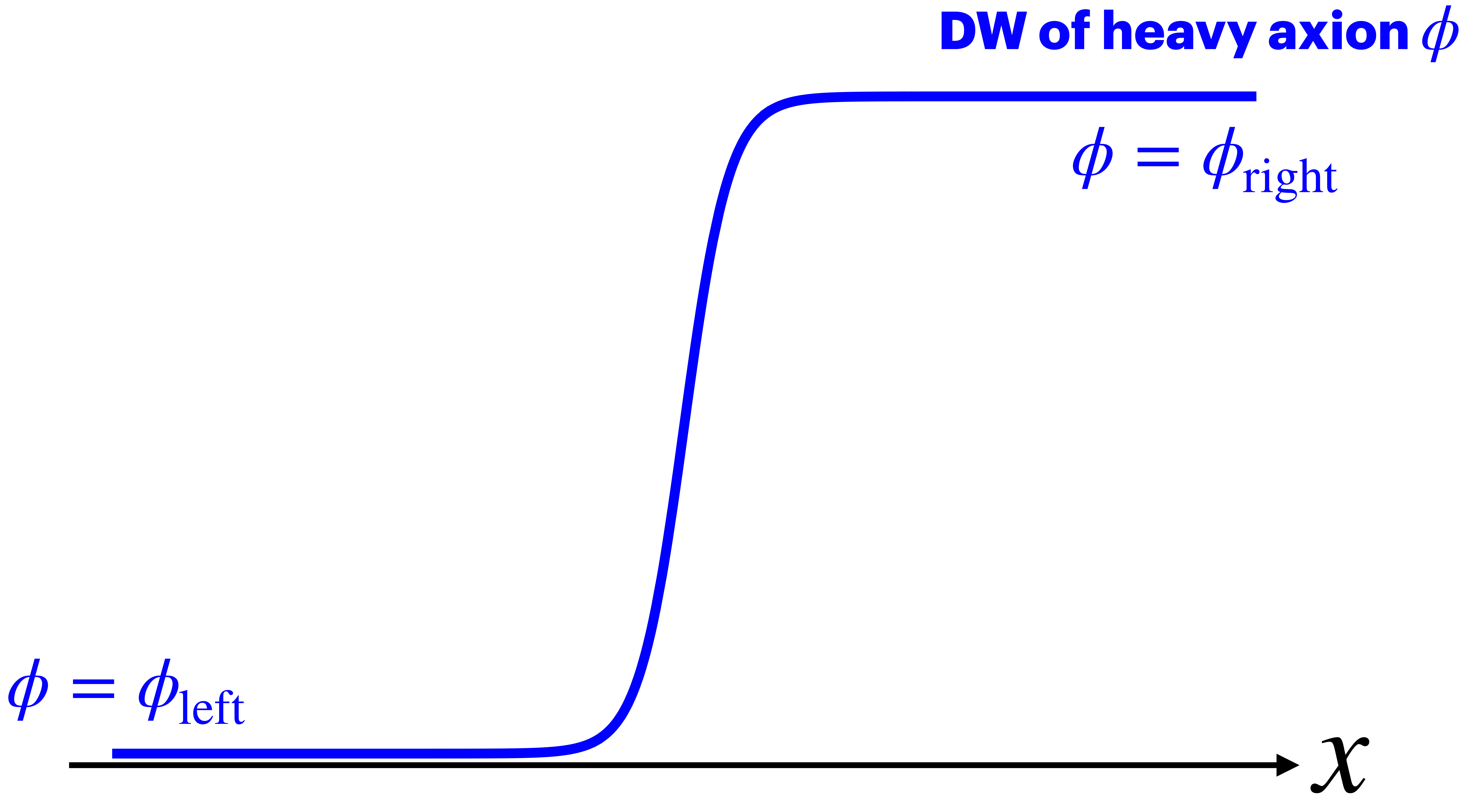
post-inflationary initial condition for $\phi_2$

Then, no strings bundles are formed, and string-wall network of $\phi_2$ remains if $|n_2| \geq 2$ (even if $|n_1| = 1$).

Mixed initial conditions makes string-wall network formation more likely.

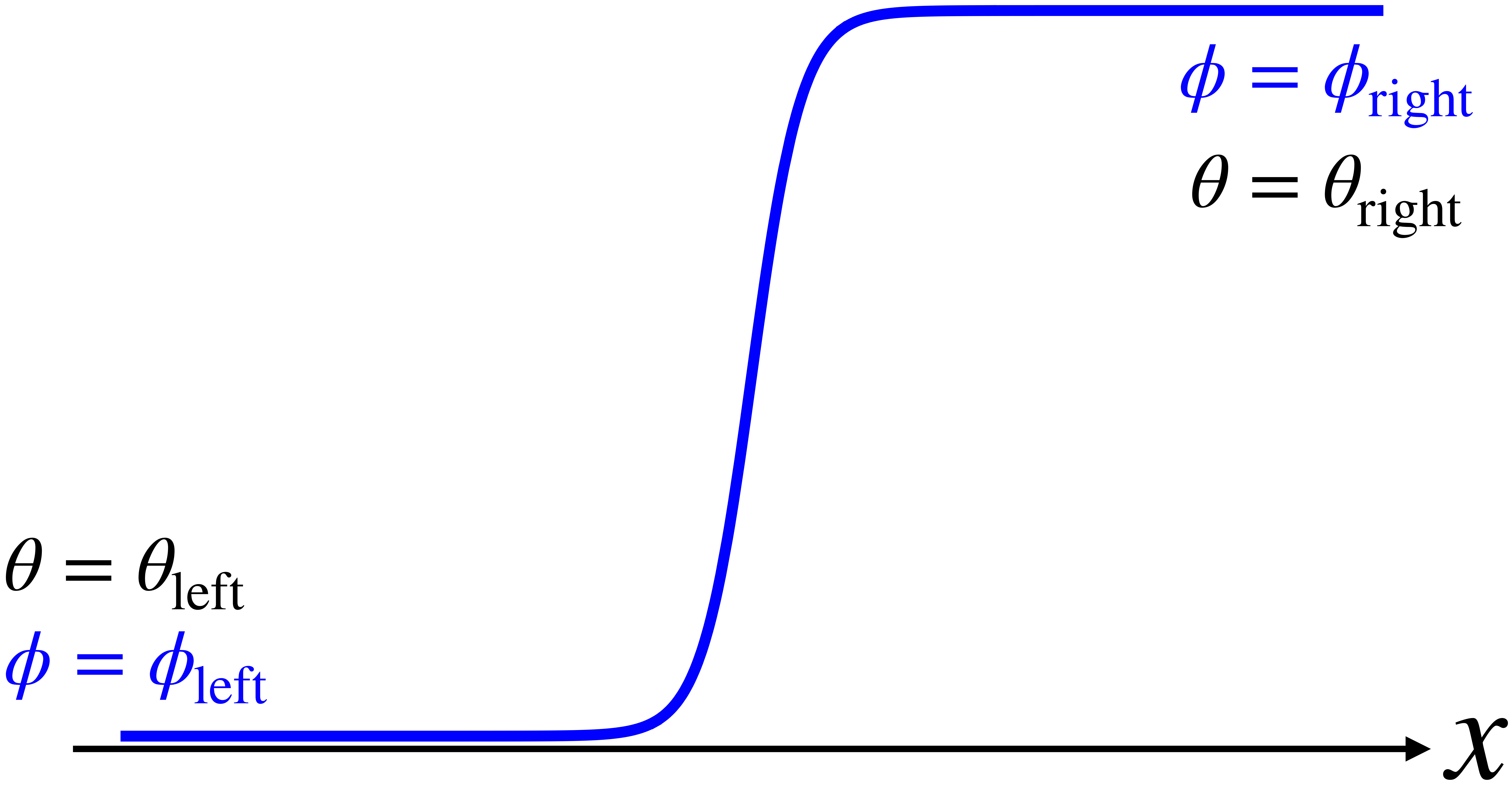
# ”Induced DW” formation due to $V_2$

Before proceeding, let me first explain what an induced domain wall is.


DW of heavy axion $\phi$
$\phi = \phi_{\rm right}$
$\phi = \phi_{\rm left}$
$x$

Couple $\phi$ to gluons: $\phi G\tilde{G}$

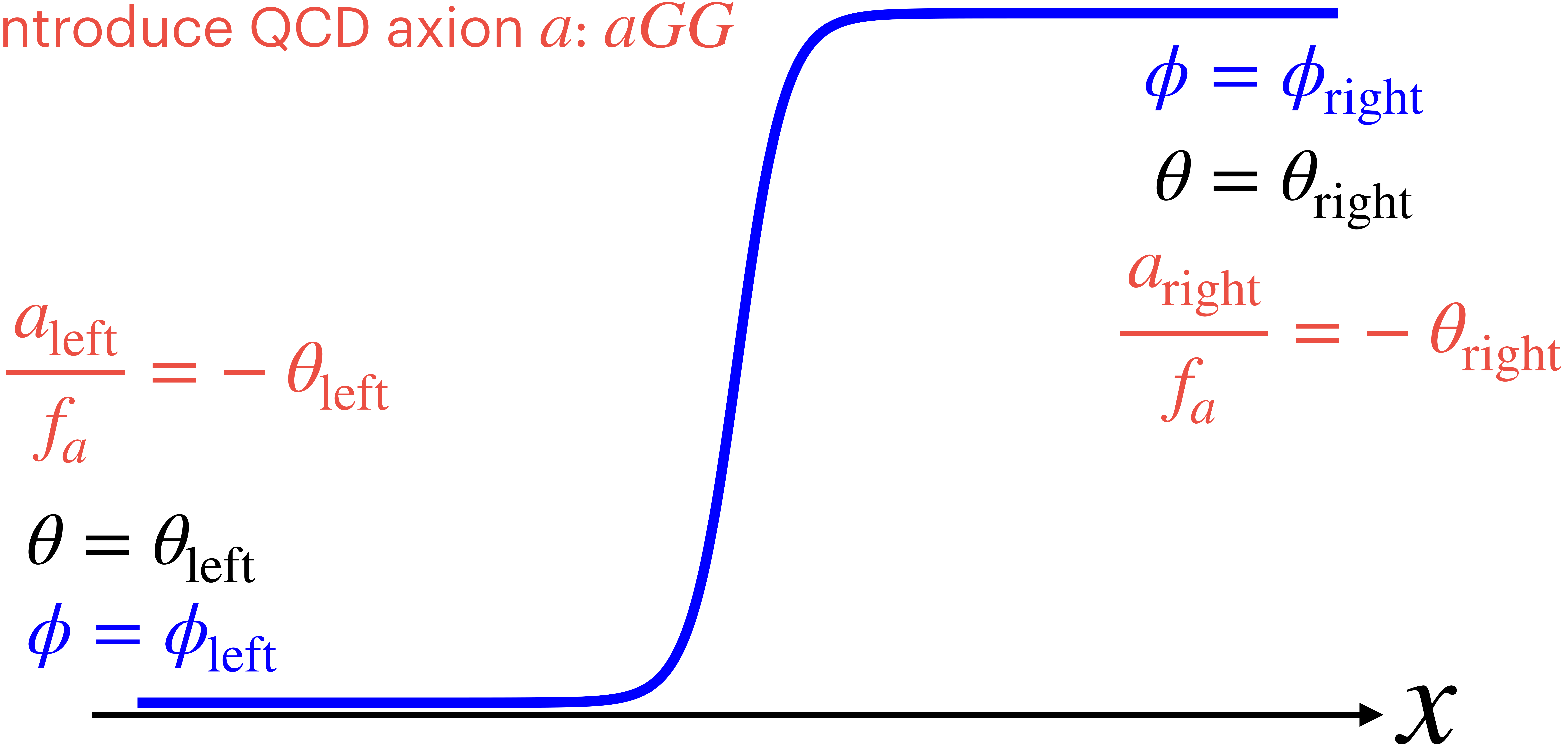
**DW of heavy axion $\phi$**



Couple $\phi$ to gluons: $\phi G\tilde{G}$

Introduce QCD axion $a$: $aG\tilde{G}$

**DW of heavy axion $\phi$**



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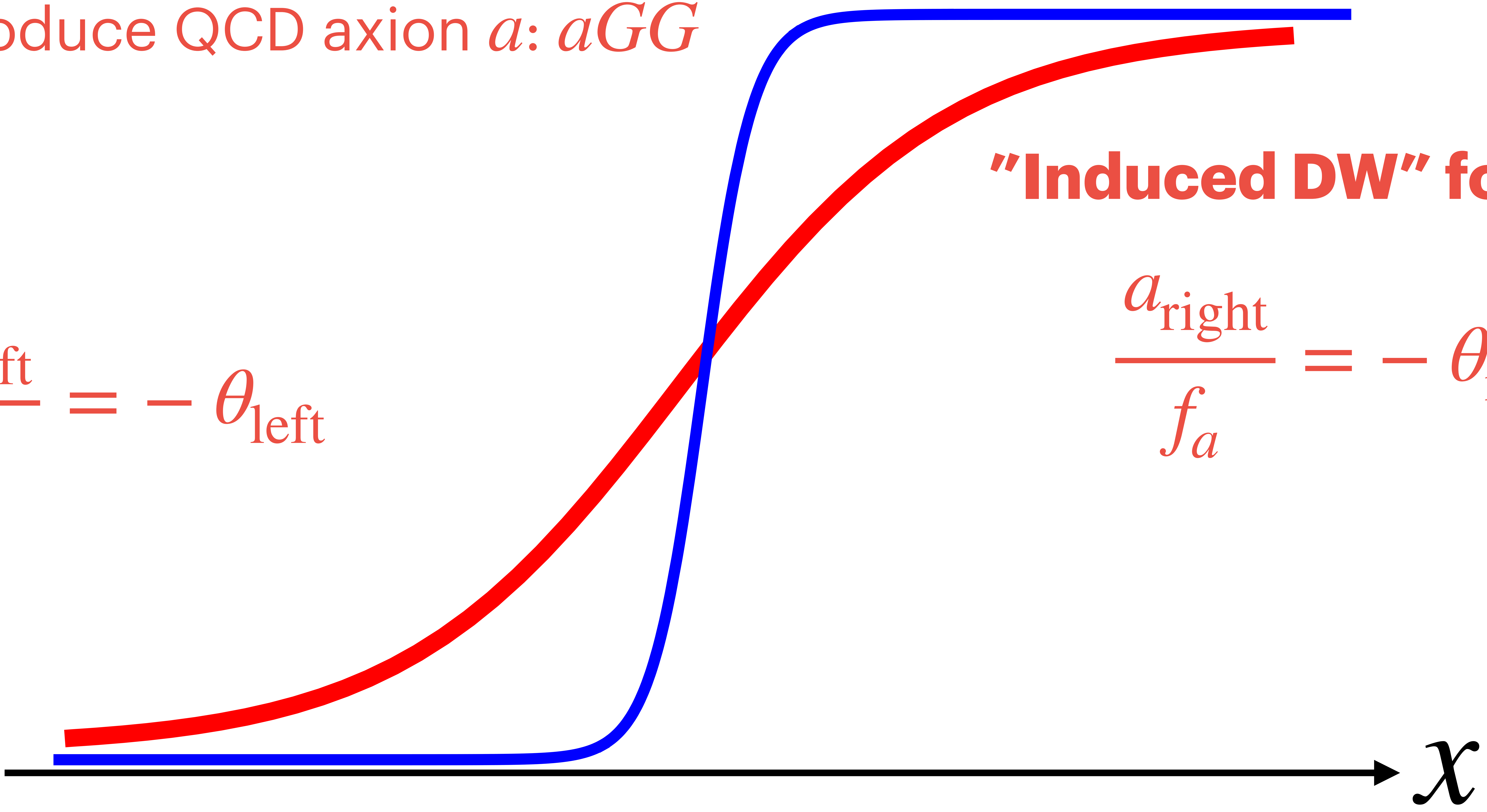
**DW of heavy axion $\phi$**

**"Induced DW" for $a$**




$$\frac{a_{\text{left}}}{f_a} = -\theta_{\text{left}}$$

$$\frac{a_{\text{right}}}{f_a} = -\theta_{\text{right}}$$

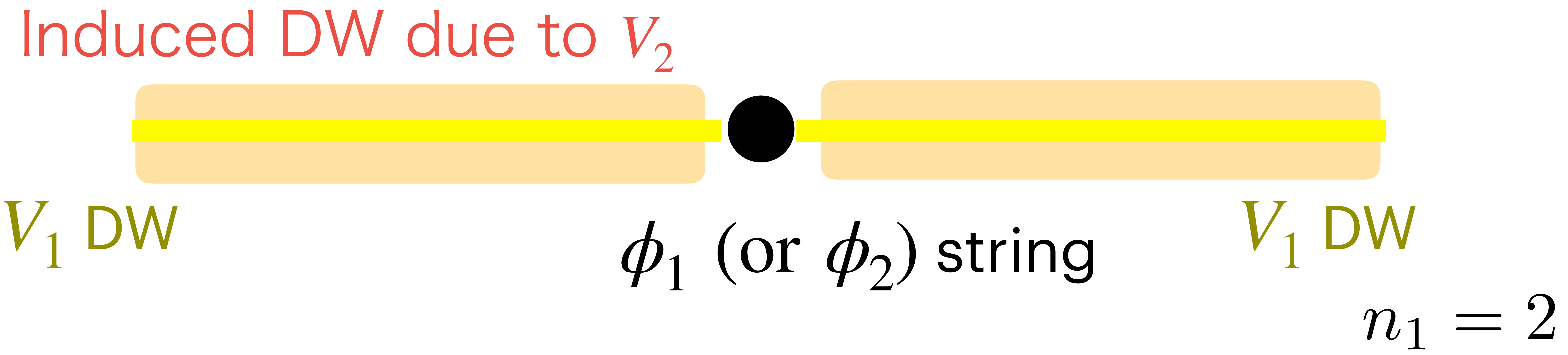
# "Induced DW" formation due to $V_2$

Now we consider the DW formation due to $V_2 \ll V_1$.

$$V_1(\phi_1, \phi_2) = \Lambda^4 \left[1 - \cos\left(n_1 \frac{\phi_1}{f_1} + n_2 \frac{\phi_2}{f_2}\right)\right]$$

$$V_2(\phi_1, \phi_2) = \Lambda'^4 \left[1 - \cos\left(n'_1 \frac{\phi_1}{f_1} + n'_2 \frac{\phi_2}{f_2}\right)\right] \qquad \Lambda' \ll \Lambda$$

Both $V_1$ and $V_2$ can be minimized in any domains, and so there is no potential bias at the minimum.



Induced DW due to $V_2$
$V_1$ DW
$\phi_1$ (or $\phi_2$) string
$V_1$ DW
$n_1 = 2$

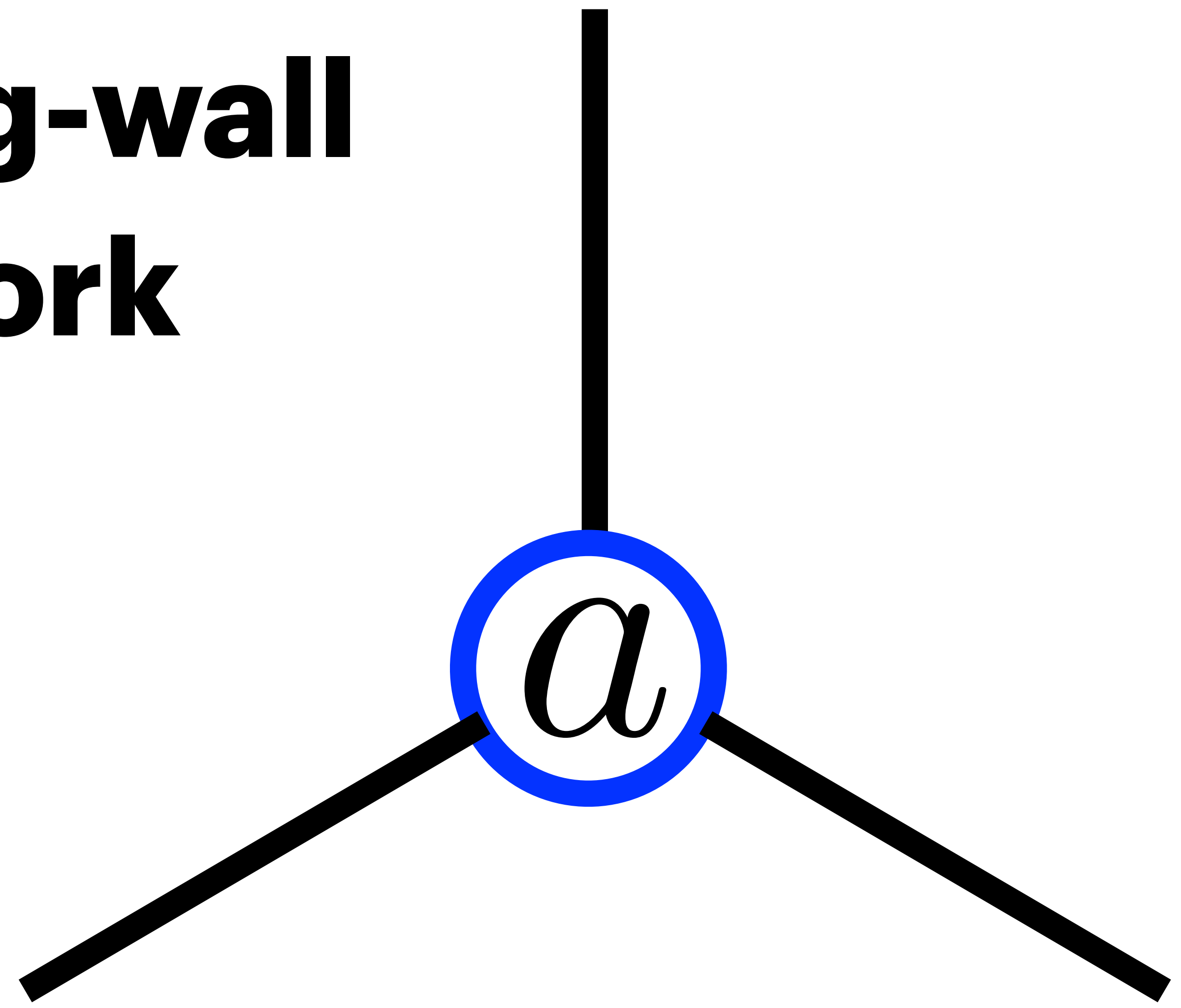
# New solution to the QCD axion DW problem

FT and Yin 2012.11576, Lee, Murai, FT and Yin 2507.07075, see also Kondo, Murayama 2507.07973

$$\mathcal{L} = -N_{\rm DW}\frac{g_s^2}{32\pi^2}\frac{a}{v_a}G\tilde{G}$$

$N_{\rm DW} = 3 \text{ or } 6$ for DFSZ axion

**String-wall network**



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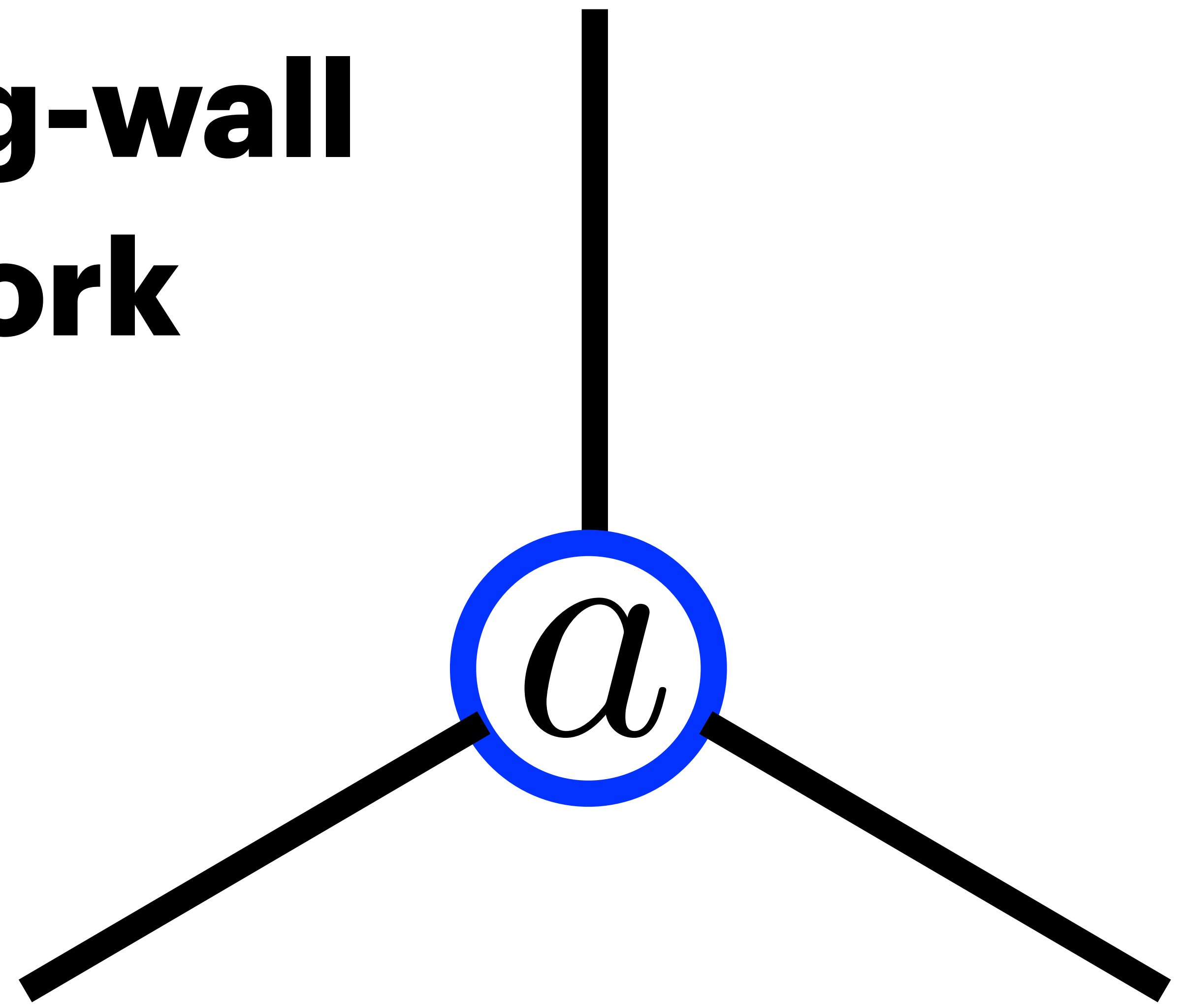
$N_{\rm DW} = 3 \text{ or } 6$ for DFSZ axion

$\varphi$ : Massless (or very light) axion

e.g. KSVZ axion

(different from $a$)

**String-wall network**



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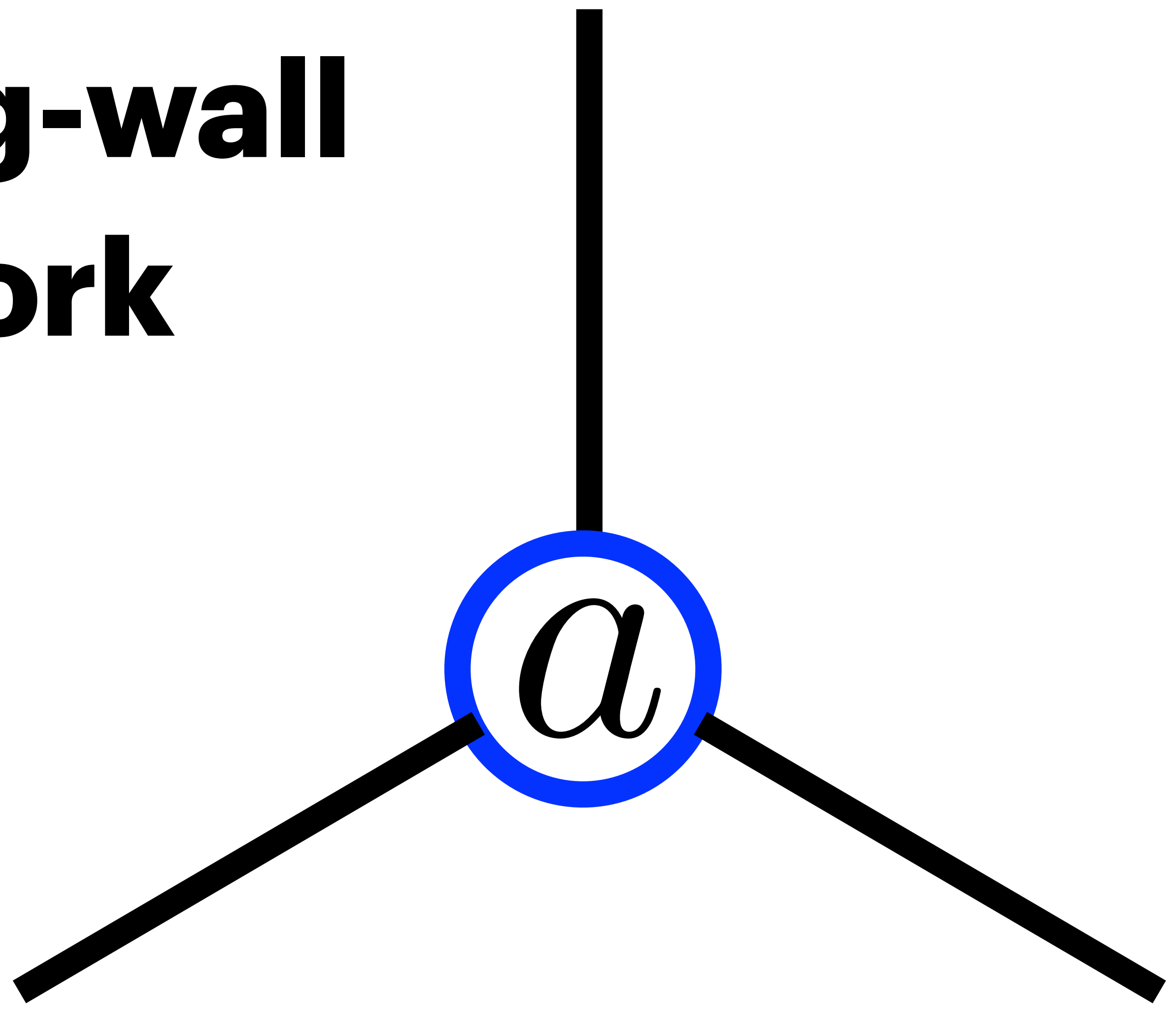
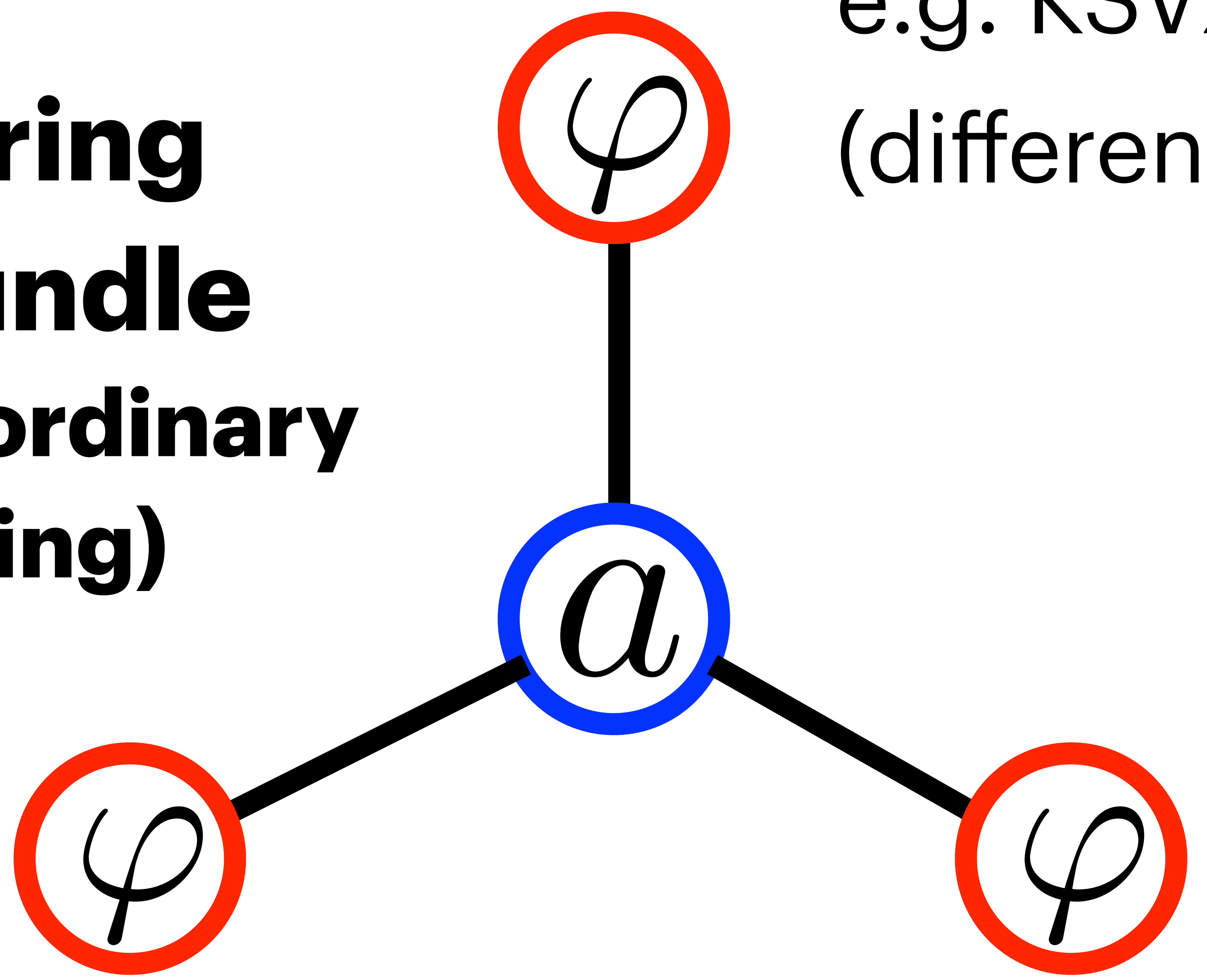



**String bundle**

**(= ordinary string)**



# New solution to the QCD axion DW problem

FT and Yin 2012.11576, Lee, Murai, FT and Yin 2507.07075,

Adding another light axion can solve the DW problem by converting the string-wall network into string bundes.

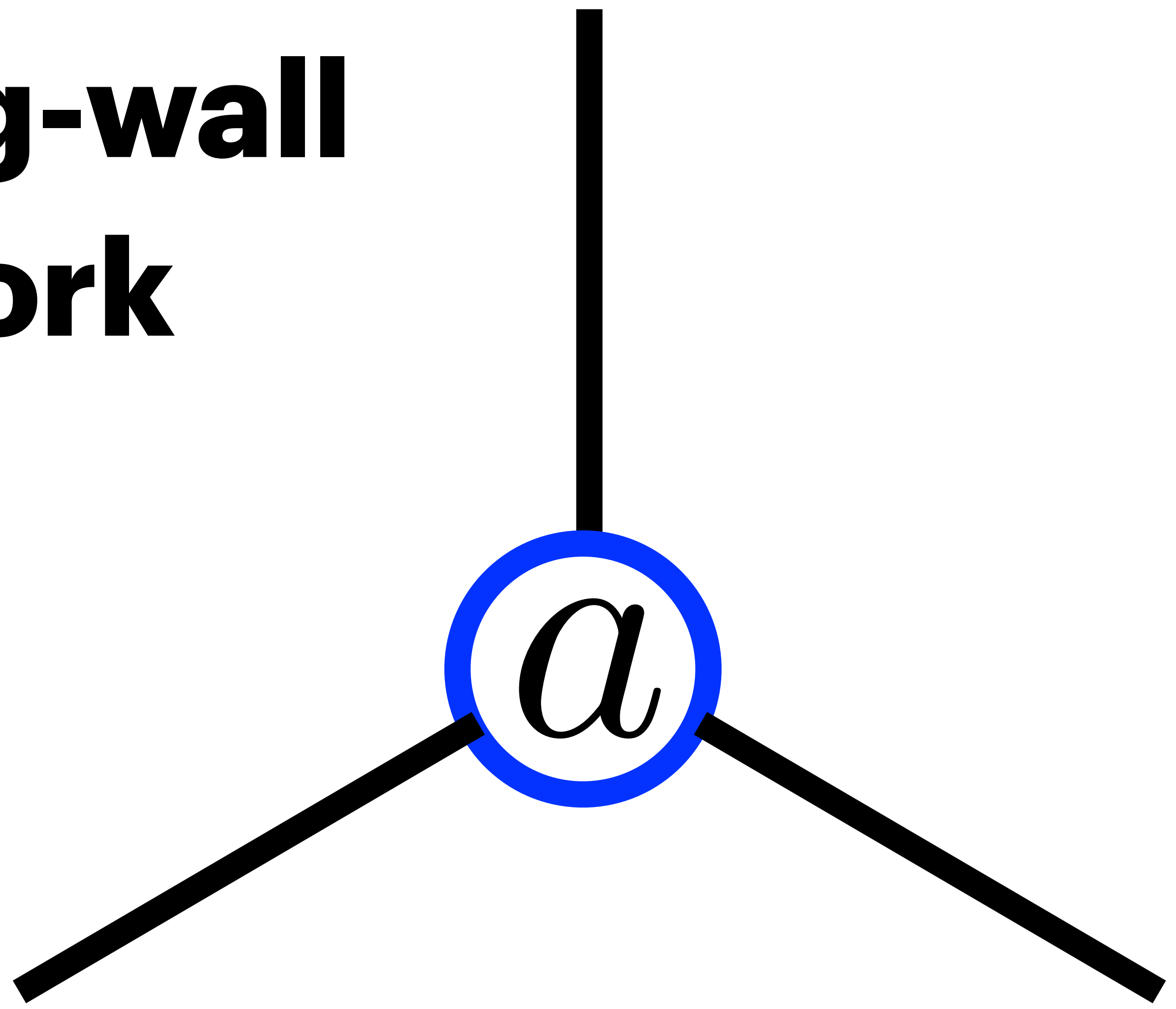
$N_{\rm DW} = 3 \text{ or } 6$ for DFSZ axion

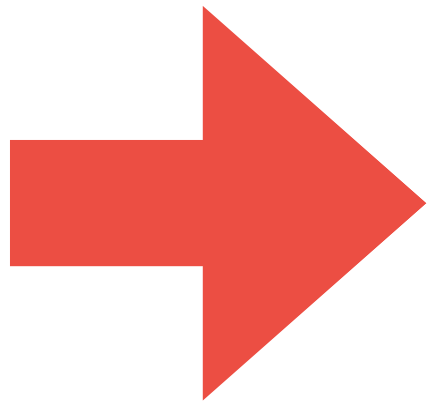
$\varphi$ : Massless (or very light) axion

e.g. KSVZ axion

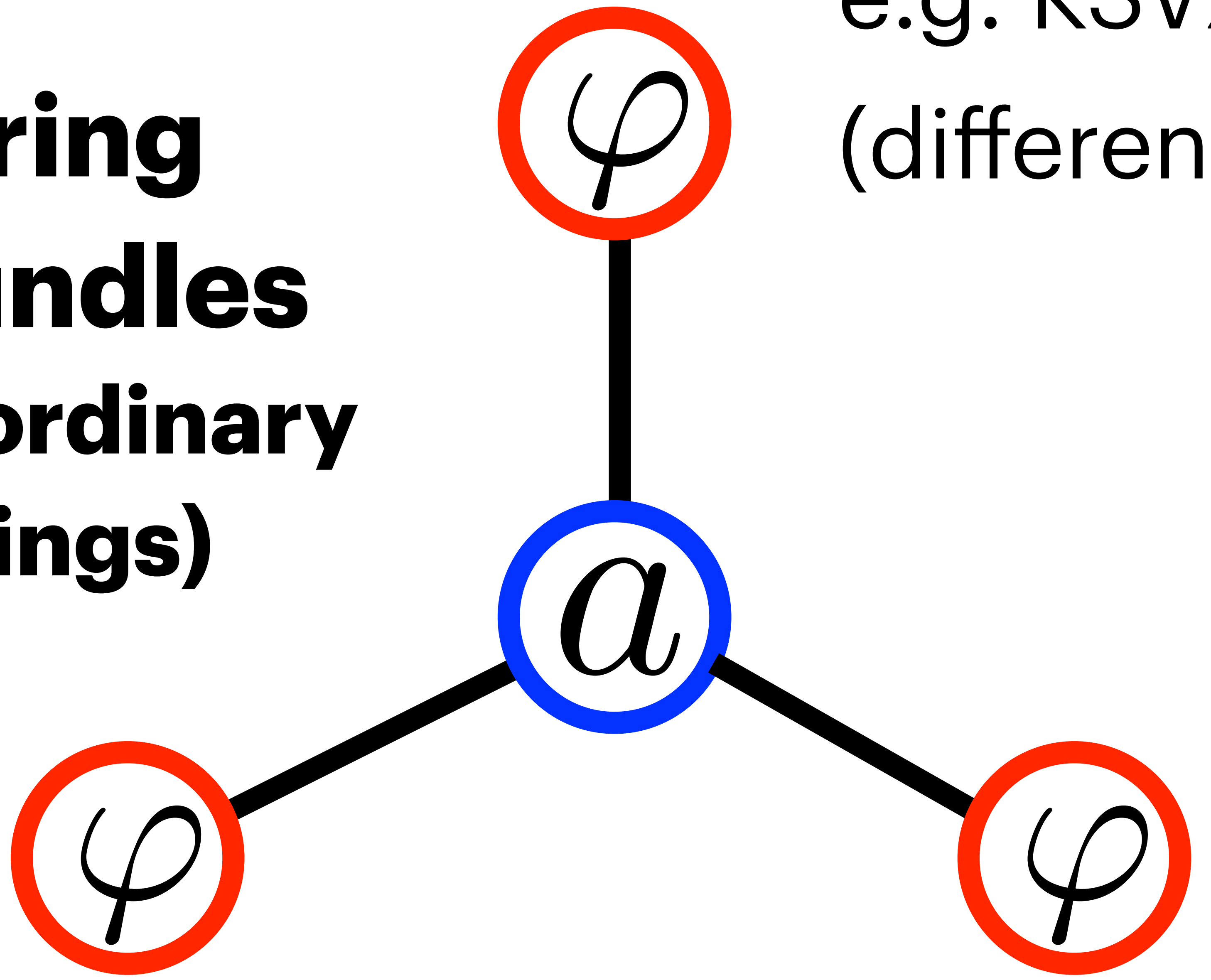
(different from $a$)

**String-wall network**



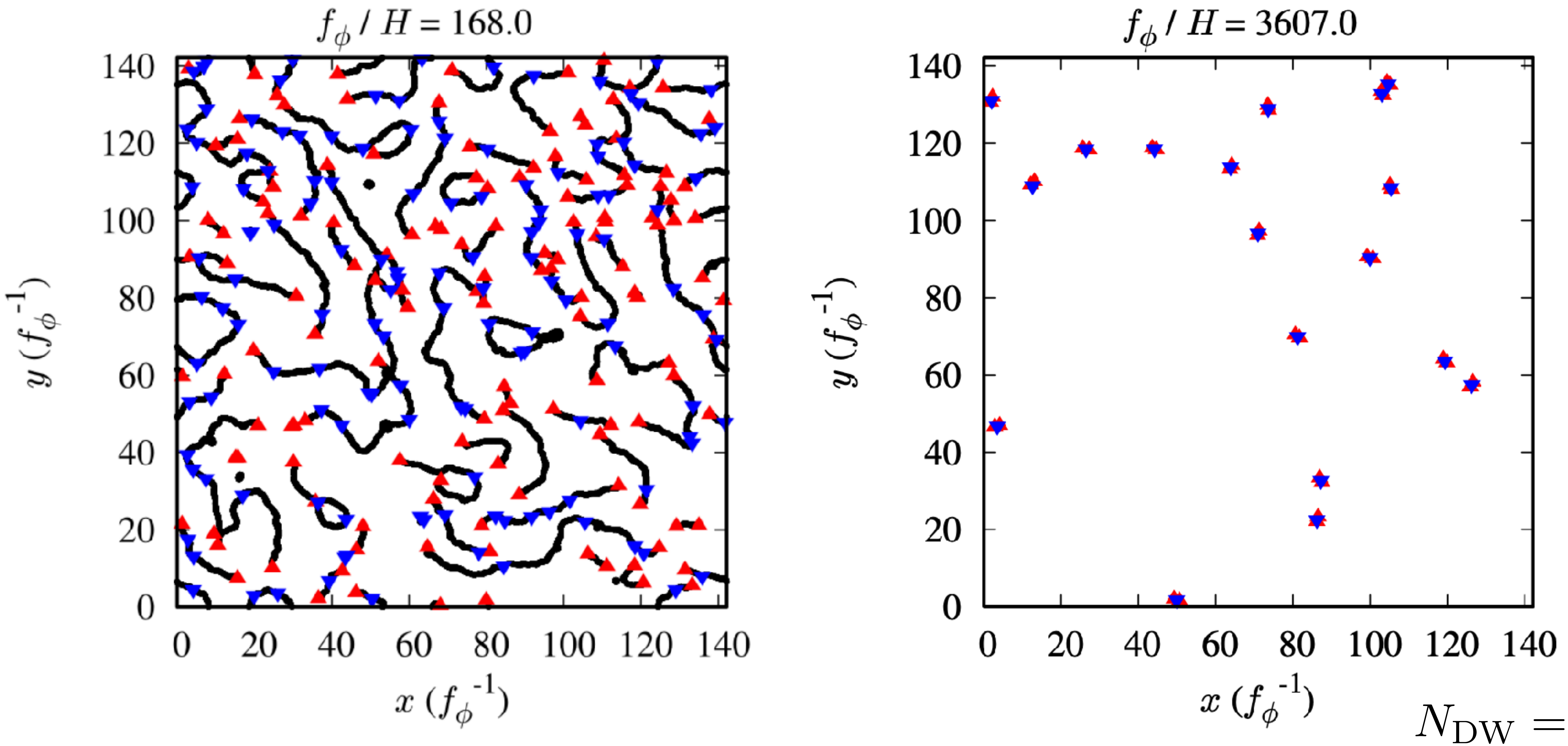



**String bundles (= ordinary strings)**



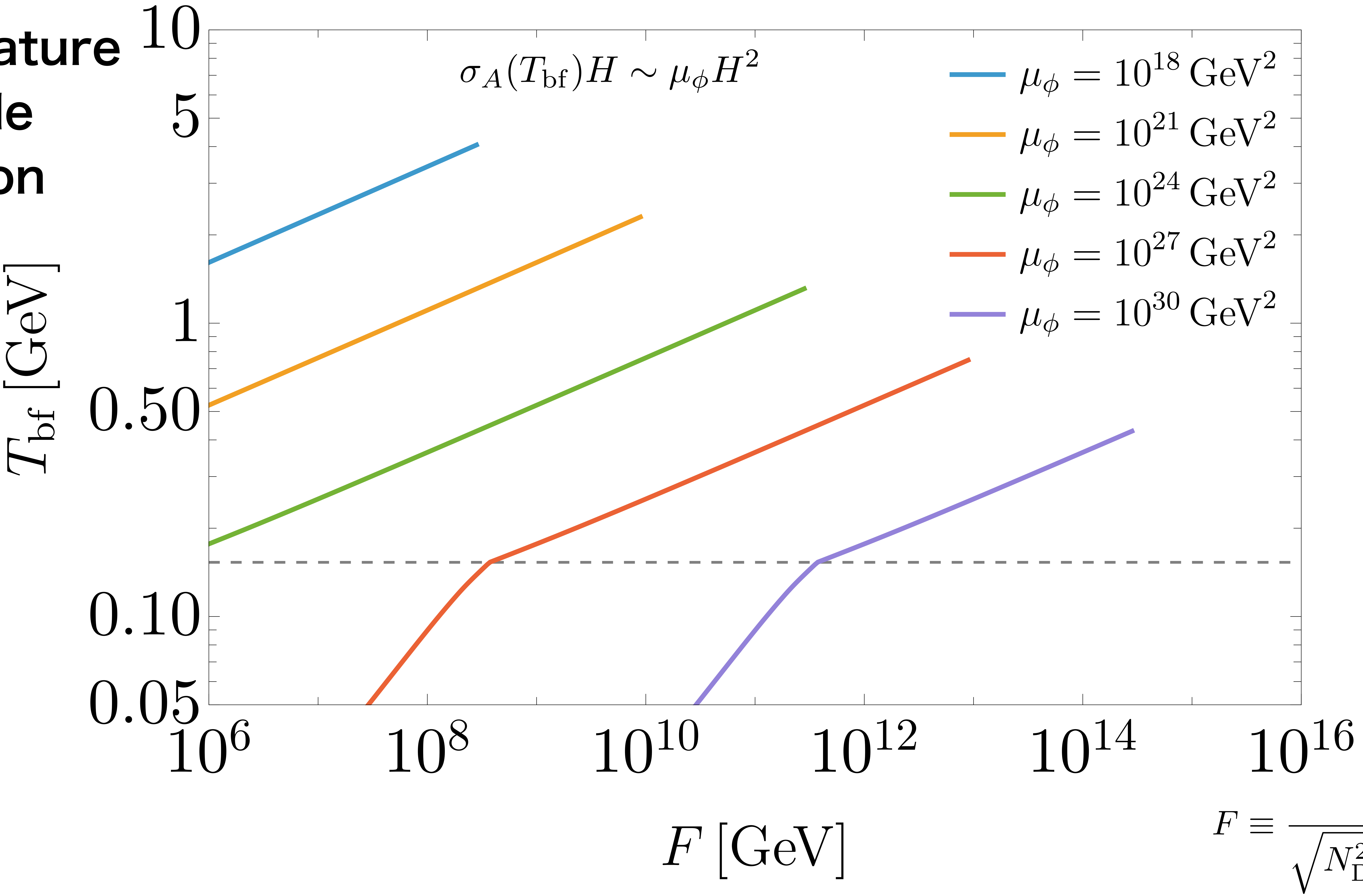
# New solution to the QCD axion DW problem

Lee, Murai, FT and Yin 2507.07075,



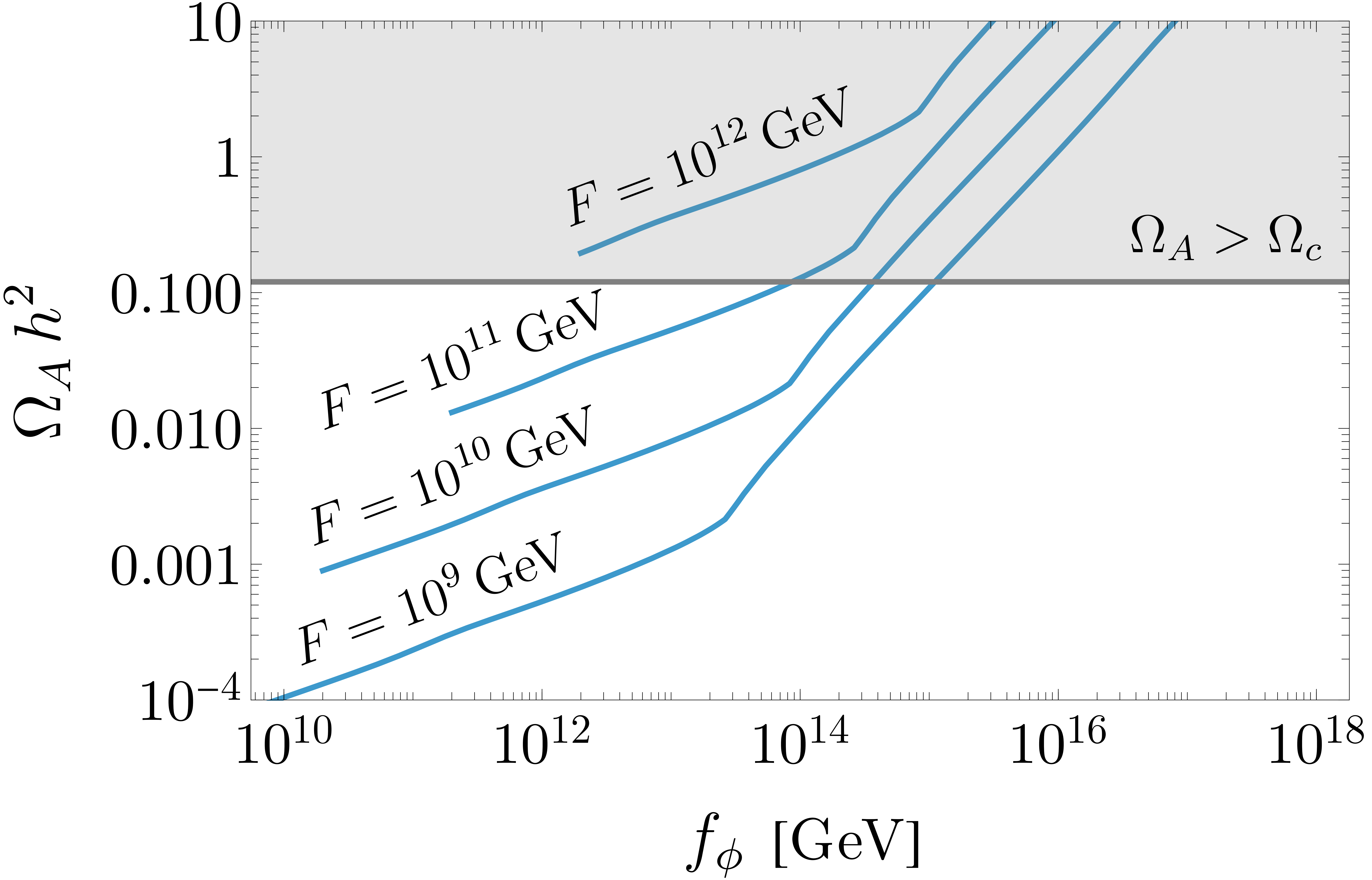

$N_{\mathrm{DW}} = 2$

Temperature at bundle formation




$$F \equiv \frac{v_a f_\phi}{\sqrt{N_{\rm DW}^2 f_\phi^2 + v_a^2}}$$

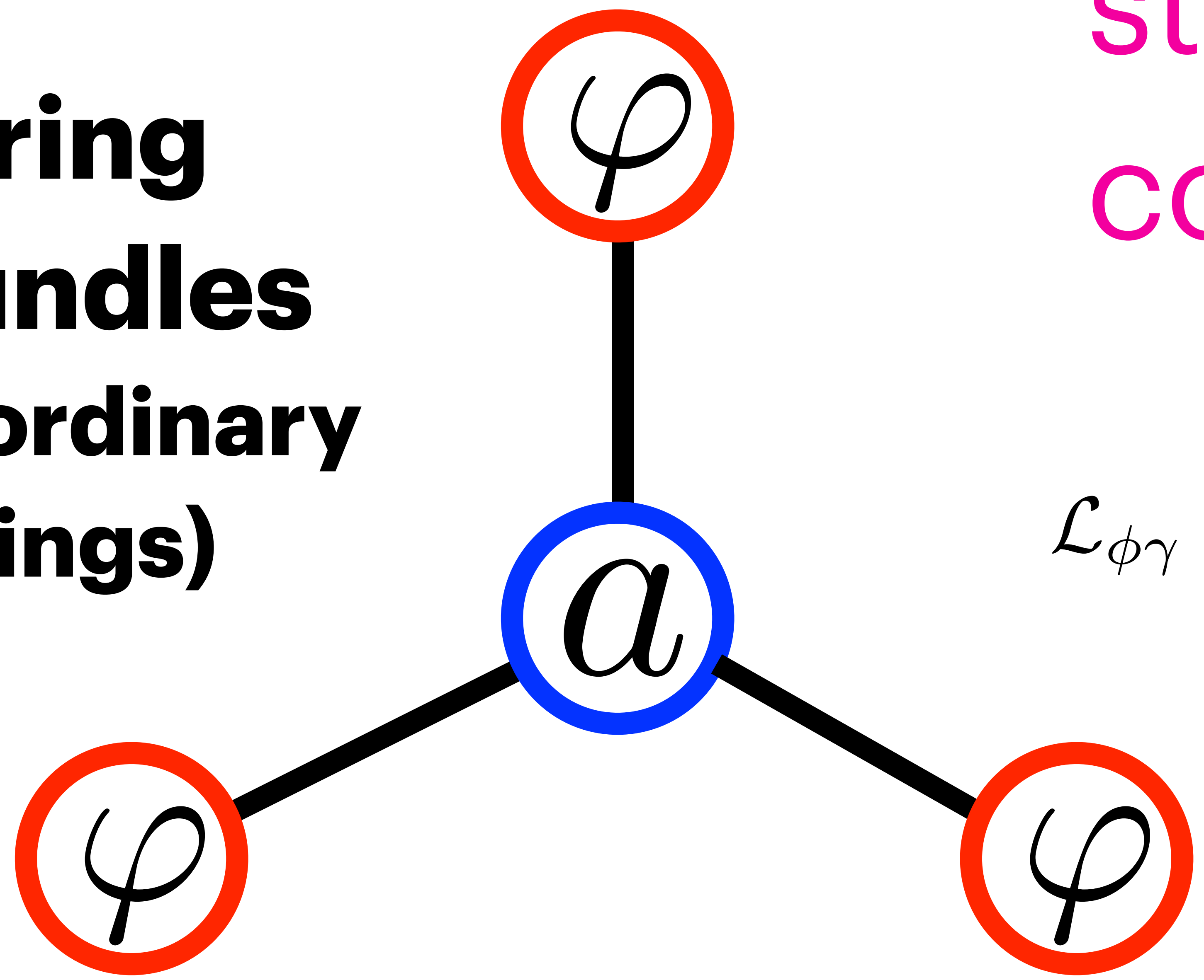
Decay constant of the QCD axion



10
1
0.100
0.010
0.001
$10^{-4}$
$\Omega_A h^2$
$10^{10}$
$10^{12}$
$10^{14}$
$10^{16}$
$10^{18}$
$f_\phi$ [GeV]
$\Omega_A > \Omega_c$
$F = 10^{12}$ GeV
$F = 10^{11}$ GeV
$F = 10^{10}$ GeV
$F = 10^{9}$ GeV

# Implications for cosmic birefringence

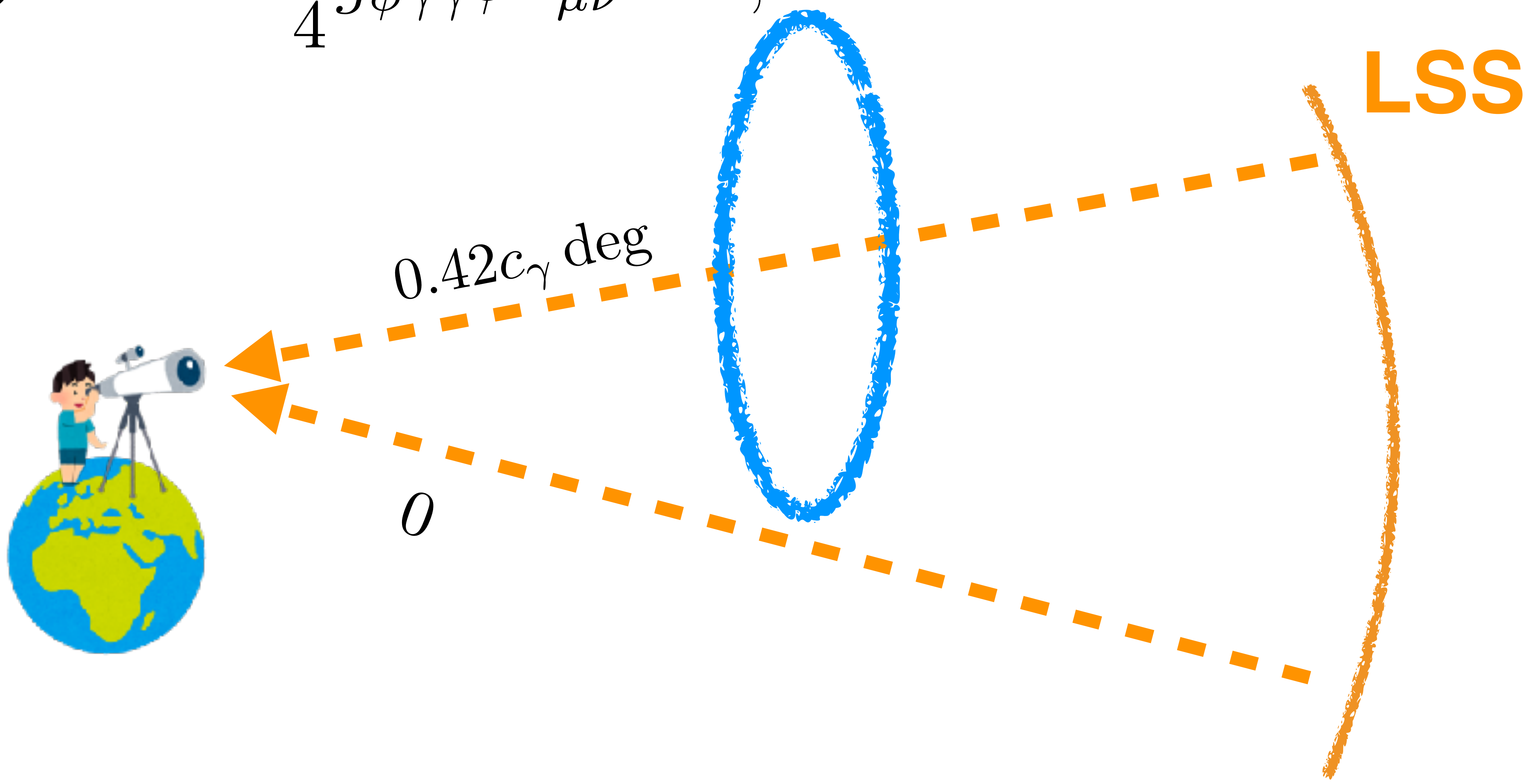
Lee, Murai, FT and Yin 2507.07075, FT and Yin 2012.11576




Unlike the usual QCD axion strings, string bundles are long-lived, and could contribute to the cosmic birefringence!

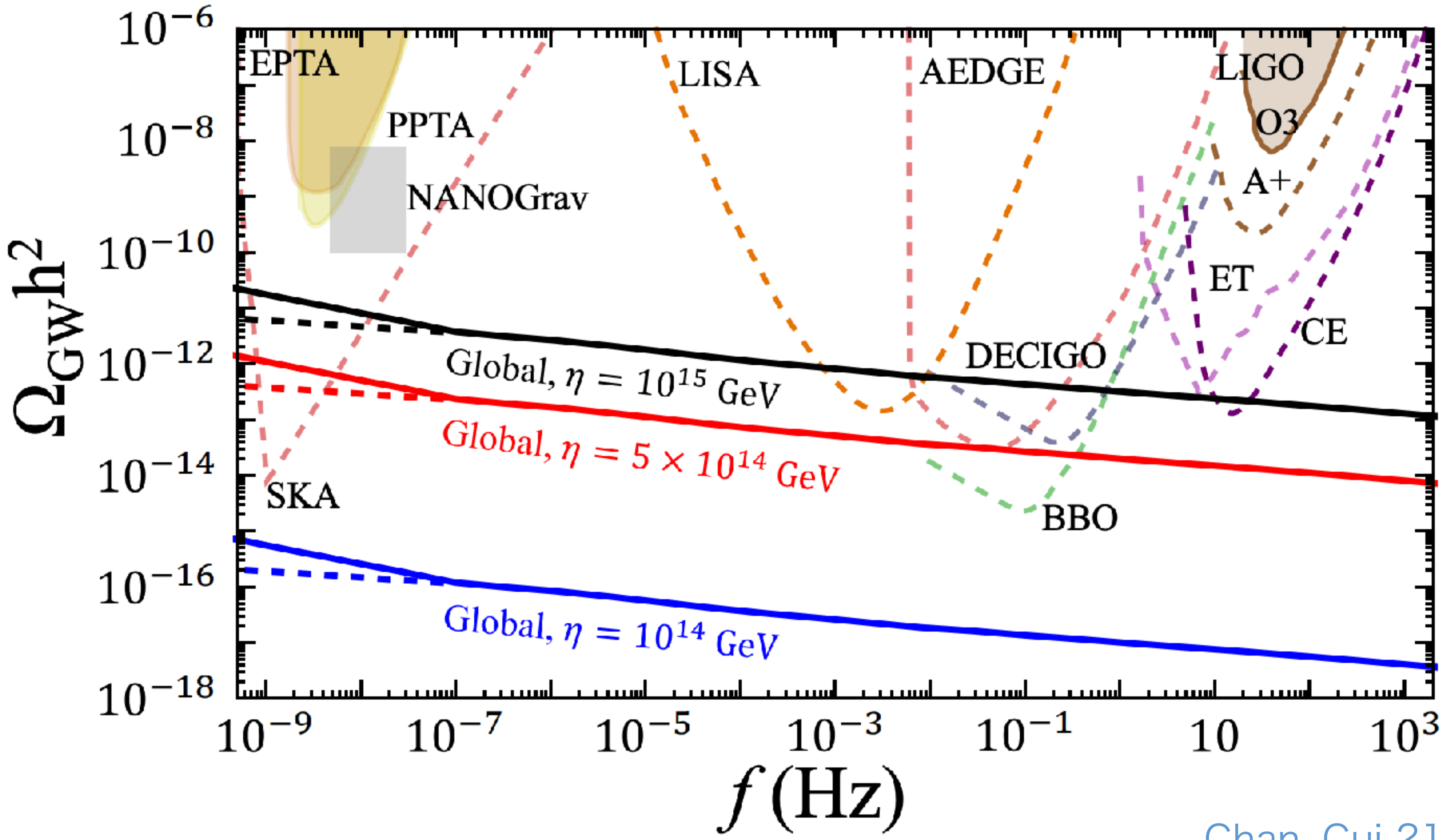
See talk by A. Long

$$\mathcal{L}_{\phi\gamma} = -c_\gamma \frac{\alpha}{4\pi}\frac{\phi}{f_\phi} F_{\mu\nu}\tilde{F}^{\mu\nu} = -\frac{1}{4} g_{\phi\gamma\gamma}\phi F_{\mu\nu}\tilde{F}^{\mu\nu},$$



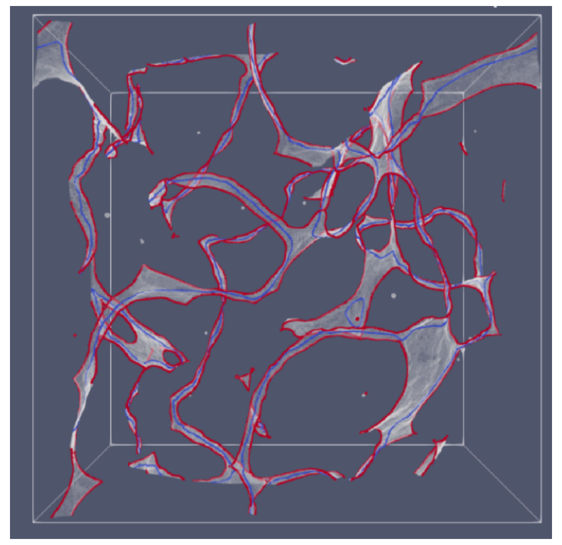

See e.g. Jain, Hagimoto, Long, Amin, 2208.08391

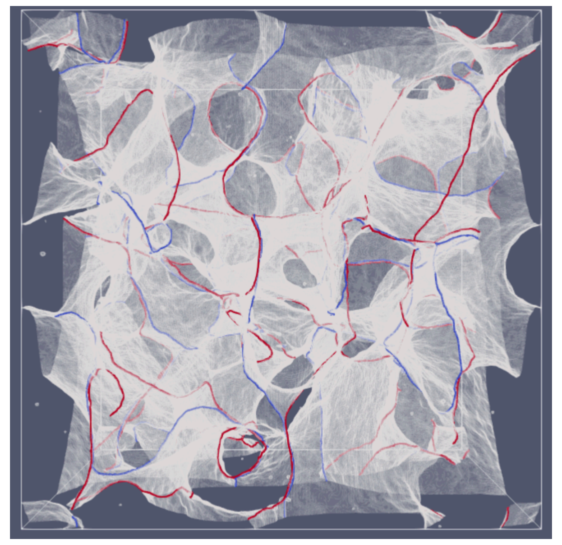
# Implications for Gravitational Waves



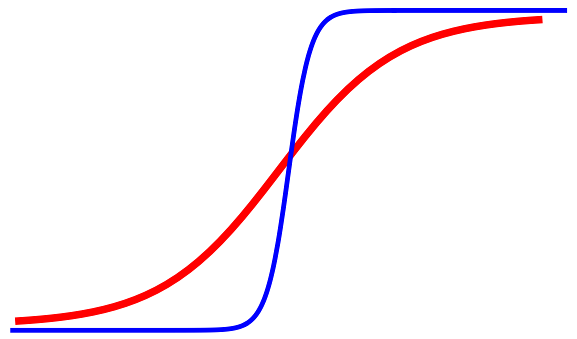

Chan, Cui 2106.09746

# Summary

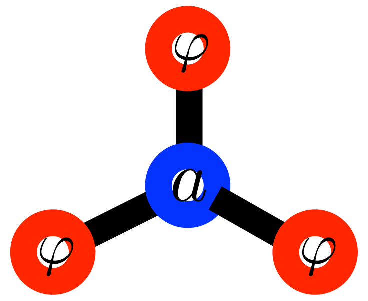




- The origin and breaking of U(1) PQ are unknown.
- A minimal extension from one to two PQ scalars often leads to stable DWs with large tension.
- Their decay can produce GWs and QCD axion DM even for small $f_a$.



- Adding another light axion can solve the QCD axion DW problem, leading to stable string bundles.




In multi-axion systems, cosmology changes drastically —
**More is different!**