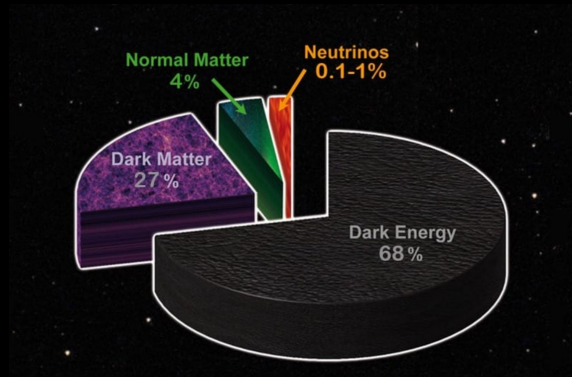
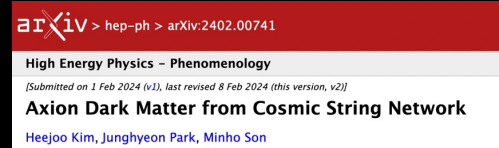


Axion mass prediction from cosmic strings

Minho Son

Korea Adv. Inst. of Sci. and Tech. (KAIST)

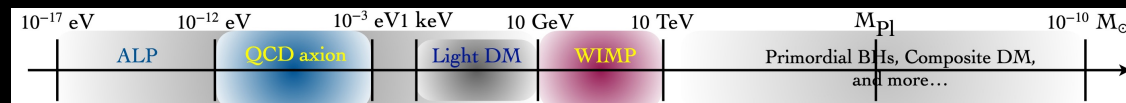
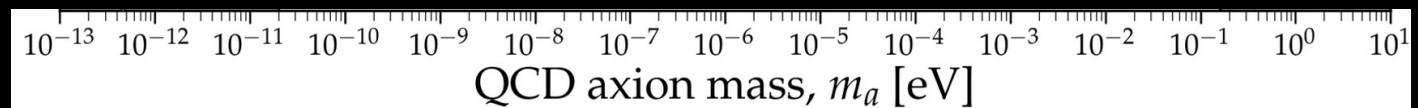
Based on



We will focus on QCD



of mass $\sim \mu\text{eV}$



Axion Dark Matter

Provides a solution to strong CP problem

Axion Dark Matter

One of the appealing qualities of the axion is that it can be dark matter

Pre-inflationary scenario

“PQ symmetry breaking before inflation”

VS

Post-inflationary scenario

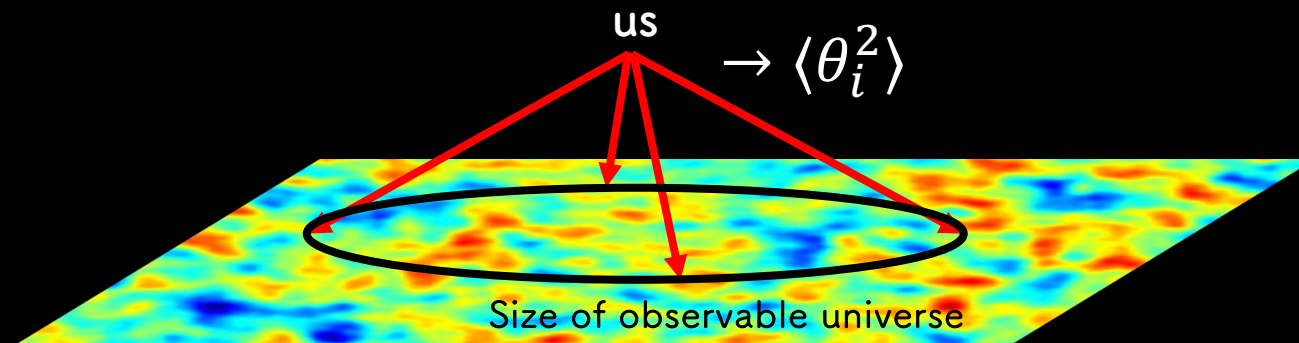
“PQ symmetry breaking after inflation”

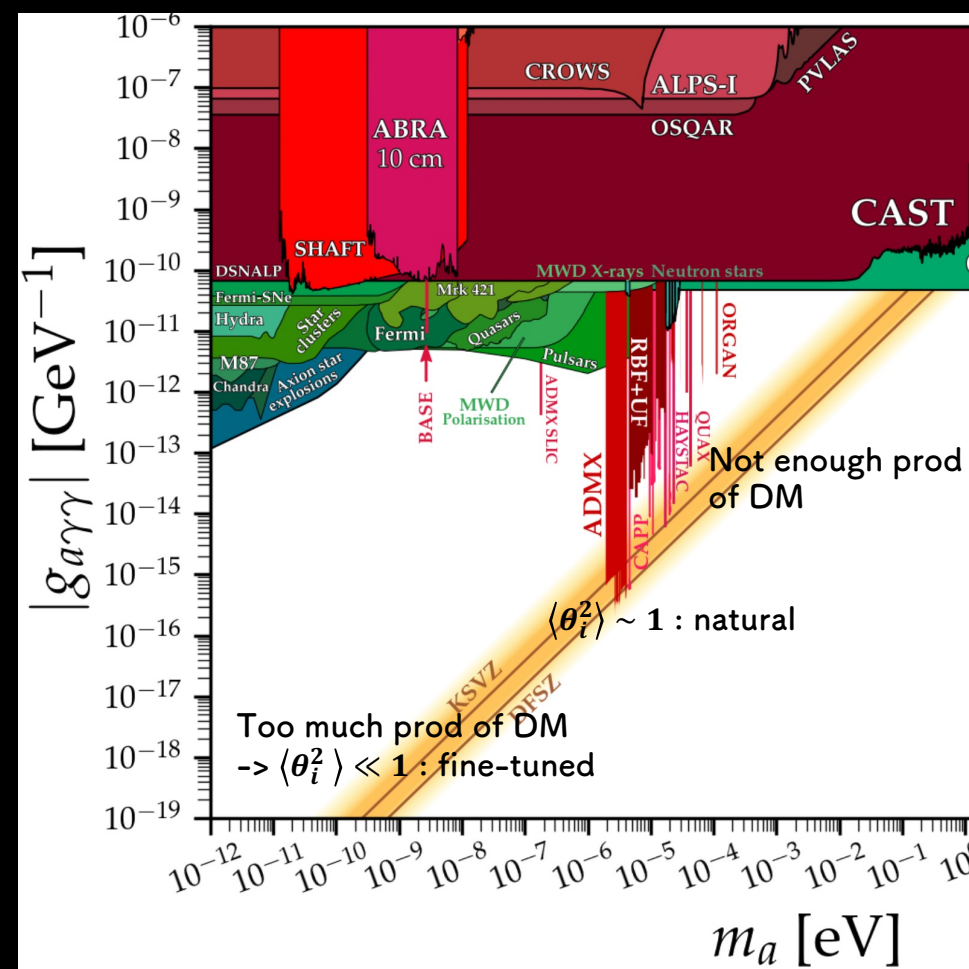
$$\Omega_a h^2 = 0.12 \left(\frac{f_a}{10^{12} \text{GeV}} \right)^{7/6} \langle \theta_i^2 \rangle$$

$$m_a \approx 5.7 \mu\text{eV} \left(\frac{10^{12} \text{GeV}}{f_a / N_W} \right)$$

Observable Universe

: includes many causally disconnected patches





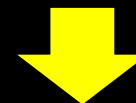
Misalignment

$$\Omega_a^{\text{mis}} h^2 = 0.12 \left(\frac{f_a}{10^{12} \text{ GeV}} \right)^{7/6} \langle \theta_i^2 \rangle$$

$$m_a \approx 5.7 \mu\text{eV} \left(\frac{10^{12} \text{ GeV}}{f_a / N_W} \right)$$

Topological defects

$$0.12 \sim \Omega_a h^2 = \Omega_a^{\text{mis}} h^2 + \Omega_a^{\text{string}} h^2 + \Omega_a^{\text{str-dw}} h^2$$



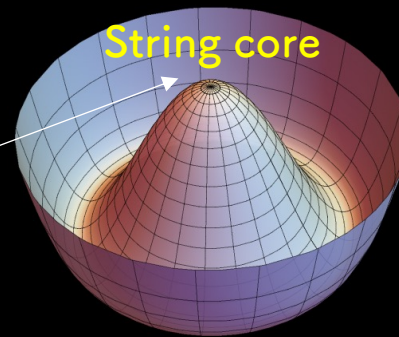
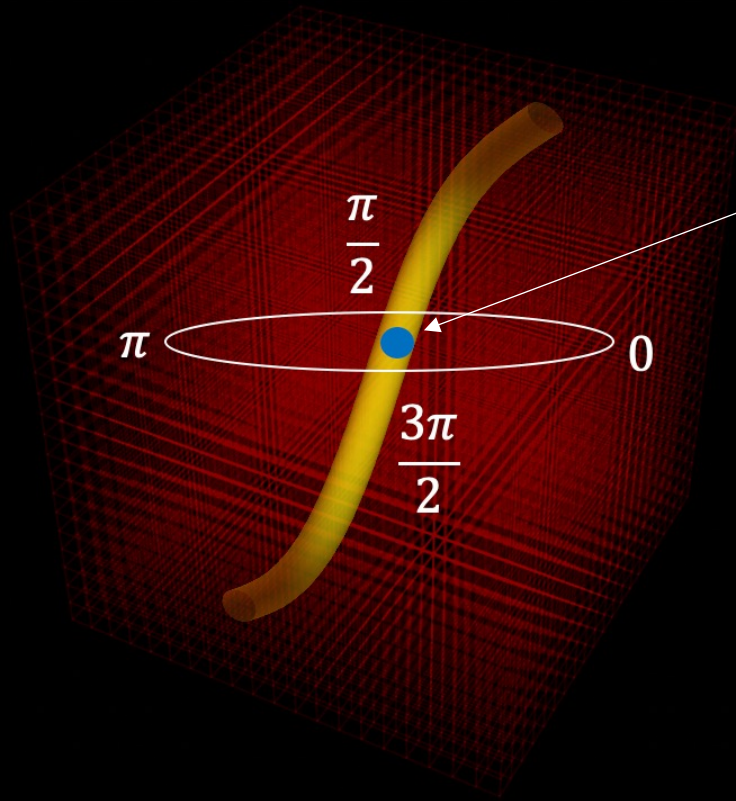
Situation of direct axion search dramatically changes depending on the size of this part

Formation of cosmic string

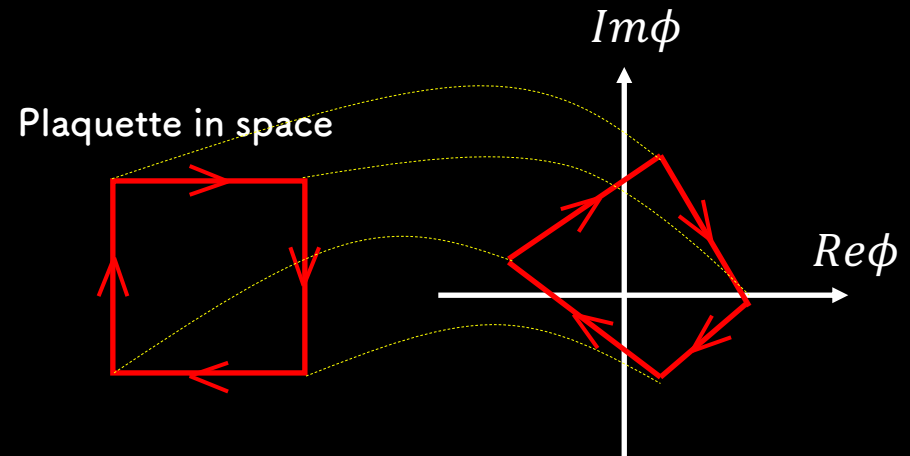
$$ds^2 = dt^2 - R^2(t)d\vec{x}^2$$

$$\ddot{\phi} + 3H\dot{\phi} - \frac{1}{R^2}\nabla^2\phi + \frac{m_r^2}{f_a^2}\left(|\phi|^2 - \frac{f_a^2}{2}\right)\phi = 0$$

Evolution of this complex scalar field is highly non-linear and it can be done only numerically



$$\phi = \frac{r(x) + f_a}{\sqrt{2}} e^{i\frac{a}{f_a}}$$



See talk by Takahashi

$$\log(m_r/H) = 2.000$$

$$\rho_{\text{tot}} = \left\langle |\dot{\phi}|^2 + |\nabla\phi|^2 + V(\phi) \right\rangle$$

3D volume (spatial) average

$$= \left\langle \frac{1}{2} \dot{a}^2 + \frac{1}{2} |\nabla a|^2 \right\rangle + \left\langle \frac{1}{2} \dot{r}^2 + \frac{1}{2} |\nabla r|^2 + V(r) \right\rangle$$

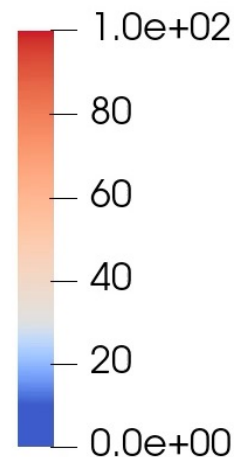
$2 \times \frac{1}{2} \dot{a}^2$: stored in axions : stored in radial modes

$$+ \left\langle \left(\frac{r^2}{2f_a^2} + \frac{r}{f_a} \right) (\dot{a}^2 + |\nabla a|^2) \right\rangle$$

String energy density

$$\rho_s = \rho_{\text{tot}} - \rho_a - \rho_r$$

$\frac{\rho_a}{H^2 f_a^2}$ (Masked)



$$\log(m_r/H) = 5.509$$

$$\rho_{\text{tot}} = \left\langle |\dot{\phi}|^2 + |\nabla\phi|^2 + V(\phi) \right\rangle$$

3D volume (spatial) average

$$= \left\langle \frac{1}{2} \dot{a}^2 + \frac{1}{2} |\nabla a|^2 \right\rangle + \left\langle \frac{1}{2} \dot{r}^2 + \frac{1}{2} |\nabla r|^2 + V(r) \right\rangle$$

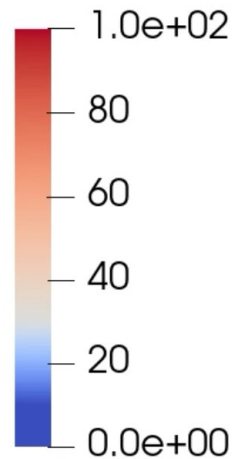
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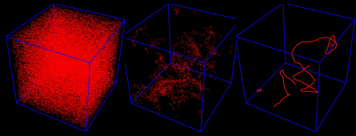
$$\rho_s = \rho_{\text{tot}} - \rho_a - \rho_r$$

$\frac{\rho_a}{H^2 f_a^2}$ (Masked)



Cosmological evolution

$$\log \frac{m_r}{H} = \log \frac{t}{t_0} \lesssim \log N$$



~ 1

~ 10

lattice simulation

$$H^{-1} \propto t$$

$$\Delta \propto R(t) = \sqrt{t}$$

$$m_r^{-1} \propto \text{const}$$

Multi-scale problem:
Even numerical simulation is very difficult and
progress is very slow

Appearance of Scaling solution

: allows us to extrapolate all the way to QCD scale

QCD PT



$t \sim 10^{-5}$ sec

~ 70

$\log \frac{m_r}{H}$

Axions decayed from
strings around this time
contribute to DM

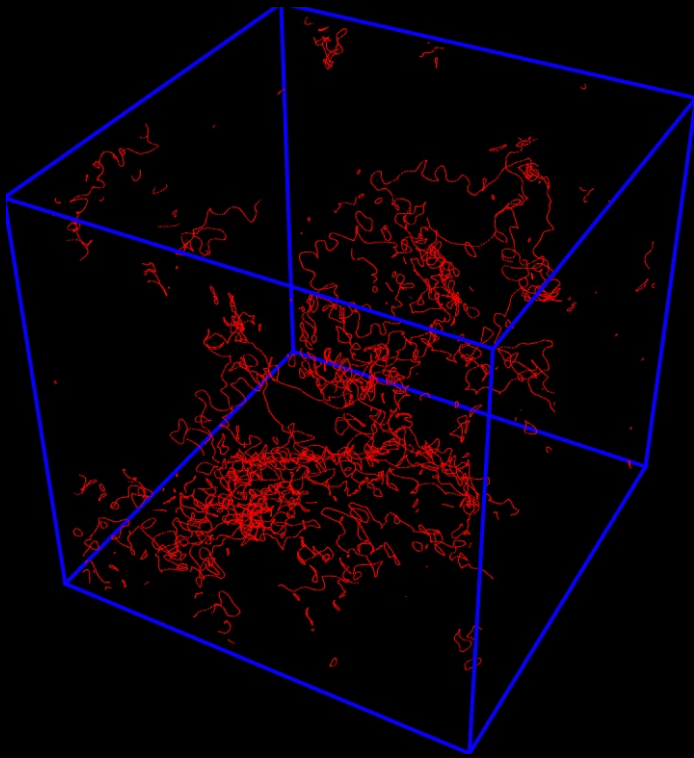
Two important scalings for axion mass prediction

$$\xi \propto \frac{\ell_{\text{tot}}(L)}{\left(L^3 / (H^{-1})^3\right)} \frac{1}{H^{-1}} = \frac{\ell_{\text{tot}}(L)t^2}{L^3}$$

: number of strings
per Hubble patch

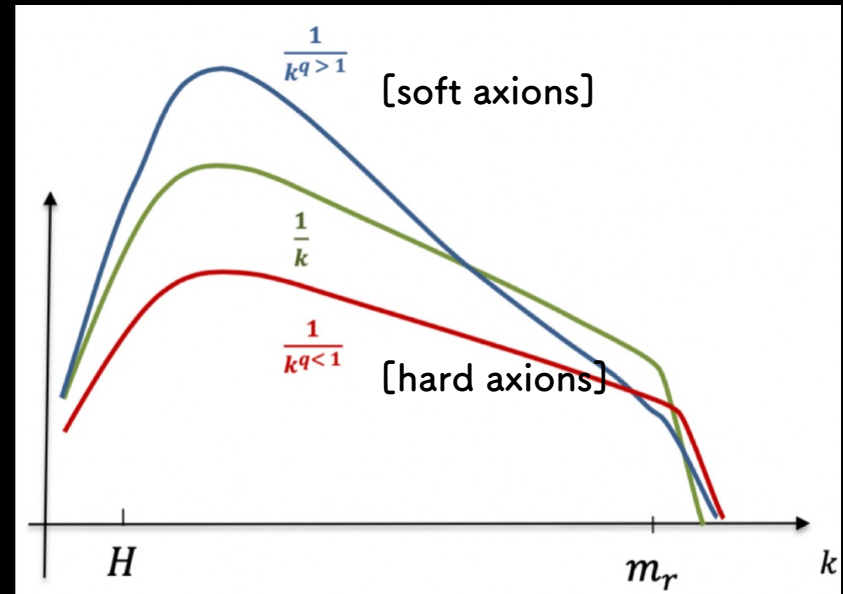
$$\frac{\partial^2 \rho_a}{\partial t \partial k} \propto \frac{1}{k^q}$$

: no new scale
between m_r and H



$$\xi = \frac{\ell_{\text{tot}}(L)t^2}{L^3}$$

: number of strings
per Hubble patch



$$\frac{\partial^2 \rho_a}{\partial t \partial k} \propto \frac{1}{k^q}$$

: no new scale
between m_r and H

Interesting situation would be
this at the QCD crossover

$$\left. \frac{n_a^{\text{str}, q>1}}{n_a^{\text{mis}, \theta_0=1}} \right|_{t_\ell} > 1$$

Axion abundance

$$n_a = \int \frac{dk}{k} \frac{\partial \rho_a}{\partial k} \quad : \text{should follow a **power law** due to absence of other non-trivial scales and sampling has to be done in scaling regime}$$

$$\frac{\partial \rho_a}{\partial k} = \int^t dt' \frac{\Gamma[t']}{H(t')} \left(\frac{R(t')}{R(t)} \right)^3 F \left[\frac{k'}{H(t')}, \frac{m_r}{H(t')} \right]$$

: instantaneous emission function

$$\Gamma[t] \sim \frac{\xi \mu_{\text{eff}}}{t^3} \sim 8\pi H^3 f_a^2 \xi \log \frac{m_r}{H}$$

$$F \sim \frac{1}{k^q}$$

Two most important scalings in ξ and spectral index q in cosmic string

$$\text{E.g.} \quad \left. \frac{n_a^{q>1}}{n_a^{\text{mis}}} \right|_{t_\ell} \propto \left(\xi_\star \log \frac{m_r}{H_\star} \right)^{\frac{1}{2} + \dots}$$

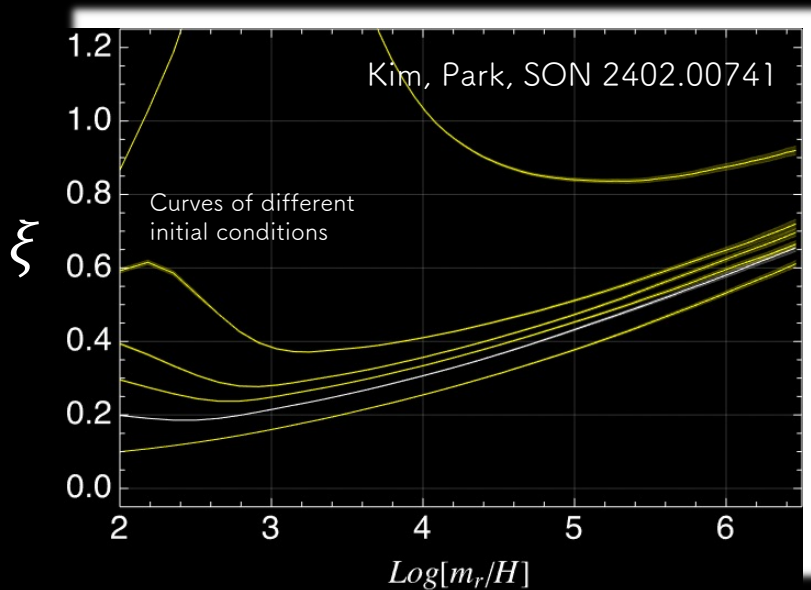
Gorghetto, Hardy, Villadoro 20'
Kim, SON, Park 2024 (redone)

Recent intriguing observations are

This is where simulation frontiers leads the theory.
These findings are not explained by theory yet

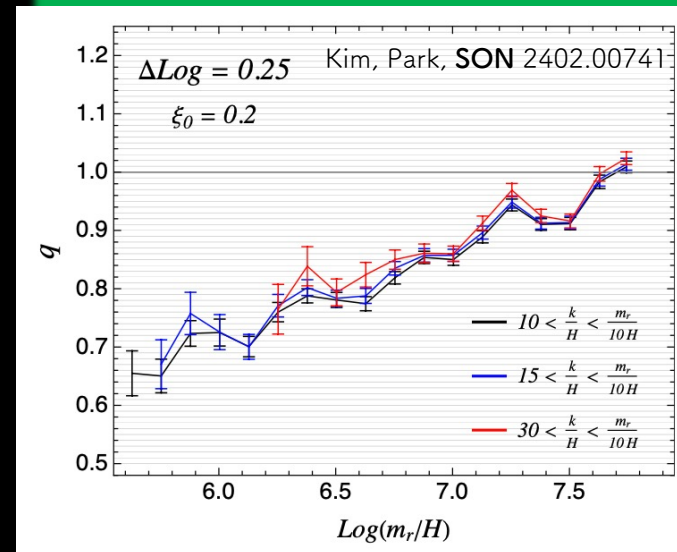
$$\xi = \beta + \alpha \log \frac{m_r}{H}$$

Gorghetto, Hardy, Villadoro 18', 20'
Benabou, Buschmann, Foster, Hook, Peterson, Willcox, Zhang, Safdi 21', 24'
Saikawa, Redondo, Vaquero, Kaltshmidt 24'



Extrapolates to

$$q = q_0 + \varepsilon \log \frac{m_r}{H}$$



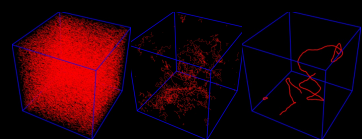
Kim, Park, SON 2402.00741

$$\xi \sim \mathcal{O}(10)$$

$$q > 1$$

$$@ m_r t = e^{70}$$

$$\left. \frac{n_a^{\text{str}, q>1}}{n_a^{\text{mis}, \theta_0=1}} \right|_{t_\ell} > 1$$



~ 1

~ 10

Supported by
scaling solution

lattice simulation

Extrapolation

$$\Omega_a^{\text{mis}} h^2 = 0.12 \left(\frac{f_a}{10^{12} \text{ GeV}} \right)^{7/6} \langle \theta_i^2 \rangle$$

$$0.12 \sim \Omega_a h^2 = \Omega_a^{\text{mis}} h^2 + \Omega_a^{\text{str}} h^2 + \Omega_a^{\text{str-dw}} h^2$$

Axions from strings in
scaling regime @QCD PT

Gorghetto, Hardy, Villadoro 18', 20'
Saikawa, Redondo, Vaquero, Kaltshmidt 24'
Kim, Park, SON 24'

: soon after this time, axions become
non-relativistic dark matter

QCD PT

$t \sim 10^{-5} \text{ sec}$

~ 70

$t_* \longleftrightarrow t_\ell$

$\log \frac{m_r}{H}$

Nonlinearity may or may not kick in

$$\frac{\sqrt{\langle a^2 \rangle}}{f_a} \geq 1 \text{ vs } \leq 1$$

Gorghetto, Hardy, Villadoro 20'

Axions from string-domain wall
network @QCD phase transition

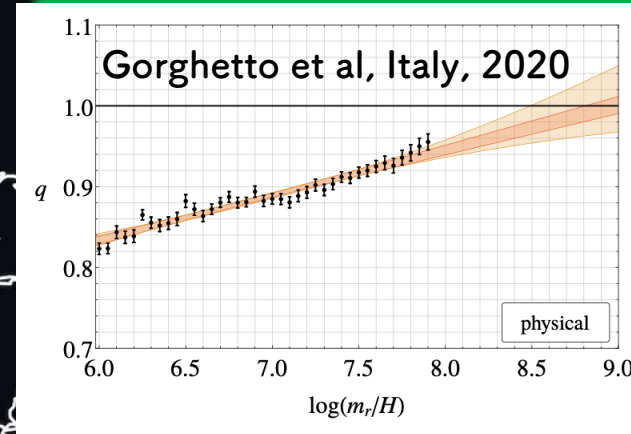
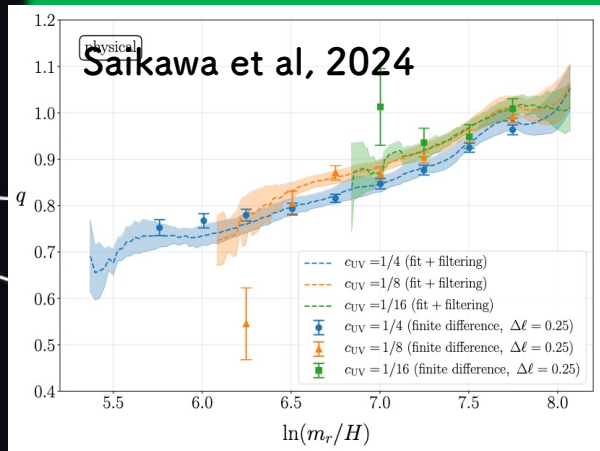
Gorghetto, Hardy, Villadoro 20'
Benabou, Buschmann, Foster, Safdi 24'

$$\text{If } \frac{\Omega_a^{\text{mis}} h^2}{\Omega_a^{\text{str-dw}} h^2} \ll \Omega_a^{\text{str}} h^2, \quad \Omega_a^{\text{str}} h^2 \approx \frac{n_a^{\text{str}}}{n_a^{\text{mis}, \theta_0=1}} \Omega_a^{\text{mis}, \theta_0=1} h^2 \leq 0.12$$

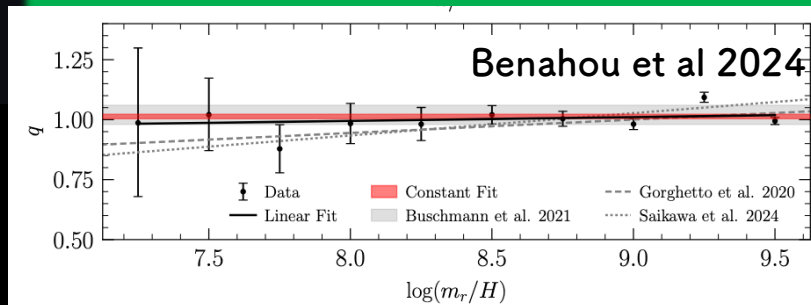
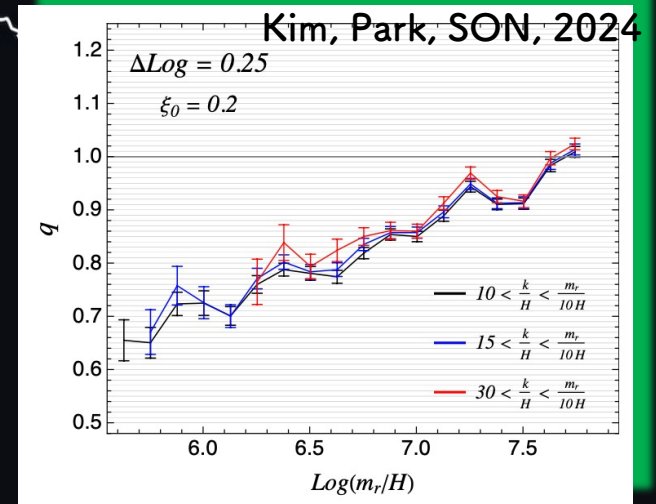
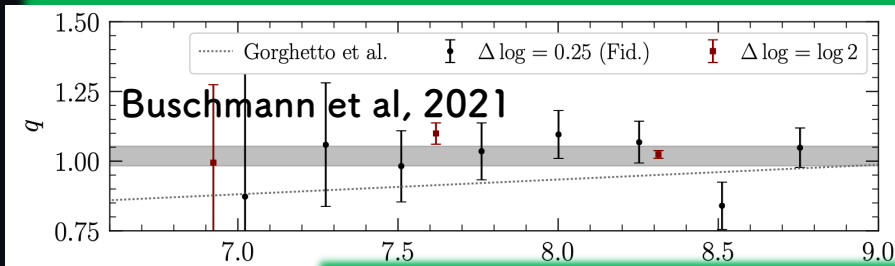
$$\rightarrow m_a \geq \boxed{} \mu\text{eV}$$

Recent results in cosmic string frontier

IR dominant axions, $q > 1$



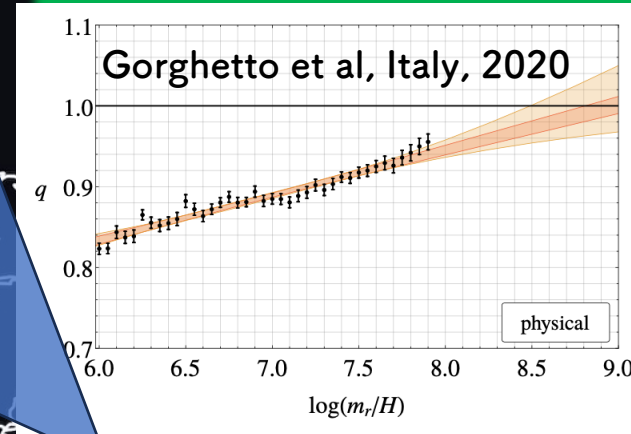
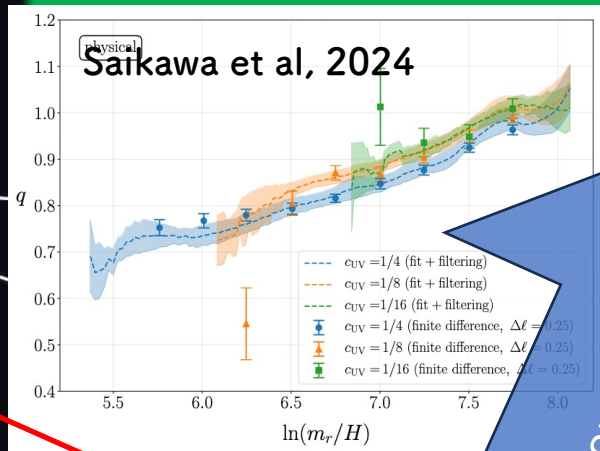
Nearly conformal strings, $q \sim 1$



non-negligible Domain walls can increase axion mass further

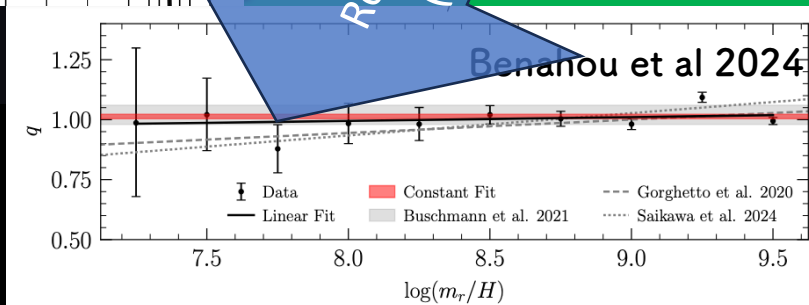
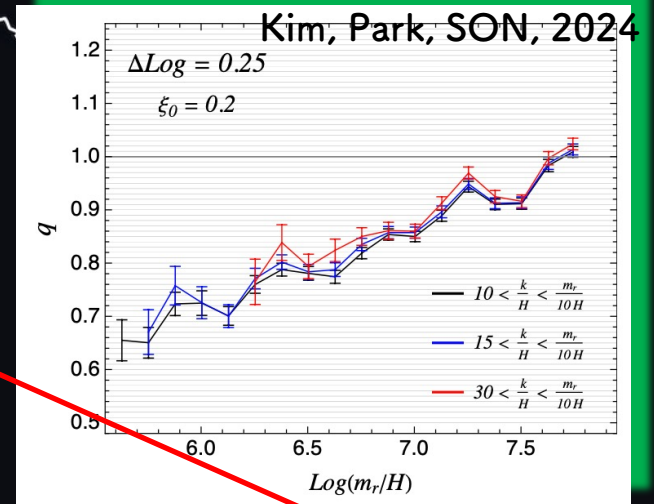
Recent results in cosmic string frontier

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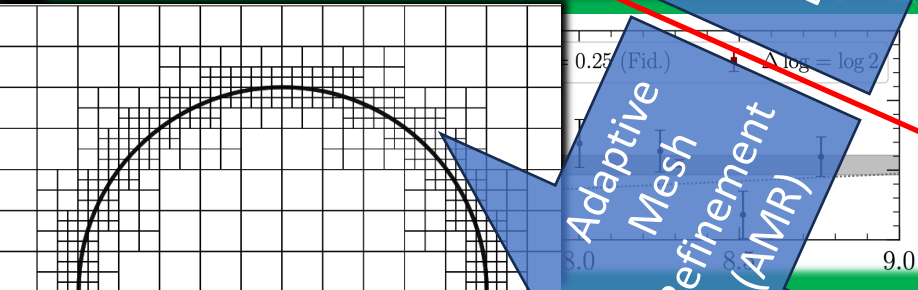


Traditional Static
Lattice Simulation

Adaptive
Mesh
Refinement
(AMR)



Nearly
conformal strings , $q \sim 1$



New update:
non-negligible Domain
walls can increase axion
mass further

Axion mass prediction

Axion Dark Matter from Cosmic String Network

Heejoo Kim[†], Junghyeon Park[†], and Minh Son[†]

[†]Department of Physics, Korea Advanced Institute of Science and Technology,
291 Daehak-ro, Yuseong-gu, Daejeon 34141, Republic of Korea

Kim, Park, SON 2402.00741

$$m_a \geq 420 \mu\text{eV}$$

Fat-string pre-evolution

$$m_a \geq 470 \mu\text{eV}$$

Thermal pre-evolution

More Axions from Strings

Marco Gorghetto^a, Edward Hardy^b, and Giovanni Villadoro^c

^a Department of Particle Physics and Astrophysics, Weizmann Institute of Science,
Herzl St 234, Rehovot 761001, Israel

^b Department of Mathematical Sciences, University of Liverpool,
Liverpool, L69 7ZL, United Kingdom

^c Abdus Salam International Centre for Theoretical Physics,
Strada Costiera 11, 34151, Trieste, Italy

Gorghetto et al 2020

$$m_a \geq 450 \mu\text{eV}$$

Fat-string pre-evolution

Spectrum of global string networks and the axion dark matter mass

Ken'ichi Saikawa,¹ Javier Redondo,^{2,3} Alejandro Vaquero,³ and Mathieu Kaltschmidt³

¹Institute for Theoretical Physics, Kanazawa University,
Kakuma-machi, Kanazawa, Ishikawa 920-1192, Japan

²Max-Planck-Institut für Physik (Werner-Heisenberg-Institut), Boltzmannstr. 8, 85748 Garching, Germany

³CAPA & Departamento de Física Teórica, Universidad de Zaragoza, C. Pedro Cerbuna 12, 50009 Zaragoza, Spain
(Dated: October 16, 2024)

Saikawa et al 2024

$$450 \mu\text{eV} \geq m_a \geq 95 \mu\text{eV}$$

Fat-string pre-evolution

Axion mass prediction from adaptive mesh refinement cosmological lattice simulations

Joshua N. Benabou,^{1,2} Malte Buschmann,³ Joshua W. Foster,^{4,5} and Benjamin R. Safdi^{1,2}

¹Berkeley Center for Theoretical Physics, University of California, Berkeley, CA 94720, U.S.A.

²Theoretical Physics Group, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, U.S.A.

³GRAPPA Institute, Institute for Theoretical Physics Amsterdam,
University of Amsterdam, Science Park 904, 1098 XH Amsterdam, The Netherlands

⁴Astrophysics Theory Department, Theory Division, Fermilab, Batavia, IL 60510, USA

⁵Kavli Institute for Cosmological Physics, University of Chicago, Chicago, IL 60637

(Dated: December 13, 2024)

Benahou et al 2024

$$280 \leftarrow 65 \mu\text{eV} \geq m_a \geq 45 \mu\text{eV}$$

Buschmann et al 2021

$$180 \mu\text{eV} \geq m_a \geq 40 \mu\text{eV}$$

Thermal pre-evolution

Axion mass prediction

Static Lattice Simulation

Axion Dark Matter from Cosmic String Network

Hyunsoo Kim,¹ Inghoan Park,¹ and Minho Son¹

Do not exceed current bound,
Only axion from strings in scaling regime
+ nonlinearity around QCD
[Domain-walls are sub-dominant]

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Fat-string pre-evolution

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Account for 100% DM abundance,
Only axion from strings in scaling regime,
Mass range due to multiple ansatz for ξ and q + stat. + syst. errors

Kenta Saikawa,¹ Salvador Guevara,² Alejandro Vaquero,³ and Mathieu Kaltschmidt³
¹KEK, Tsukuba, 305-8565, Japan
²RIKEN, Wako, 351-0192, Japan
³CITA & Departamento de Física Teórica, Universidad de Zaragoza, C. Pedro Cerbasi 12, 50009 Zaragoza, Spain
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Axions from strings scaling regime + string-domain walls,

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$$\xi_c^{\text{lin}} = -0.19(3) + 0.205 \ell$$

$$\xi_c^{\text{sat}} = \frac{-0.25(15) + 0.23(6)\ell}{1 + 0.02(4)\ell}$$

$$\ell = \log \frac{m_r}{H}$$

$$q = q_0 + q_1/\ell^2 \text{ (Plateau)}$$

$$q = q_0 + q_1\ell \text{ (Linear)}$$

TABLE II: Axion dark matter mass predicted at $\ln(m_r/H) = 70$ and its error budget. For each extrapolation of q (plateau extrapolation with $q = q_0 + q_1/\ell^2$ and linear extrapolation with $q = q_0 + q_1\ell$), we consider two models of ξ_c .

Parameter	$m_a [\mu\text{eV}]$ (Plateau extrapolation)		$m_a [\mu\text{eV}]$ (Linear extrapolation)	
	ξ_c^{lin}	ξ_c^{sat}	ξ_c^{lin}	ξ_c^{sat}
q	141–198	102–144	439	311
ξ	167–173	95–151	431–447	237–381
x_0	149–197	111–137	429–450	305–316
f_L	168–172	122–125	435–443	308–313
n_{QCD}	138–175	101–127	379–449	268–317

Note that plateau (linear) model predicts smaller (larger) axion mass

Plateau ansatz :

$$95 \mu\text{eV} \lesssim m_a \lesssim 198 \mu\text{eV}$$

$$f_a \approx (2.9 - 6.0) \times 10^{10} \text{ GeV}$$

Linear ansatz :

$$237 \mu\text{eV} \lesssim m_a \lesssim 450 \mu\text{eV}$$

$$f_a \approx (1.3 - 2.4) \times 10^{10} \text{ GeV}$$

$$\rightarrow 95 \mu\text{eV} \lesssim m_a \lesssim 450 \mu\text{eV}$$

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Mass range due to multiple ansatz for ξ and q + stat. + syst. errors

Kenta Saikawa^a, Saikat Ghosh^a, Alejandro Vaquero^a, and Mathieu Kaltschmidt^a
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(Dated: October 16, 2024)

Saikawa et al 2024

$$450 \mu\text{eV} \geq m_a \geq 95 \mu\text{eV}$$

Fat-string pre-evolution

AMR

Axion mass prediction from adaptive mesh refinement cosmological lattice simulations

Joshua N. Benabou^{1,2}, Malte Buschmann³, Joshua W. Foster^{4,5} and Benjamin R. Safdi^{1,2}

Account for 100% DM abundance,
Mass range due to the range of q from fitting,
Axions from strings scaling regime + string-domain walls,

Benabou et al 2024

$$280 \leftarrow 65 \mu\text{eV} \geq m_a \geq 45 \mu\text{eV}$$

Buschmann et al 2021

$$180 \mu\text{eV} \geq m_a \geq 40 \mu\text{eV}$$

Thermal pre-evolution

Exact scaling behavior has a dramatic impact on axion mass

String number density per Hubble

$$\xi = \beta + \alpha \log \frac{m_r}{H} \quad \text{vs} \quad \xi \sim \mathcal{O}(1)$$

A counter-study claiming a constant scaling of a unity
Correia, Hindmarsh, Lizarraga, Lopes-Eiguren,
Rummukainen, Urrestilla, 2410.18064

Moving frame vs Rest frame

$$\xi_r = \xi \langle \gamma^{-1} \rangle$$

Spectral index of axion power spectrum

$$q = q_0 + q_1 \log \frac{m_r}{H} \quad , q = q_0 + \frac{q_1}{\log^2 \frac{m_r}{H}} \quad , q = \mathcal{O}(1)$$

✓ stronger disagreement in here

New independent scaling solution being consistent with ξ and q
would be helpful for clarifying disagreements

New scaling in string fluctuations

arXiv > hep-ph > arXiv:2411.08455

High Energy Physics – Phenomenology

[Submitted on 13 Nov 2024]

More Scalings from Cosmic Strings

Heejoo Kim, Minho Son

Observed a new scaling consistent with spectral index of axion power spectrum

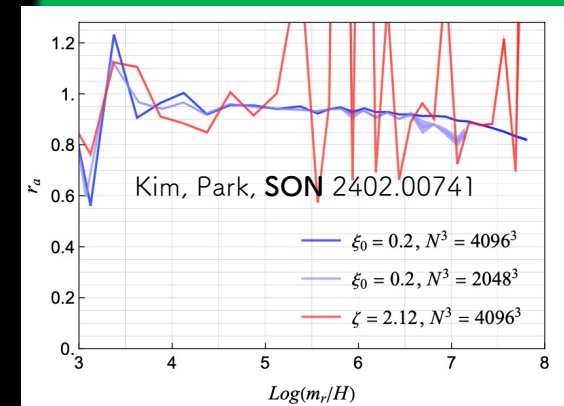
$$q = q_0 + q_1 \log \frac{m_r}{H}$$

New brand-new scaling consistent with q could be hinted from this observation

****Branching fraction of strings to axion**

$$r_a = \frac{\Gamma_a}{\Gamma_a + \Gamma_r}$$

$$R^{-4} \partial_t (R^4 \rho_a) \sim \Gamma_a$$



$$\rightarrow \Gamma = \Gamma_a + \Gamma_r \sim \Gamma_a$$

We speculate that axion power-law spectrum is originated from string power-law

Looking into Kim, SON, 2411.08455

String power spectrum

Long string

Self-avoiding 3D
random walks

Oscillations larger than $\sim H^{-1}$
is frozen in expanding Univ,
and those are well-described
by Random walk strings

$\sim H^{-1}$: correlation length

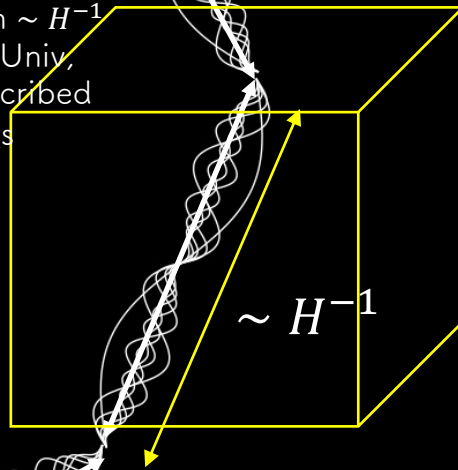


Looking into Kim, SON, 2411.08455

String power spectrum

Self-avoiding 3D random walks

Oscillations larger than $\sim H^{-1}$ is frozen in expanding Univ., and those are well-described by Random walk strings



We are interested in oscillations shorter than $\sim H^{-1}$ that can contribute to axions

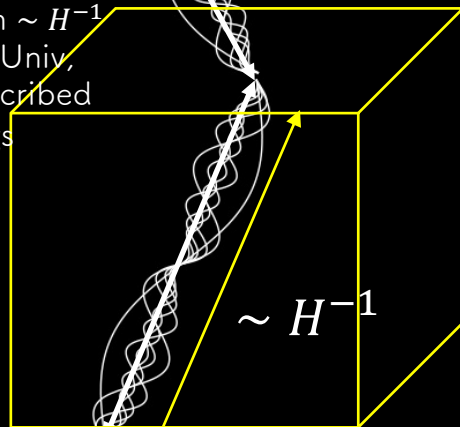
Looking into Kim, SON, 2411.08455

String power spectrum

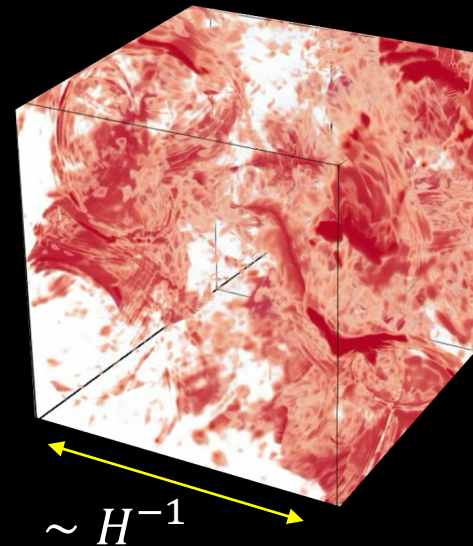
Speculation :
Axion power-law spectrum should be originated
from string power-law behavior

Self-avoiding 3D random walks

Oscillations larger than $\sim H^{-1}$
is frozen in expanding Univ.,
and those are well-described
by Random walk strings



Axion energy density (red color) from cosmic
strings inside one Hubble volume in real simulation



Spectra of
string fluctuations

Axion spectra

Looking into String power spectrum

$\vec{\gamma}(s)$

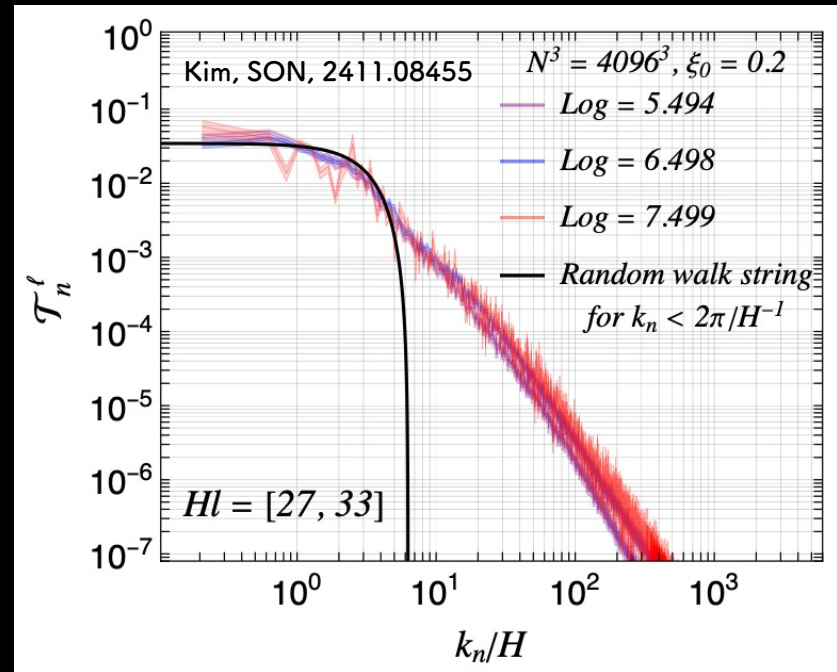
$$\frac{d\vec{\gamma}(s)}{ds} = \vec{t}(s) = \sum_{n=-\infty}^{\infty} \vec{T}_n^{\ell} e^{i\frac{2\pi n}{\ell}s}$$

$$\langle \vec{T}_n^{\ell} \cdot \vec{T}_{n'}^{\ell} \rangle = \mathcal{T}_n^{\ell} \delta_{(n+n')0}$$

Self-avoiding 3D random walks

Oscillations larger than $\sim H^{-1}$
is frozen, and those are well-
described by Random walk
strings

$\sim H^{-1}$



Assume string fluctuation follows the power law
similarly to axions

$$\mathcal{T}_n^{\ell} \sim \frac{1}{k^p}$$

String power spectrum

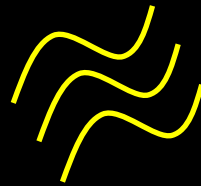
$$\vec{\gamma}(s)$$

$$\Gamma_a^{\text{str}}(k) \propto \frac{1}{k^p}$$

Kim, **SON**, 2411.08455

This work:
Explicitly analyze the spectra
of string fluctuations

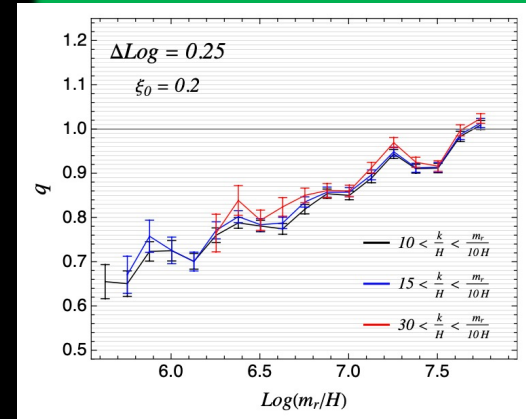
Radiation
into Axions



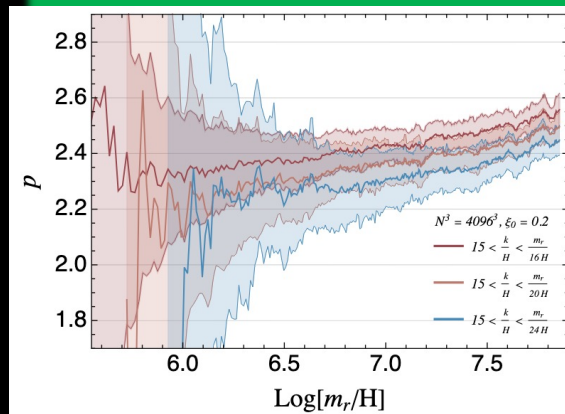
$$\Gamma_a \sim \frac{1}{k^q}$$

Traditional:
All literature analyze the axion
spectra away from string cores

Kim, Park, SON 2402.00741

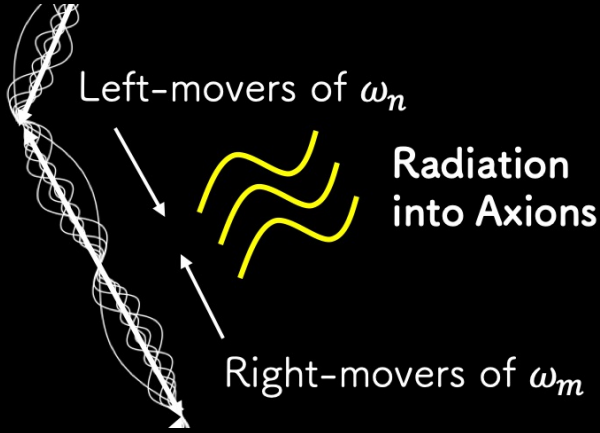


Kim, **SON**, 2411.08455



Question :
How q is related to p ?

Analytic connection between strings and axions



1. No Expansion of Universe
2. Almost straight string
3. Kalb-Ramond (KR) interaction in thin string Limit

$$\frac{\Gamma_a^{\text{str.}}(k)}{m_r^2 f_a^2} = 8\pi^3 \xi \left(\frac{m_r}{H}\right)^{-2} \int_0^\infty \frac{d(H\ell)}{H\ell_{\text{ave.}}} \frac{\rho(\ell)}{H} \sum_{n=1}^\infty n H \delta\left(k - \frac{2\pi n}{\ell}\right) \sum_{\substack{|m| < n, \\ m+n \text{ even}}} \mathcal{T}_{\frac{n+m}{2}}^\ell \mathcal{T}_{\frac{n-m}{2}}^\ell,$$

$$\Gamma_a^{\text{str}}(k = \frac{2\pi n}{\ell}) \propto n \sum_{m=1}^{n-1} \frac{1}{(n+m)^p (n-m)^p}$$

$$\mathcal{T}_n^\ell \sim \begin{cases} B_\ell \left(\frac{k_n}{H}\right)^{-p} & : k_{IR} < k < k_{UV} \\ 0 & : \text{otherwise} \end{cases}$$

$$\rightarrow \Gamma_a^{\text{str}}(k) \propto k^{2-2p} \times k^{p-1} = \frac{1}{k^{p-1}}$$

$$: \text{a few} \times (2k_{IR}) < k < k_{UV}$$

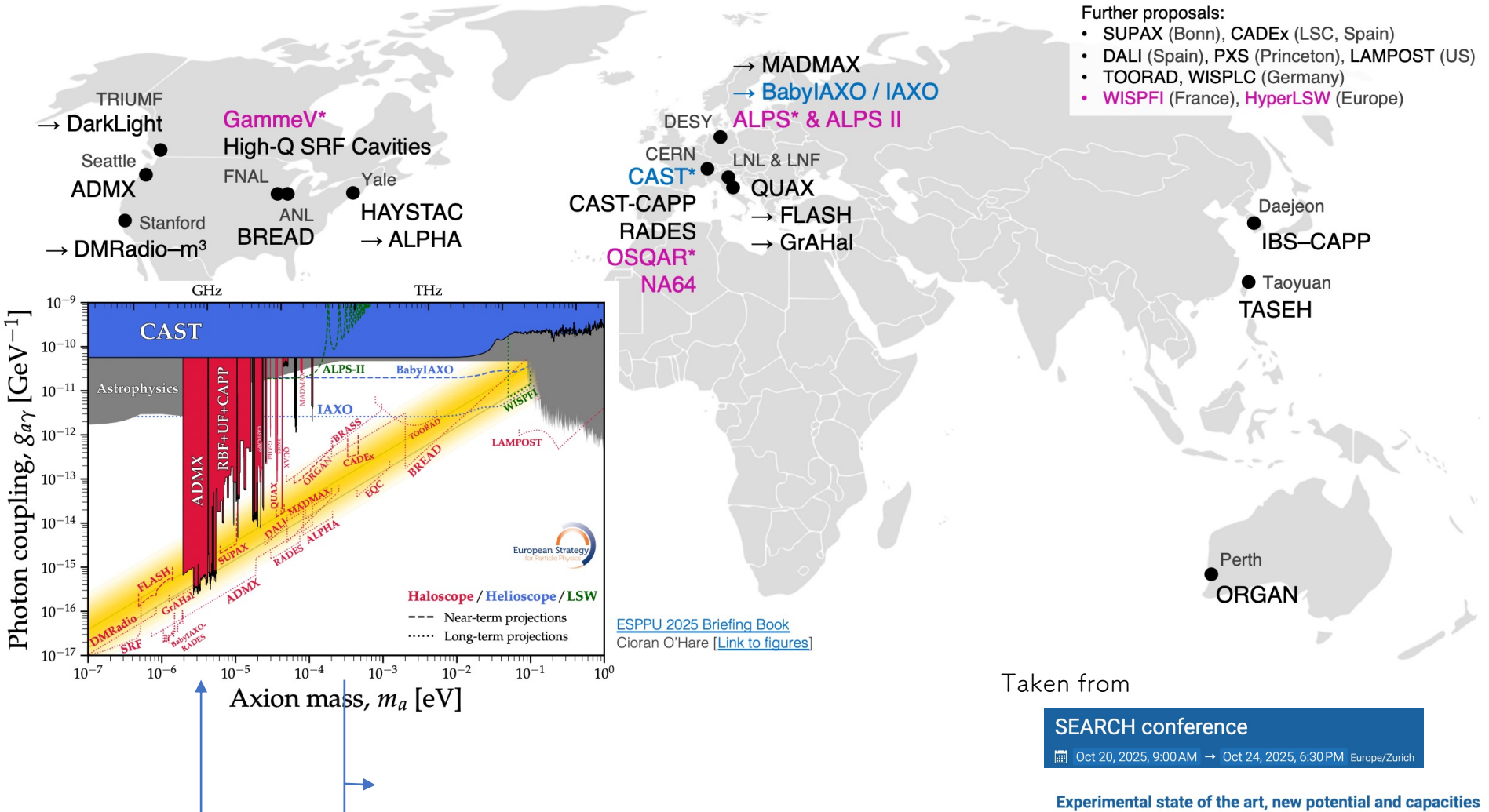


$$q = p - 1 - \Delta_{\text{non-pert}}^{\text{str.}}$$

Part missed in
our derivation

Low-mass Axion / ALP detection

Haloscopes (relic axions), Helioscopes (solar axions), Light shining through the wall (lab axions), '→' Future / proposed experiments, *Concluded exps (incomplete list)



Taken from

SEARCH conference

Oct 20, 2025, 9:00 AM → Oct 24, 2025, 6:30 PM Europe/Zurich

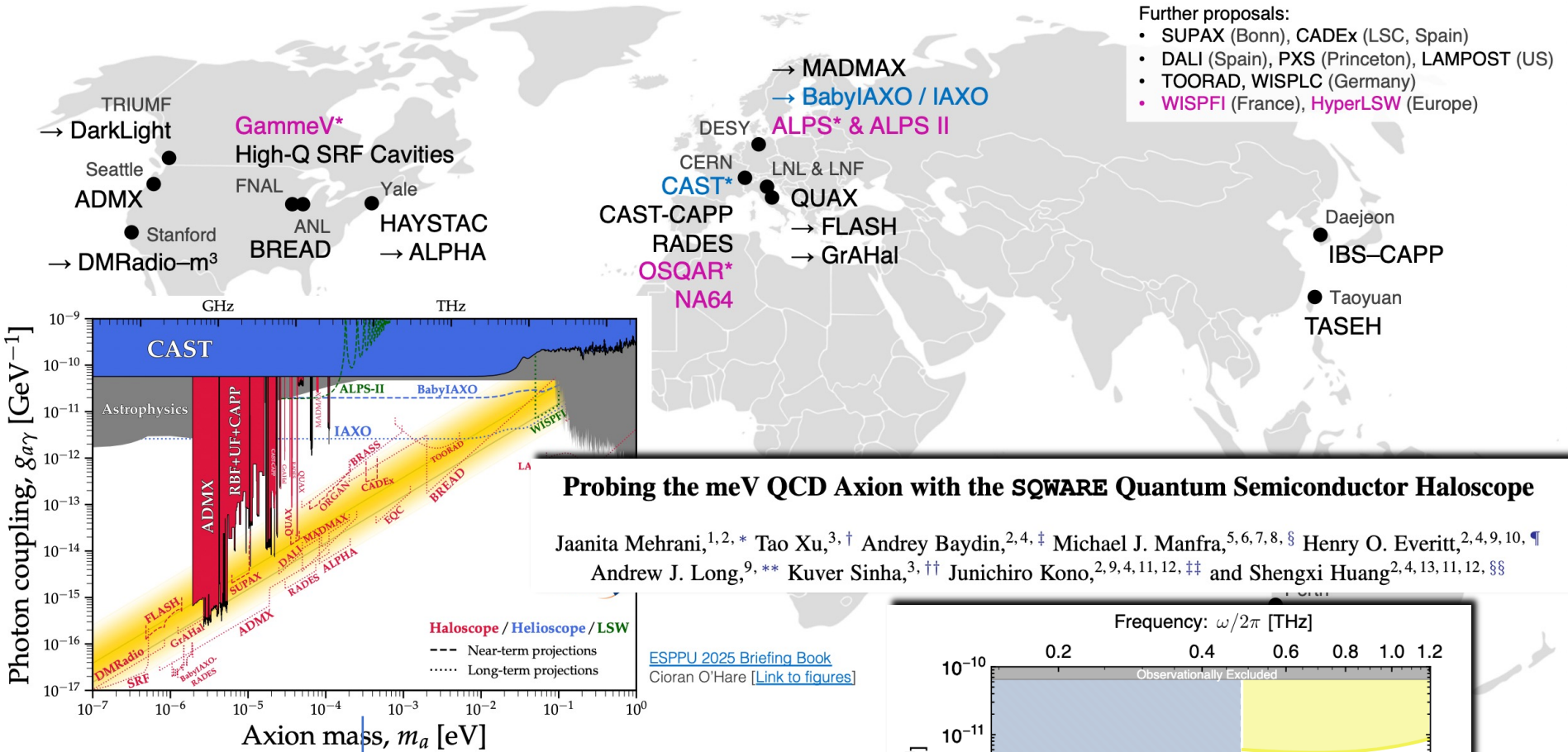
Experimental state of the art, new potential and capacities

Speaker: Andreas Hoecker (CERN)

* CAPP → DMAG (Dark Matter Axion Group)

Low-mass Axion / ALP detection

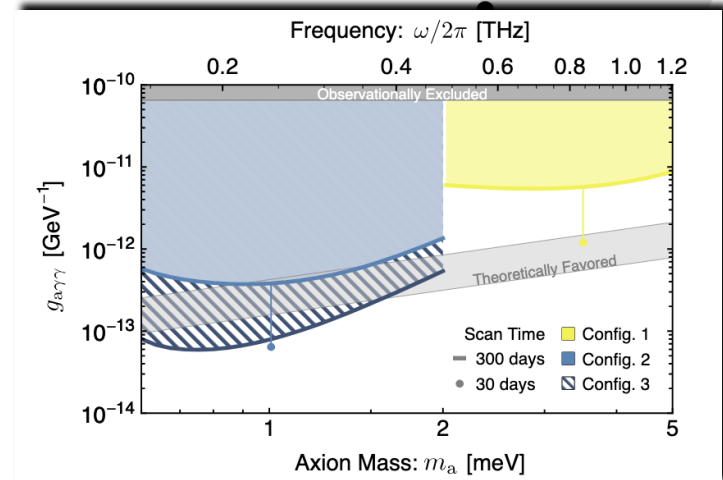
Haloscopes (relic axions), **Helioscopes** (solar axions), **Light shining through the wall** (lab axions), '→' Future / proposed experiments, *Concluded expts (incomplete list)



Probing the meV QCD Axion with the SQUARE Quantum Semiconductor Haloscope

Jaanita Mehrani,^{1,2,*} Tao Xu,^{3,†} Andrey Baydin,^{2,4,‡} Michael J. Manfra,^{5,6,7,8,§} Henry O. Everitt,^{2,4,9,10,¶} Andrew J. Long,^{9,**} Kuver Sinha,^{3,††} Junichiro Kono,^{2,9,4,11,12,‡‡} and Shengxi Huang^{2,4,13,11,12,§§}

ESPPU 2025 Briefing Book
Cioran O'Hare [\[Link to figures\]](#)



In near future

- ✓ Improve lattice resolution at least by 4x -> 16K each dimension

Technically huge jump. Super-challenging

Question : q is very close to touch the line of unity.

will q keep growing above unity or get saturated ? 16K should help for this.

Supported by
Static Lattice Simulations

VS

Supported by
AMR (Adaptive Mesh Refinement)

- ✓ Numerical testing the collapse mechanism (while solving axion quality) of string-domain wall system with $N > 1$ might be interesting

Question : is commercial method of gauged discrete symmetry actually working ?

Lu, Reece, Sun, 2312.07650

- ✓ The statistical analysis of the string time evolution should be useful to understand the interaction between axion and strings.

Question : any hint for better EFT description ?