

Linear-Time Computation of the Frobenius Normal Form for Symmetric Toeplitz Matrices via Graph-Theoretic Decomposition

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We introduce a linear-time algorithm for computing the Frobenius normal form (FNF) of symmetric Toeplitz matrices by utilizing their inherent structural properties through a graph-theoretic approach. Previous results of the authors established that the FNF of a symmetric Toeplitz matrix is explicitly represented as a direct sum of symmetric irreducible Toeplitz matrices, each corresponding to connected components in an associated weighted Toeplitz graph. Conventional matrix decomposition algorithms, such as Storjohann's method (1998), typically have cubic-time complexity. Moreover, standard graph component identification algorithms, such as breadth-first or depth-first search, operate linearly with respect to vertices and edges, translating to quadratic-time complexity solely in terms of vertices for dense graphs like weighted Toeplitz graphs. Our method uniquely leverages the structural regularities of weighted Toeplitz graphs, achieving linear-time complexity strictly with respect to vertices through two novel reductions: the α -type reduction, which eliminates isolated vertices, and the β -type reduction, applying residue class contractions to achieve rapid structural simplifications while preserving component structure. These reductions facilitate an efficient recursive decomposition process that yields linear-time performance for both graph component identification and the resulting FNF computation. This work highlights how structured combinatorial representations can lead to significant computational gains in symbolic linear algebra.

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