

Probing CP violation for electroweak baryogenesis in 2HDM



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JHEP 10 (2024) 041, arXiv: 2408.06863 [hep-ph]

Collaborators: Shinya Kanemura

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Introduction

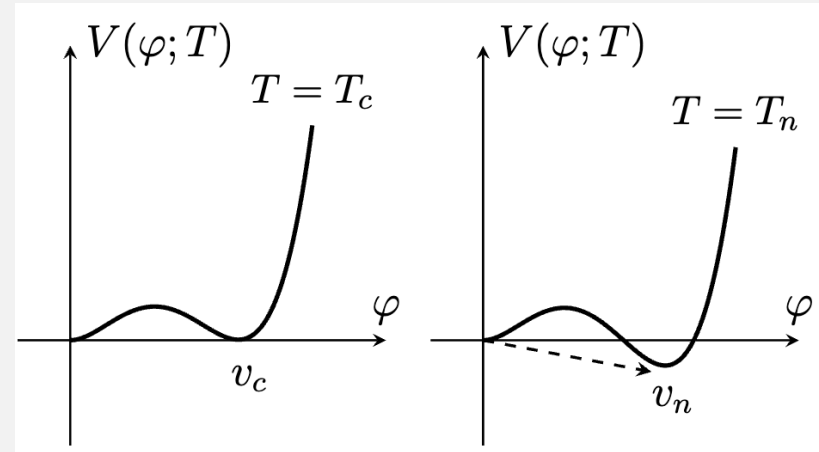
- **SM was established by the Higgs discovery in 2012.** ATLAS, CMS (2012)
- **Some remaining problems**
e.g., baryon asymmetry of the Universe (BAU) $\eta_B^{obs} = \frac{n_B - n_{\bar{B}}}{s} \simeq 8 \times 10^{-11}$ PDG (2022)
- **Sakharov three conditions for baryon asymmetry** Sakharov (1967)
 - ① Baryon # violation
 - ② C and CP violation
 - ③ Departure from thermal equilibrium
- **A promising scenario for baryogenesis: Electroweak baryogenesis** Kuzmin, Rubakov and Shaposhnikov (1985)
 - ① Sphaleron process
 - ② EW interaction with CP phase
 - ③ EW first order phase transition

Electroweak baryogenesis

- Expanding bubble walls are created at first order PT.

Coleman, Callan and Coleman (1977)
and many works

First order PT is realized
by tunneling process (vacuum decay).



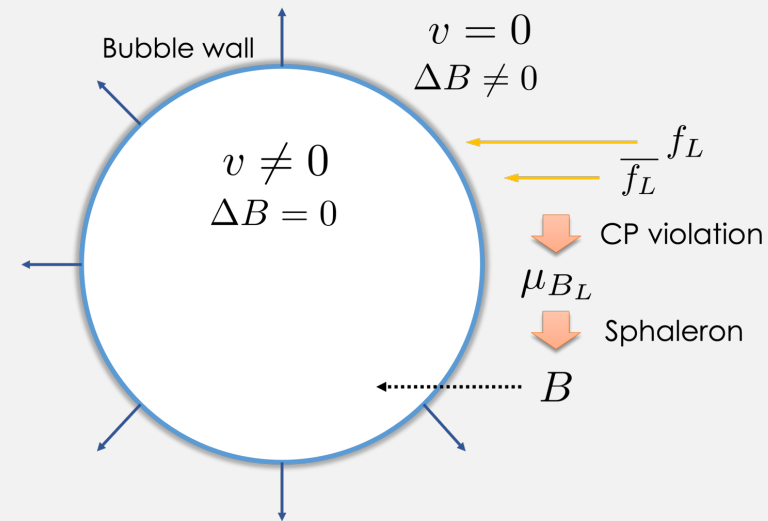
- Non-equilibrium sphaleron process around the bubble wall

Outside : baryon number violated
Inside : baryon number conserved

$$\Gamma_{\text{sph}}^{\text{brk}} \propto e^{-v/T} \quad \text{Moore (1999)}$$

- Generated baryon number is conserved inside the bubble.

$$\Gamma_{\text{sph}}^{\text{brk}}(T_n) < H(T_n) \Rightarrow v_n/T_n \gtrsim 1 \rightarrow \text{Strongly first order PT}$$

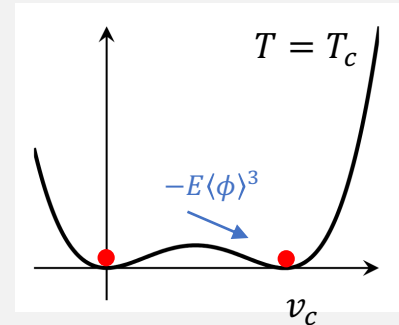


Is EWBG possible in the SM ?

• Difficulties in the Standard Model

- SM cannot realize strongly first order EWPT.

$$V(\langle\phi\rangle, T) = \frac{1}{2}m_T^2\langle\phi\rangle^2 - ET\langle\phi\rangle^3 + \frac{\lambda}{4}\langle\phi\rangle^4 + \dots \quad E \simeq \frac{1}{4\pi v^3} (2m_W^3 + m_Z^3)$$



$$\frac{v_n}{T_n} \sim \frac{v_c}{T_c} \simeq \frac{2E}{\lambda} = \frac{4Ev^2}{m_h^2} \gtrsim 1 \quad \rightarrow \quad m_h \lesssim \sqrt{4Ev^2} \simeq 48 \text{ GeV}$$

Shaposhnikov (1987)
Dine et al. (1992), (1992)

Crossover like for $m_h \gtrsim 60 \text{ GeV}$ (lattice results)

Kajantie et al. (1996);
D'Onofrio and Rummukainen (2016);

- SM Jarlskog invariant (from CKM matrix)

$$J_{CP} = \text{Im}[V_{ud}V_{us}^*V_{cs}V_{cs}^*] = 3.12 \times 10^{-5} \quad \text{PDG (2024)}$$



$$\eta_B < O(10^{-26})$$

Gavela et al. (1994);
Huet and Sather (1995);

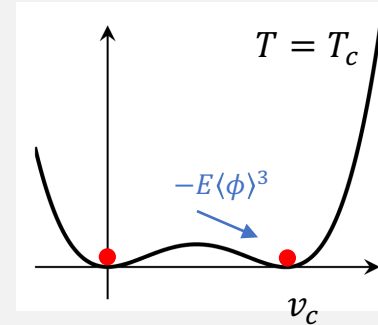
CPV and first order EWPT in 2HDM

- EWPT can be changed by new physics effects.

e.g.) Two Higgs doublet model (2HDM)

$$\mathcal{L} \sim -\lambda \phi_{\text{SM}}^\dagger \phi_{\text{SM}} \phi^\dagger \phi \rightarrow E \simeq \frac{1}{4\pi v^3} (2m_W^3 + m_Z^3 + (\lambda v^2)^{3/2})$$

Funakubo et al. (1994); Davies et al. (1994); Cline and Lemieux (1997) and more



- New CP violation in new physics

e.g.) CPV in 2HDM (SU(2) doublets: Φ_1 and Φ_2)

- Relative phase b/w Φ_1 and Φ_2
- New interaction b/w Φ_2 and SM fermions

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Progress of Theoretical Physics, Vol. 49, No. 2, February 1973

CP-Violation in the Renormalizable Theory of Weak Interaction

Makoto KOBAYASHI and Toshihide MASKAWA

Department of Physics, Kyoto University, Kyoto

(Received September 1, 1972)

Constraints and testability

- New physics effects making EWPT first order

e.g.) Triple Higgs coupling

$$\lambda_{hhh} \simeq h \text{---} \lambda_{\text{SM}} \begin{array}{c} \nearrow h \\ \searrow h \end{array} + \text{---} \lambda \begin{array}{c} \nearrow \phi \\ \searrow \lambda \end{array} \dots$$

CMS (2022), ATLAS (2023),
de Blas *et al.* (2020)

	LHC	HL-LHC	ILC(1TeV)	FCC-ee/eh/hh
$\frac{\Delta\lambda_{hhh}}{\lambda_{hhh}}$	~600%	~50%	~10%	~5%

- O(10)% deviation for first order EWPT [Kanemura, Okada and Senaha \(2005\), and more](#)

e.g.) Gravitational waves signatures

[Grojean and Servant \(2007\),](#)
[Kakizaki, Kanemura, and Matsui \(2015\), and more](#)

- CP violating observables as a probe of EWBG

- Electric dipole moments

Relation b/w EWBG and EDM

[Aiko, Endo, Kanemura and YM, JHEP 07 \(2025\) 236](#)

EDMs	Current bounds	Expected limits
Electron	$4.1 \times 10^{-30} \text{ e cm}$ JILA (2023)	$O(10^{-33}) \text{ e cm}$ Vutha, et al. (2018)
Neutron	$1.8 \times 10^{-26} \text{ e cm}$ Abel, et al. (2020)	$O(10^{-28}) \text{ e cm}$ nEDM (2019)
Proton	$2.1 \times 10^{-25} \text{ e cm}$ Sahoo (2017)	$O(10^{-29}) \text{ e cm}$ Alarcon, et al. (2022)

- Direct CP violation in $H^\pm$ decays via loop-induced $H^\pm W^\mp Z$ vertex

Probe of CPV in the Higgs sector [Kanemura and YM, JHEP 10 \(2024\) 041](#)

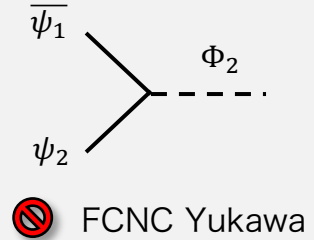
Outline

- Introduction ✓
- Electroweak baryogenesis and EDM
- $H^\pm W^\mp Z$ vertex probing CP violation in 2HDM

EDM predicted in 2HDM

- Two regimes to avoid FCNC

Discrete Z_2 symmetry: $\Phi_2 \rightarrow -\Phi_2$



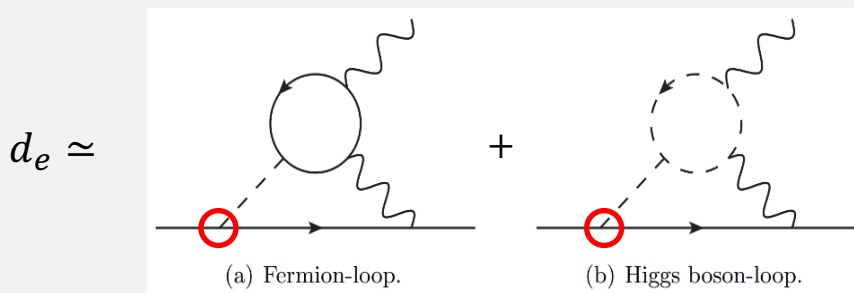
	Softly broken Z_2 2HDM	General 2HDM
FCNC Yukawa	Forbidden	Needs to be small (e.g. MFV)
# of new CP phase	1	Many (more than 10)
eEDM to explain BAU	$O(10^{-28}) e \text{ cm}$	Depends on Yukawa structure

Fromme et al. (2006); Dorsch et al. (2017);
Basler et al. (2021), and more

Fuyuto, Hou, and Senaha (2019);
Enomoto, Kanemura and YM, JHEP 01 (2022) 104,
Enomoto, Kanemura and YM, JHEP 09 (2022) 121, and more

- What is the leading contributions for EDM ?

Fuyuto, Hou and Senaha (2019);
Kanemura Kubota and Yagyu (2020); and more



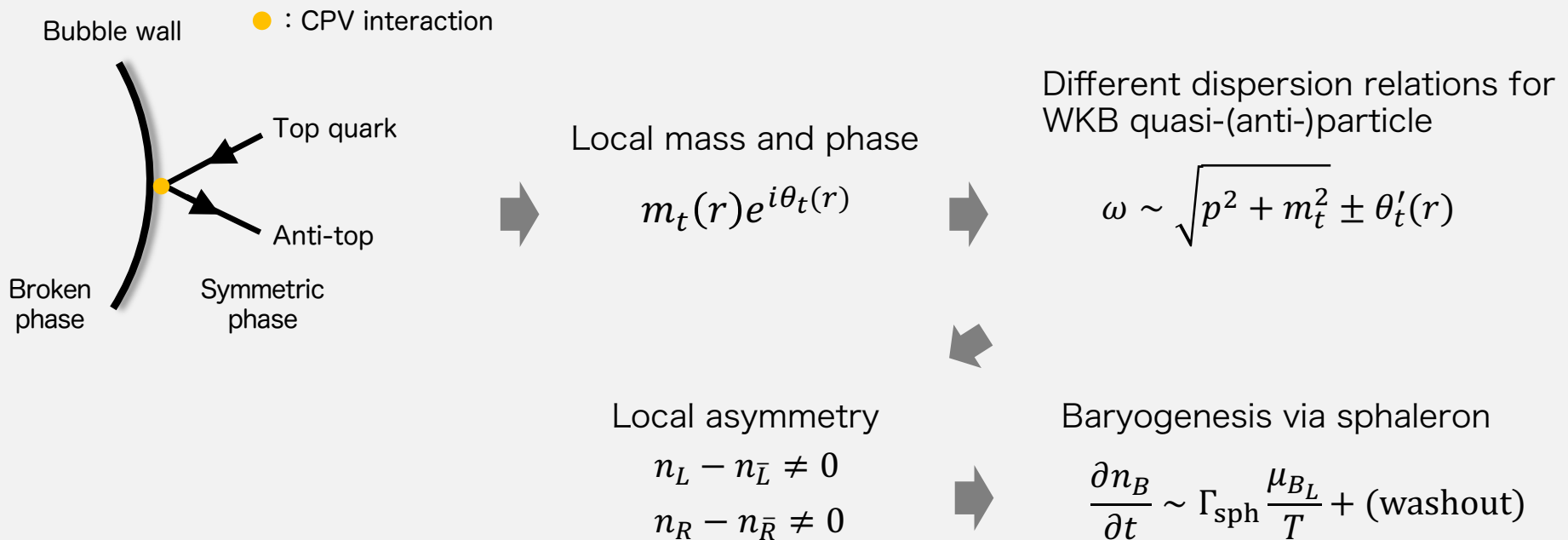
○ : couplings b/w electron and additional Higgs

- However, this CPV coupling is not important for BAU (Why? -> see next)

Top transport scenario

- CPV force acting on top and anti-top $\propto y_t$

Cline, Joyce, and Kainulainen (2000);
Fromme and Huber (2007); and more



- Which EDMs constrain this scenario in g2HDM, and how?

General two Higgs doublet model

- **Most general potential** Higgs basis

$$\Phi_1 = \begin{pmatrix} G^\pm \\ \frac{1}{\sqrt{2}}(v + h_1 + iG^0) \end{pmatrix} \quad \Phi_2 = \begin{pmatrix} H^\pm \\ \frac{1}{\sqrt{2}}(h_2 + ih_3) \end{pmatrix}$$

$$V = -\mu_1^2 \Phi_1^\dagger \Phi_1 + M^2 \Phi_2^\dagger \Phi_2 - (\mu_3^2 \Phi_1^\dagger \Phi_2 + \text{h.c.})$$

Davidson and Haber (2005)

$$+ \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1)$$

$$+ \left\{ \left(\frac{1}{2} \lambda_5 \Phi_1^\dagger \Phi_2 + \lambda_6 \Phi_1^\dagger \Phi_1 + \lambda_7 \Phi_2^\dagger \Phi_2 \right) \Phi_1^\dagger \Phi_2 + \text{h.c.} \right\} \quad (\mu_3^2, \lambda_5, \lambda_6, \lambda_7 \in \mathbb{C})$$

- **Most general Yukawa sector**

$$\mathcal{L}_Y = - \sum_{k=1,2} (\overline{Q}_L Y_{k,u}^\dagger \tilde{\Phi}_k u_R + \overline{Q}_L Y_{k,d} \Phi_k d_R + \overline{L}_L Y_{k,l} \Phi_k e_R + \text{h.c.})$$

$$Y_{1,u} = \text{diag}(y_u, y_c, y_t)$$

$$Y_{1,d} = \text{diag}(y_d, y_s, y_b)$$

$$Y_{1,l} = \text{diag}(y_e, y_\mu, y_\tau)$$

- Y_2 is general complex matrix

e.g.) Up type

$$Y_{2,u} = \begin{pmatrix} \rho_{uu} & \rho_{cu} & \rho_{tu} \\ \rho_{uc} & \rho_{cc} & \rho_{tc} \\ \rho_{ut} & \rho_{ct} & \rho_{tt} \end{pmatrix}$$

General two Higgs doublet model

- Stationary conditions and mass spectra

$$\frac{\partial V}{\partial h_i} = 0 \Leftrightarrow \mu_1^2 = \frac{1}{2}\lambda_1 v^2, \quad \mu_3^2 = \frac{1}{2}\lambda_6 v^2$$

$$\frac{\partial^2 V}{\partial h_i \partial h_j} = \mathcal{M}_{ij}^n = \begin{pmatrix} \lambda_1 v^2 & \text{Re}[\lambda_6] v^2 & -\text{Im}[\lambda_6] v^2 \\ \text{Re}[\lambda_6] v^2 & M^2 + \frac{1}{2}(\lambda_3 + \lambda_4 + \lambda_5) v^2 & -\frac{1}{2}\text{Im}[\lambda_5] \\ -\text{Im}[\lambda_6] v^2 & -\frac{1}{2}\text{Im}[\lambda_5] & M^2 + \frac{1}{2}(\lambda_3 + \lambda_4 - \lambda_5) v^2 \end{pmatrix}$$

$$m_{H^\pm}^2 = M^2 + \frac{1}{2}\lambda_3 v^2$$

- Mass eigenstate for neutral scalar bosons

125 GeV Higgs

Orthogonal matrix R $\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = R \begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix}, \quad R^T \mathcal{M}^n R = \text{diag}(m_{H_1}, m_{H_2}, m_{H_3})$

- Rephasing invariants in the model $\Phi_2 \rightarrow e^{i\theta} \Phi_2$

Potential: $\text{Im}[\lambda_5^* \lambda_6^2], \text{Im}[\lambda_5^* \lambda_7^2], \text{Im}[\lambda_6^* \lambda_7]$

Yukawa: $\text{Im}[\lambda_5 \rho_{tt}^2], \text{Im}[\lambda_6 \rho_{tt}], \text{Im}[\lambda_7 \rho_{tt}]$ (and other ρ_{ij} related invariants)

Essential CP violation

- **Discovered 125GeV Higgs is SM like.** ATLAS, Nature (2022); CMS, Nature (2022);

e.g.) $H_1 ZZ$ coupling $\kappa_Z \simeq 1 \Rightarrow |\lambda_6| \ll 1$

$\Rightarrow \text{Im}[\lambda_5^* \lambda_7^2], \text{Im}[\lambda_5 \rho_{tt}^2], \text{Im}[\lambda_7 \rho_{tt}]$

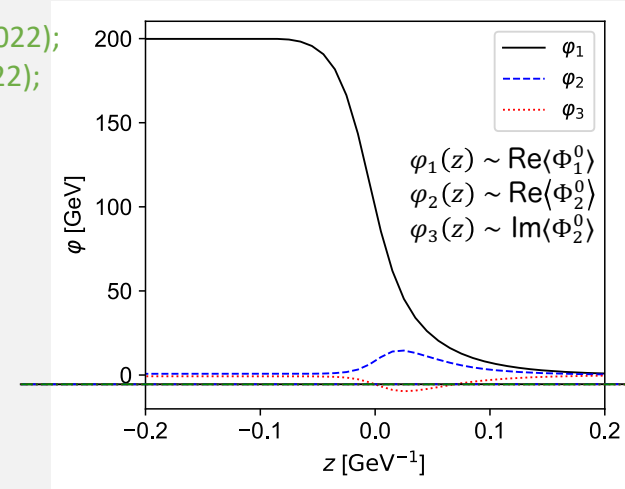
- **CPV force as a function of VEVs φ**

$$S_{\text{CPV}} \propto \frac{y_t |\rho_{tt}|}{2} \{ (\varphi_1 \varphi_2' - \varphi_2 \varphi_1') \sin(\arg[\rho_{tt}]) + (\varphi_3 \varphi_1' - \varphi_1 \varphi_3') \cos(\arg[\rho_{tt}]) \}$$

- If $\lambda_6 \simeq \lambda_7 \simeq 0$, tree level potential approximately has $\mathbb{Z}_2$ symmetry ($\Phi_2 \rightarrow -\Phi_2$).
 $\Rightarrow$ VEV of Φ_2 is suppressed, $\varphi_2, \varphi_3 \ll 1$.
- For sufficient BAU, $\text{Im}[\lambda_7 \rho_{tt}]$ is necessary.

- **Minimal setup** $\rho_{ij} = 0$ (except for ρ_{tt}) and $\lambda_4 = \lambda_5 = \lambda_6 = 0$
 $(\lambda_4 = \lambda_5 \text{ is for T parameter})$

$\Rightarrow m_{H_2} = m_{H_3} = m_{H^\pm} \equiv m_\Phi$ One available CP phase: $\arg[\lambda_7 \rho_{tt}]$



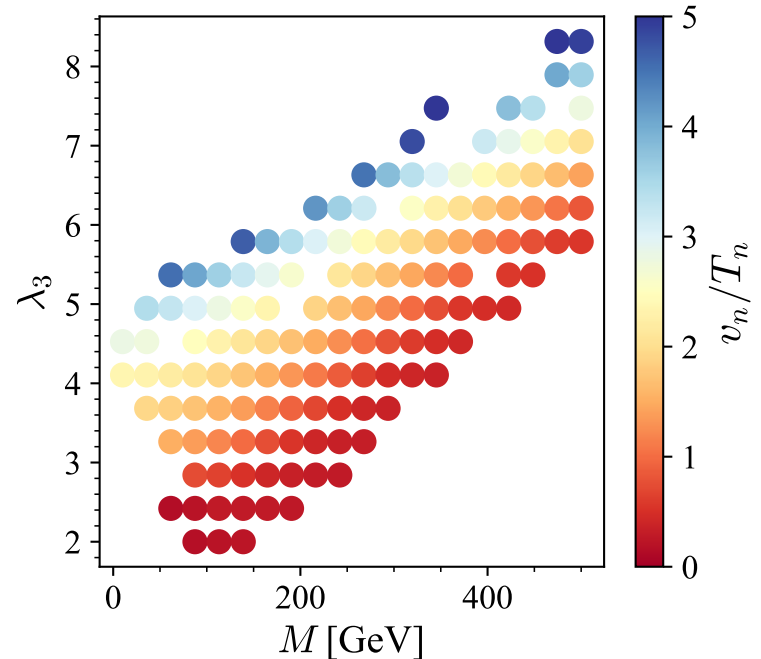
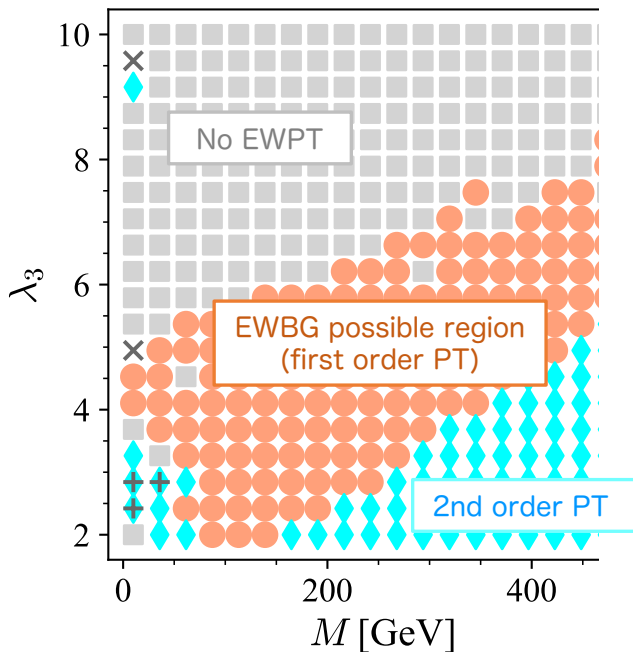
Electroweak phase transition

• Several fates of the vacuum

(In analyses, we used Cosmotransitions [Wainwright \(2011\)](#))

c.f.) $m_{H^\pm}^2 = M^2 + \frac{1}{2}\lambda_3 v^2$ and $m_{H_2} = m_{H_3} = m_{H^\pm}$

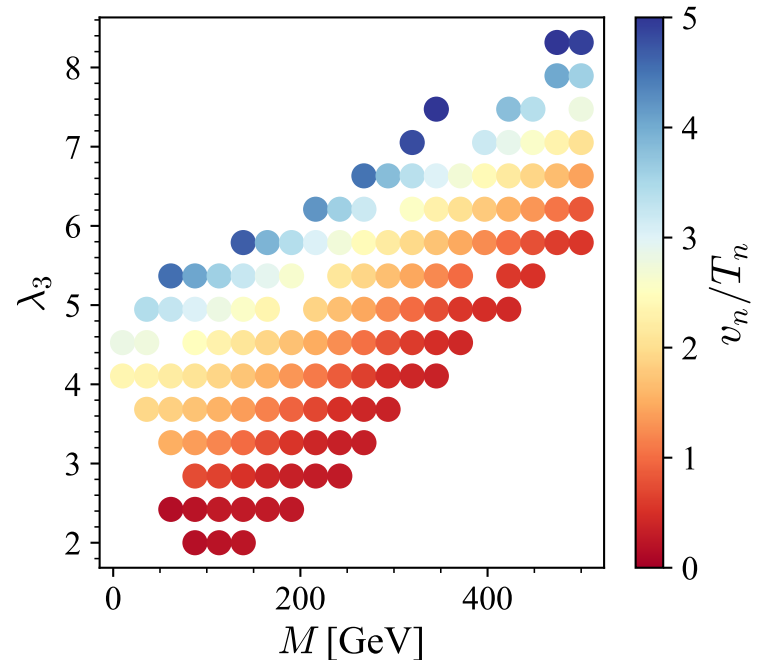
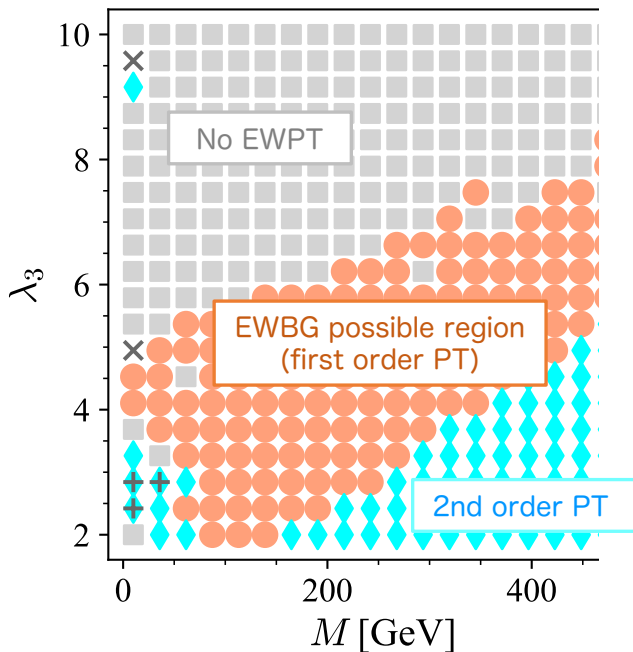
- (a) **Pink filled circle:** The phase transition from phase A to phase B is first order, B is the vacuum at $T = 0$, B is the electroweak vacuum, and the VEV in A is *below* than 1 GeV.
- (b) **Gray +:** The phase transition $A \rightarrow B$ is first order, B is the vacuum at $T = 0$, B is the electroweak vacuum, and the VEV in A is *larger* than 1 GeV.
- (c) **Gray ×:** The phase transition $A \rightarrow B$ is first order, B is the vacuum at $T = 0$, and B is *not* the electroweak vacuum.
- (d) **Black dot:** The phase transition $A \rightarrow B$ is first order, and B is *not* the vacuum at $T = 0$.
- (e) **Blue diamond:** The phase transition $A \rightarrow B$ is second order.
- (f) **Gray box:** No phase transition is returned, including the case of $\Gamma/H^4 < 1$.



Electroweak phase transition

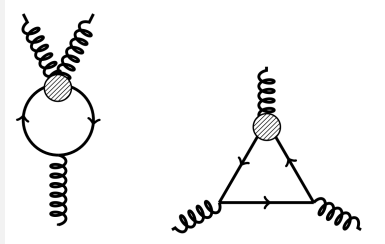
- For successful EWBG, at least,
 - $O(1)$ large λ_3 coupling
 - Satisfying $v_n/T_n > 1$
 are needed.

c.f.) $m_{H^\pm}^2 = M^2 + \frac{1}{2}\lambda_3 v^2$ and $m_{H_2} = m_{H_3} = m_{H^\pm}$



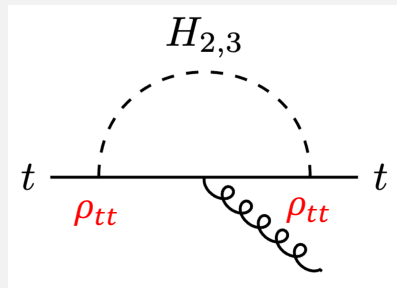
EDMs in the minimal setup

- Top chromo EDM induces Weinberg op. and light fermion EDMs by RGE running.



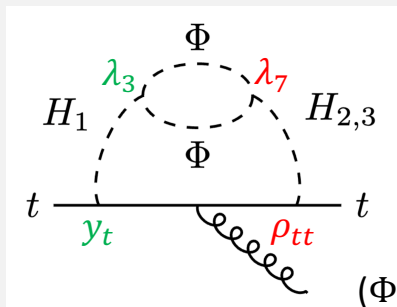
Kamenik et al. (2012);
Hisano, Tsumura and Yang (2012);
Kaneta et al. (2023);
and more works

- At 1 loop level
e.g.)



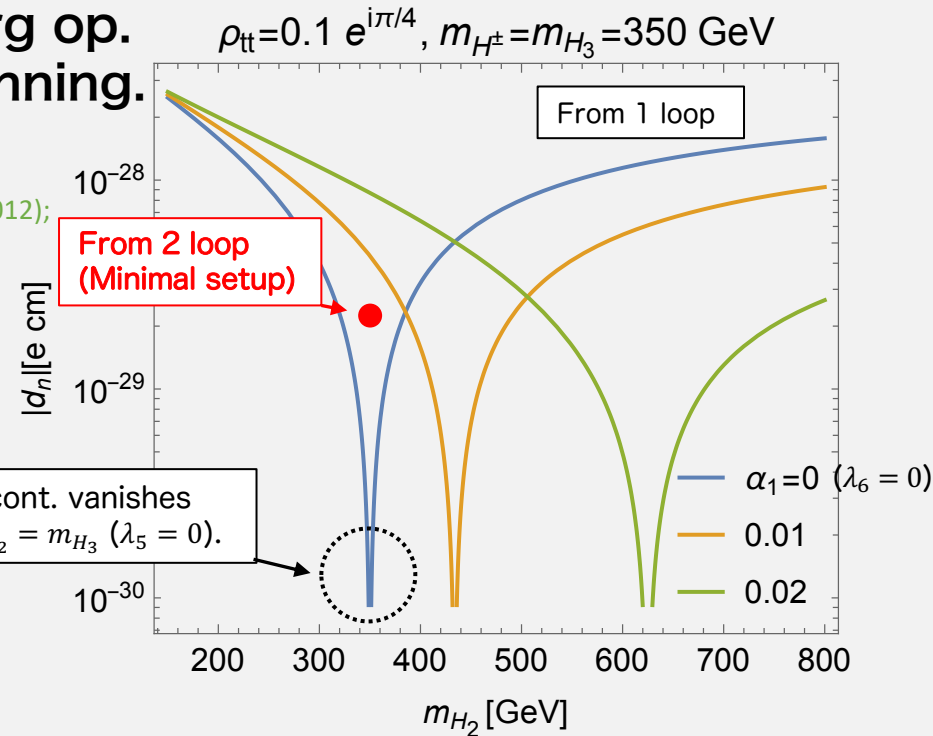
$$\propto \text{Im}[\lambda_5 \rho_{tt}^2]$$

- With the minimal setup, 2 loop diagrams are leading.
e.g.)



$$\propto \text{Im}[\lambda_7 \rho_{tt}]$$

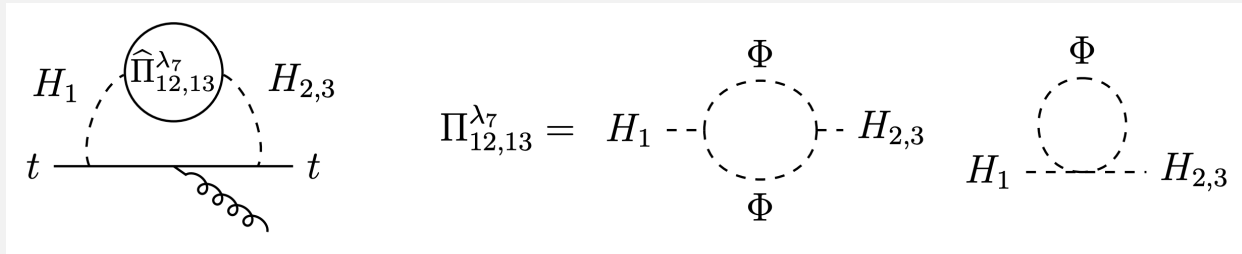
($\Phi = H_2, H_3, H^\pm$)



At the red point,
 $\lambda_7 = e^{i\pi/4}, -\mu_2^2 = 30^2 \text{ GeV}^2$ are taken.

Renormalization

- UV divergence in mixing-self-energy diagrams



- Effective potential renormalization

$$\left. \frac{\partial V}{\partial \varphi_i} \right|_{\varphi=v_{EW}} = 0, \quad \left. \frac{\partial^2 V}{\partial \varphi_i \partial \varphi_j} \right|_{\varphi=v_{EW}} = \mathcal{M}_{ij} \quad \Rightarrow \quad \begin{aligned} \Gamma_i^{(1)}(p^2=0) + \delta\Gamma_i^{(1)} &= 0 \\ \Gamma_{ij}^{(2)}(p^2=0) + \delta\Gamma_{ij}^{(2)} &= 0 \end{aligned}$$

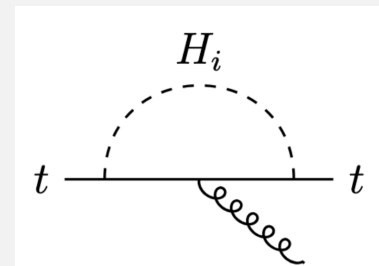
- Other renormalization schemes? e.g.) $\overline{\text{MS}}$ scheme $d_t^{\overline{\text{MS}},(2)} \stackrel{(?)}{=} d_t^{\text{EP},(2)} + \Delta d_t^{(2)}$

• Scheme conversion $\lambda_6^{\overline{\text{MS}},(1)} = \lambda_6^{\text{EP},(1)} + \Delta\lambda_6^{(1)}$

$\Rightarrow \lambda_6^{\text{EP},(1)} = 0$ does not mean $\lambda_6^{\overline{\text{MS}},(1)} = 0$.

• Finally, $d_t^{\overline{\text{MS}},(2)} = d_t^{\text{EP},(2)} + \Delta d_t^{(2)} - \Delta d_t^{(2)} + O(\hbar^3)$

from one-loop diagram



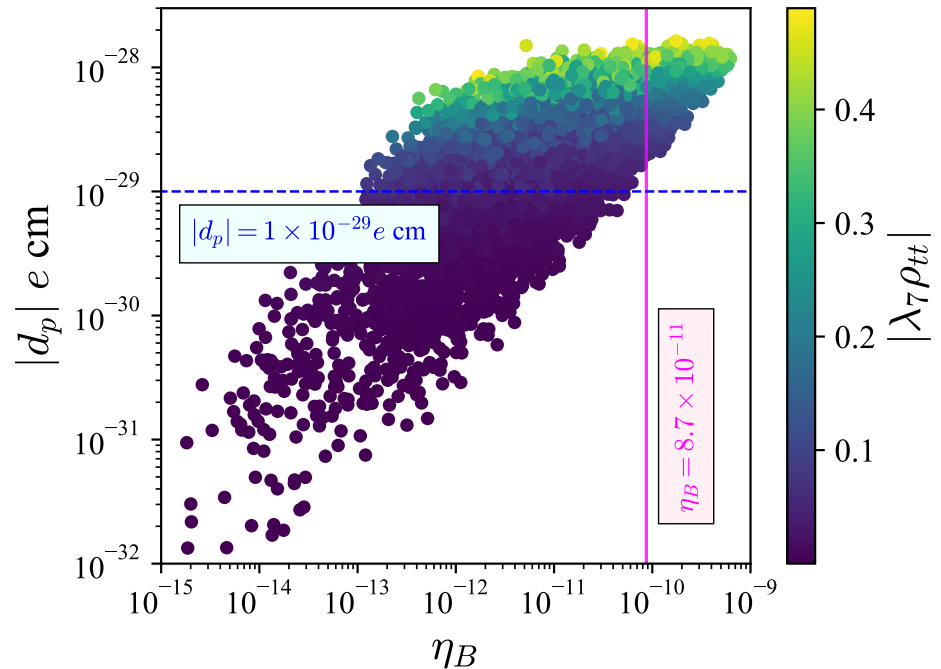
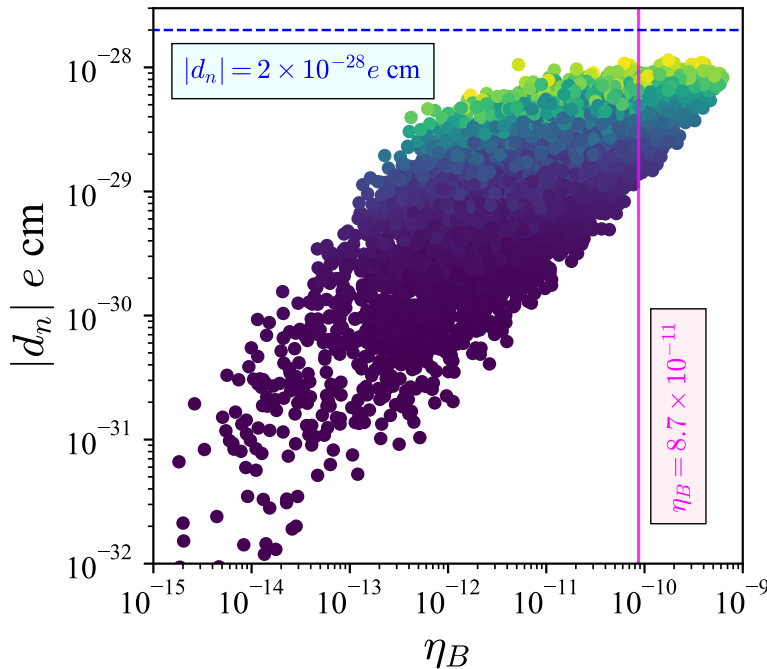
Correlation b/w EDMs and BAU

- Scanning parameter space

$$m_\Phi = [200, 500] \text{ GeV}, \mu_2^2 = [-m_\Phi^2, 0], |\rho_{tt}| = [0, 0.5]$$

$$|\lambda_7| = [0, 1], \lambda_2 = [0, 1], \arg[\lambda_7 \rho_{tt}] = -\pi/2, v_w = [0.1, 1/\sqrt{3}]$$

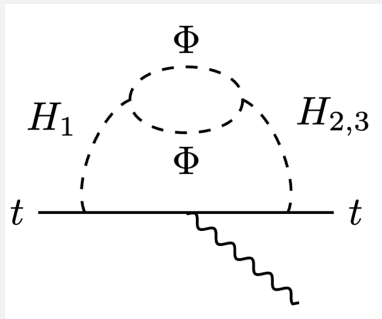
- Neutron and proton EDMs



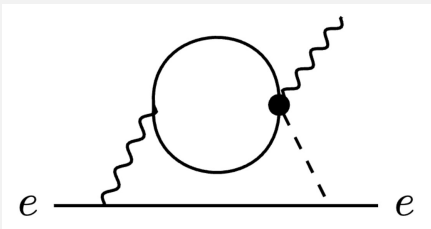
Correlation b/w EDMs and BAU

• Electron EDM induced by top EDM

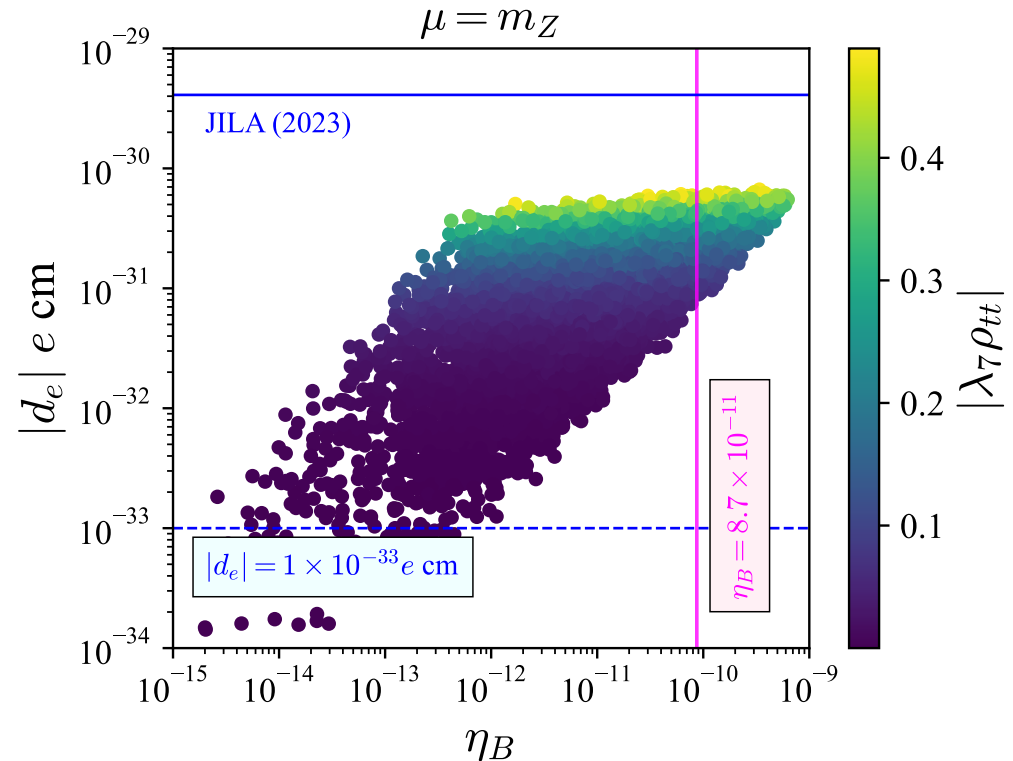
- Dipole operators for top are induced below $\Lambda \simeq m_\Phi$.



- Matching to d_e at $\mu \sim m_Z$, where top, Higgs, W and Z are decoupled.



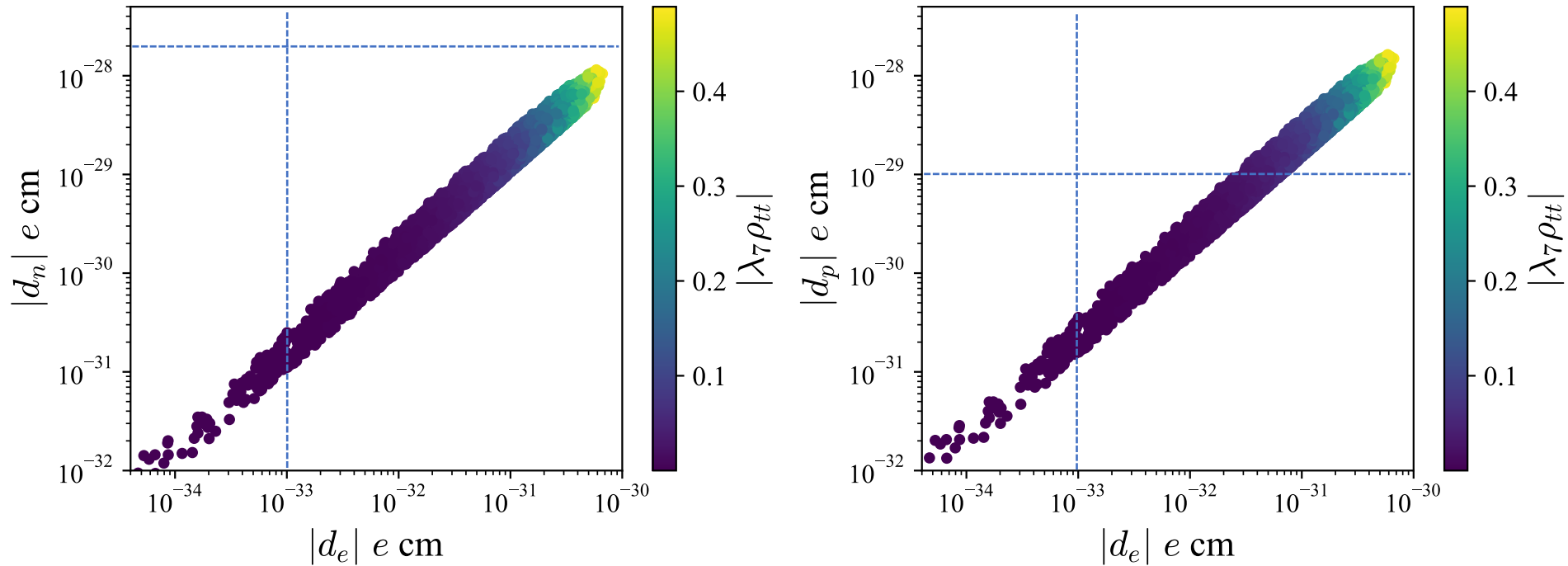
Cirigliano et al. (2016)
Fuyuto and Ramsey-Musolf (2017)



- Small effects by other couplings (e.g. ρ_{ee}) can change EDMs, but not η_B very much.

Correlation among EDMs

- Strong correlation among EDMs (dashed: future prospect bounds)

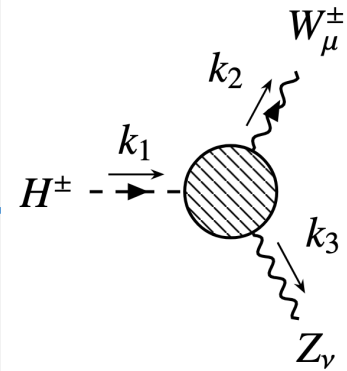


- All of CPV quantities are correlated by $\text{Im}[\lambda_7 \rho_{tt}]$.
⇒ Characteristic prediction of our scenario

Outline

- Introduction ✓
- Electroweak baryogenesis and EDM ✓
- $H^\pm W^\mp Z$ vertex probing CP violation in 2HDM

$H^\pm W^\mp Z$ vertex



- Custodial symmetry in the SM**

$$\mathbb{M}_{\text{SM}} = (\tilde{\Phi}, \Phi) = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^- & \phi^0 \end{pmatrix}$$

$$\mathcal{L} = \text{Tr}[(D_\mu \mathbb{M}_{\text{SM}})^\dagger (D^\mu \mathbb{M}_{\text{SM}})] - V(\text{Tr}[\mathbb{M}_{\text{SM}}^\dagger \mathbb{M}_{\text{SM}}])$$

$$\mathbb{M}_{\text{SM}} \rightarrow L \mathbb{M}_{\text{SM}} R^\dagger \quad L \in \text{SU}(2)_L, R \in \text{SU}(2)_R \quad L=R \text{ symmetry} \rightarrow \text{Custodial symmetry}$$

Sikivie et al. (1980)

- ρ parameter (relation b/w W and Z bosons mass) $\rho = m_W^2 / m_Z^2 c_W^2 = 1$ (Tree level)

- An important vertex : $H^\pm W^\mp Z$ vertex**

Grifols and Mendez (1980)

- WZ system (two bosons system) $\sim \mathbf{3} \times \mathbf{3} = \mathbf{1}_S + \mathbf{3}_A + \mathbf{5}_S$

$\rho = 1$ models	Scalar reps. (L,R)	Quintuplet $H_5^\pm$	Triplet $H_3^\pm$	$H^\pm W^\mp Z$ vertex
Georgi-Machacek model	(3,3)	✓	✓	$H_5^\pm W^\mp Z$ (tree-level)
2HDM	(2,2)*2	✗	✓	$H_3^\pm W^\mp Z$ (loop-induced)

- Relation to CP violation (2HDM)**

Pomarol and Vega (1994)

- To have CP phase in the potential, the custodial symmetry must be broken.

Custodial symmetry

- **Bi-linear form** $\mathbb{M}_1 = (\tilde{\Phi}_1, \Phi_1), \mathbb{M}_2 = (\tilde{\Phi}_2, \Phi_2) \text{diag}(e^{-i\chi}, e^{i\chi})$

Pomarol and Vega (1994)

$$SU(2)_L: \mathbb{M}_i \rightarrow U(x)\mathbb{M}_i, \quad U(1)_Y: \mathbb{M}_i \rightarrow \mathbb{M}_i \exp(-ig'Y\alpha(x)\sigma_3)$$

Haber and Neil (2011)

- **Global transformation**
$$\begin{aligned} \mathbb{M}_1 &\rightarrow L\mathbb{M}_1 R^\dagger \\ \mathbb{M}_2 &\rightarrow L\mathbb{M}_2 R^\dagger \end{aligned} \quad L \in SU(2)_L, R \in SU(2)_R$$

- **After EWSB,** $\langle \mathbb{M}_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix}, \quad \langle \mathbb{M}_2 \rangle = 0$

$\Rightarrow \langle \mathbb{M}_1 \rangle$ and $\langle \mathbb{M}_2 \rangle$ are invariant

for transformation with $L = R$ **(Custodial symmetry)**

- **Gauge invariant quantities**

$$SU(2)_L \times U(1)_Y \text{ inv.} \quad \left[\begin{array}{l} \text{Tr}[\mathbb{M}_1^\dagger \mathbb{M}_1] = 2|\Phi_1|^2 \\ \text{Tr}[\mathbb{M}_2^\dagger \mathbb{M}_2] = 2|\Phi_2|^2 \\ \text{Tr}[\mathbb{M}_1^\dagger \mathbb{M}_2] = e^{-i\chi} \Phi_1^\dagger \Phi_2 + \text{h.c.} \\ \text{Tr}[\mathbb{M}_1^\dagger \mathbb{M}_2 \sigma_3] = e^{-i\chi} \Phi_1^\dagger \Phi_2 - \text{h.c.} \end{array} \right] \quad SU(2)_L \times SU(2)_R \text{ inv.}$$

Custodial symmetry

- Focus on Higgs potential

$$\begin{aligned}
 V = & \text{(global } SU(2)_L \times SU(2)_R \text{ inv. terms)} \\
 & + i \operatorname{Im}[\mu_3^2 e^{-i\chi}] \operatorname{Tr}[\mathbb{M}_1^\dagger \mathbb{M}_2 \sigma_3] - \frac{1}{4} (\lambda_4 - \operatorname{Re}[\lambda_5 e^{-2i\chi}]) \operatorname{Tr}[\mathbb{M}_1^\dagger \mathbb{M}_2 \sigma_3]^2 \\
 & + \frac{i}{2} \operatorname{Im}[\lambda_5 e^{-2i\chi}] \operatorname{Tr}[\mathbb{M}_1^\dagger \mathbb{M}_2] \operatorname{Tr}[\mathbb{M}_1^\dagger \mathbb{M}_2 \sigma_3] \\
 & + \frac{i}{2} (\operatorname{Im}[\lambda_6 e^{-i\chi}] \operatorname{Tr}[\mathbb{M}_1^\dagger \mathbb{M}_1] + \operatorname{Im}[\lambda_7 e^{-i\chi}] \operatorname{Tr}[\mathbb{M}_2^\dagger \mathbb{M}_2]) \operatorname{Tr}[\mathbb{M}_1^\dagger \mathbb{M}_2 \sigma_3]
 \end{aligned}$$

- Two custodial symmetry Gerard and Herquet (2007)

$\chi = 0, \pi$	$\lambda_4 = \lambda_5, \operatorname{Im}[\lambda_6] = \operatorname{Im}[\lambda_7] = 0$	(Usual)	Cf.) ($\lambda_6 = 0$ limit) $\lambda_4 + \lambda_5 \propto m_{H_2}^2 - m_{H^\pm}^2$
$\chi = \pi/2, 3\pi/2$	$\lambda_4 = -\lambda_5, \operatorname{Re}[\lambda_6] = \operatorname{Re}[\lambda_7] = 0$	(Twisted)	$\lambda_4 - \lambda_5 \propto m_{H_3}^2 - m_{H^\pm}^2$

- Relation with CP violation Pomarol and Vega (1994)

- Rephasing invariants in the potential $\lambda_5 \operatorname{Im}[\lambda_6^2], \lambda_5 \operatorname{Im}[\lambda_7^2], \operatorname{Im}[\lambda_6^* \lambda_7]$
- CP phases are in the potential $\Leftrightarrow$ Custodial symmetry is violated by the potential

$H^\pm \rightarrow W^\pm Z$ decay

- **Equivalence theorem for $m_{H^\pm}/m_W \gg 1$**

Cornwall, Levin and Tiktopoulos (1974); Lee, Quigg and Thacker (1977) and more

- ● violate usual custodial symmetry
- ● violate twisted custodial symmetry

- **Decay amplitude**

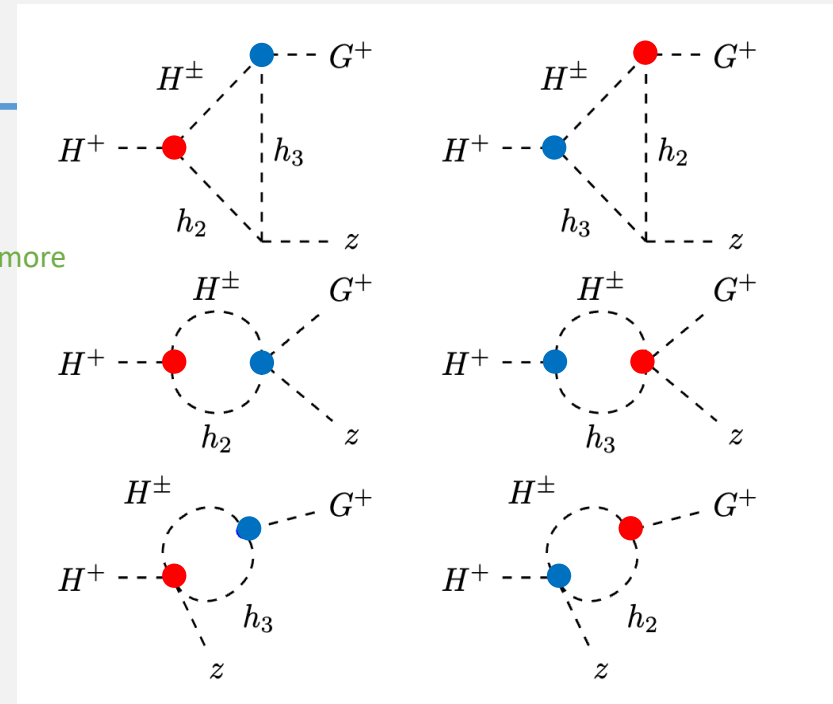
$$\mathcal{M} = \frac{i \text{Re}[\lambda_7]}{16\pi^2 v} (m_{H^\pm}^2 - m_{H_3}^2) \times (\text{loop functions}) \leftarrow \text{known part}$$

$$+ \frac{\text{Im}[\lambda_7]}{16\pi^2 v} (m_{H^\pm}^2 - m_{H_2}^2) \times (\text{loop functions}) \leftarrow \text{new part}$$

cf.) ρ parameter $\rho = m_W^2/m_Z^2 \cos^2 \theta_W$ $\rho_{\text{exp}} \simeq 1$ Global fit value from PDG

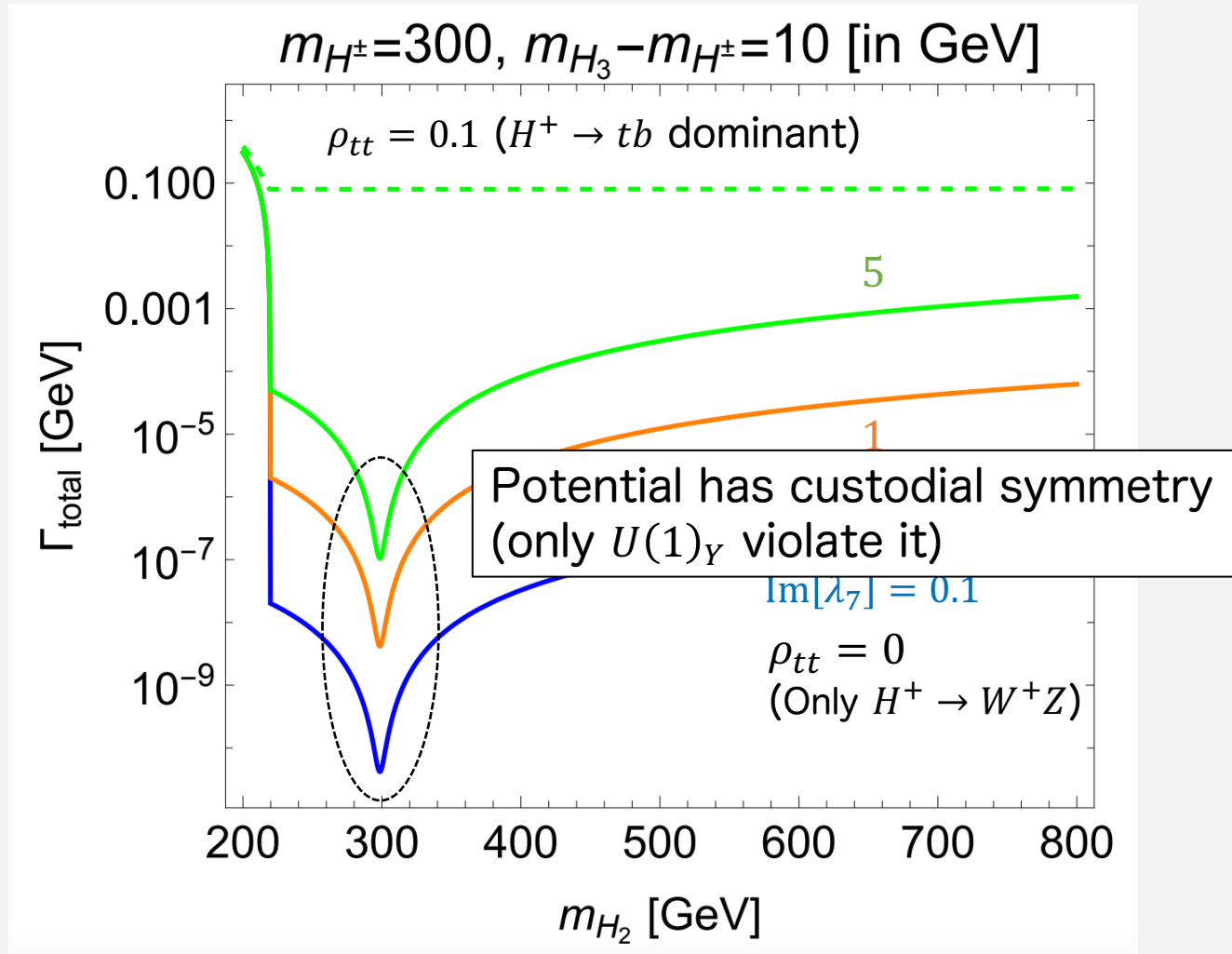
$m_{H_2}^2 = m_{H^\pm}^2$ or $m_{H_3}^2 = m_{H^\pm}^2$, $\Delta\rho = 0$ (1 loop level) Pomarol and Vega (1994)

$\Rightarrow$ Custodial symmetry violation by λ_7 is insensitive at the LO calculation



Decay rate for $H^\pm \rightarrow W^\pm Z$

- Non-zero $\text{Im}[\lambda_7]$ and $\lambda_4 + \lambda_5 \propto m_{H_2}^2 - m_{H^\pm}^2$ case



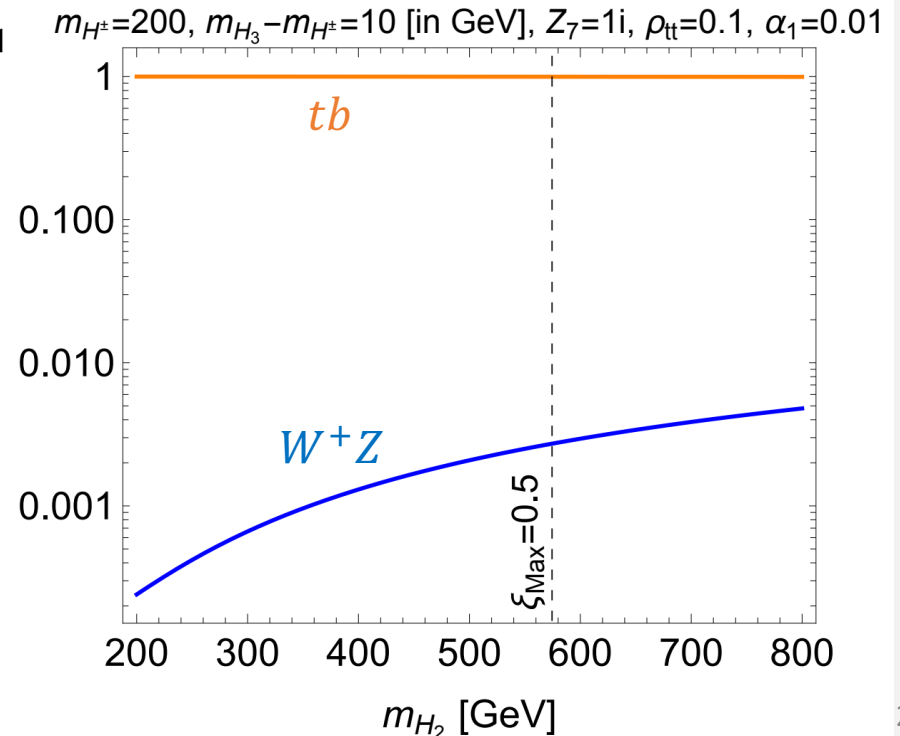
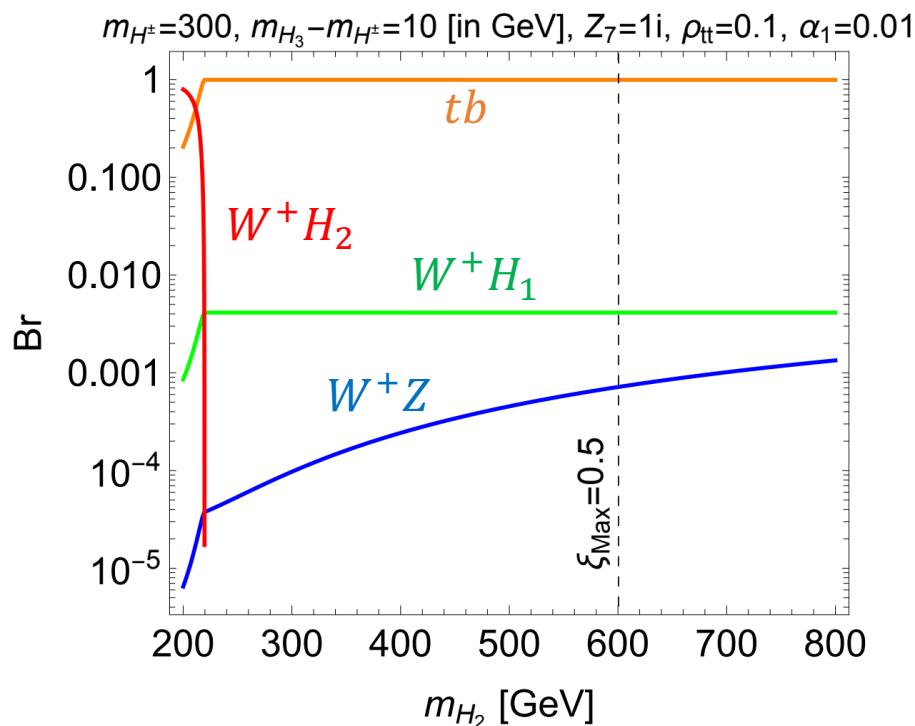
Branching ratio

cf.) Mixing angle

$$H_1 = h_1 \cos \alpha_1 - h_2 \sin \alpha_1$$

$$H_2 = h_1 \sin \alpha_1 + h_2 \cos \alpha_1$$

- Branching ratio for $H^\pm \rightarrow XY$ ($\rho_{ij} = 0$ except for ρ_{tt})
 - Custodial symmetry violation $\propto m_{H_2} - m_{H^\pm}$
 - For $m_{H^\pm} < m_{H_2}$, main modes are $H^\pm \rightarrow tb, WZ, WH_1$
 - If $m_{H^\pm} < m_W + m_{H_1}$, $\text{Br}(H^\pm \rightarrow W^\pm Z)$ can be efficiently large



Testing CP violation

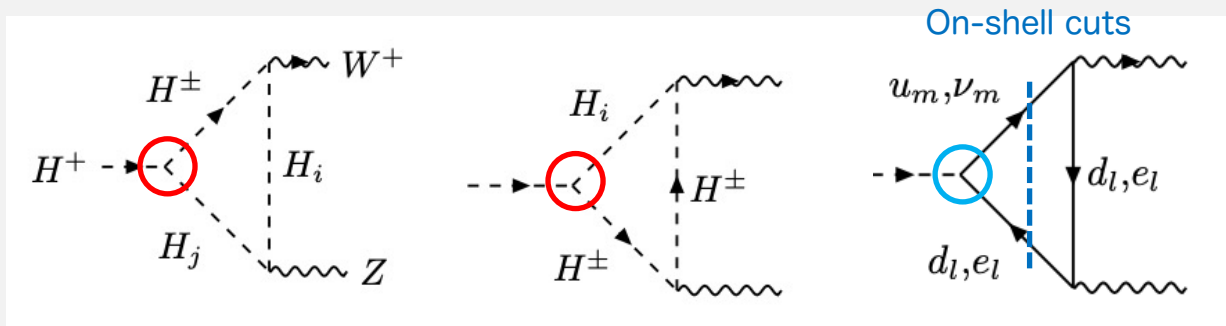
- Difference b/w $H^+ \rightarrow W^+ Z$ and $H^- \rightarrow W^- Z$

$$\Delta \equiv \Gamma(H^+ \rightarrow W^+ Z) - \Gamma(H^- \rightarrow W^- Z)$$

- Interference of scalar and fermion contributions

$$\Delta \simeq \rho_{tt}^R \lambda_7^I (m_{H^\pm}^2 - m_{H_2}^2) \times f_2 \operatorname{Im}[f_1^*] + \rho_{tt}^I \lambda_7^R (m_{H^\pm}^2 - m_{H_3}^2) \times f_2 \operatorname{Im}[f_1]$$

f_1 : loop function in fermion loop f_2 : loop function in scalar loop



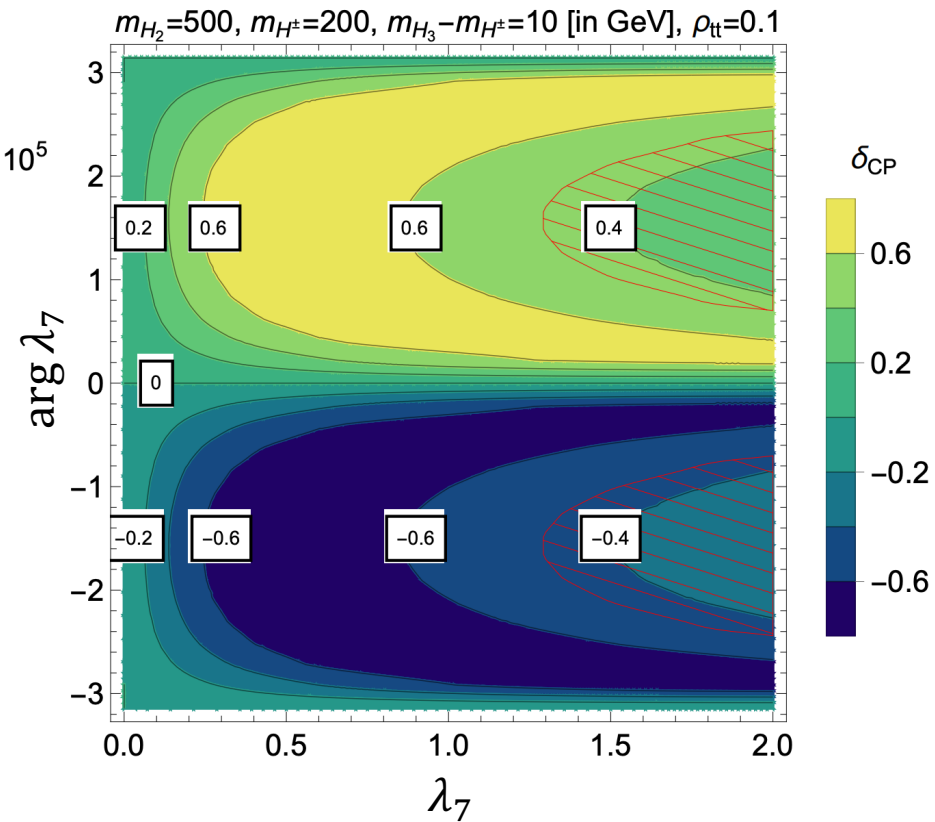
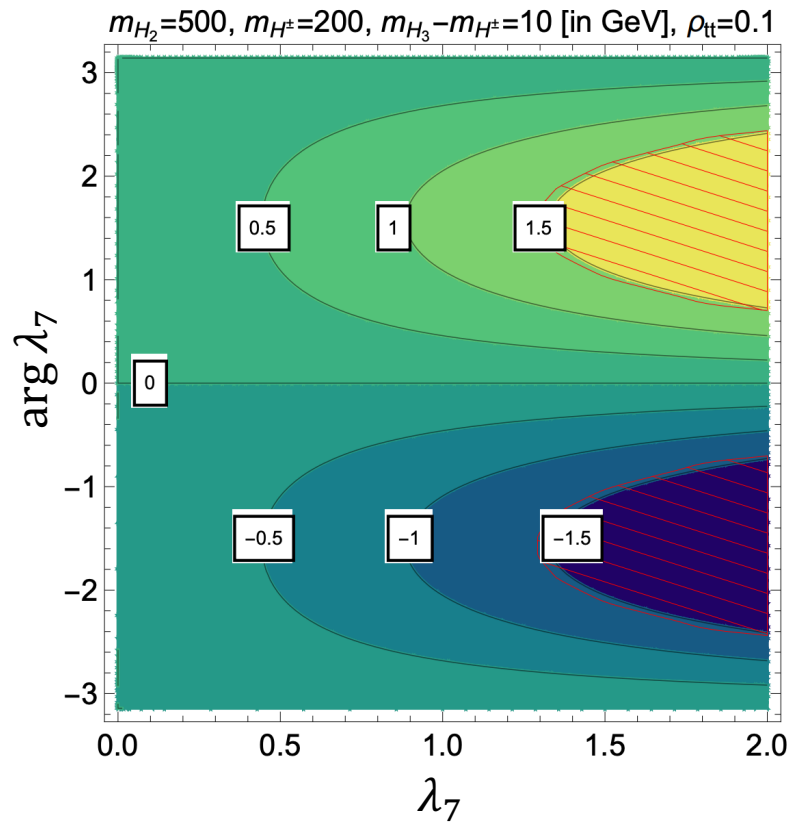
so-called “direct CP violation”

- Asymmetry b/w $H^+ \rightarrow W^+ Z$ and $H^- \rightarrow W^- Z$ is sensitive to $\operatorname{Im}[\rho^f \lambda_7]$.

Testing CP violation

Left: $\Delta \equiv \Gamma(H^+ \rightarrow W^+ Z) - \Gamma(H^- \rightarrow W^- Z)$

Right: $\delta_{CP} \equiv \frac{\Delta}{\Gamma(H^+ \rightarrow W^+ Z) + \Gamma(H^- \rightarrow W^- Z)}$

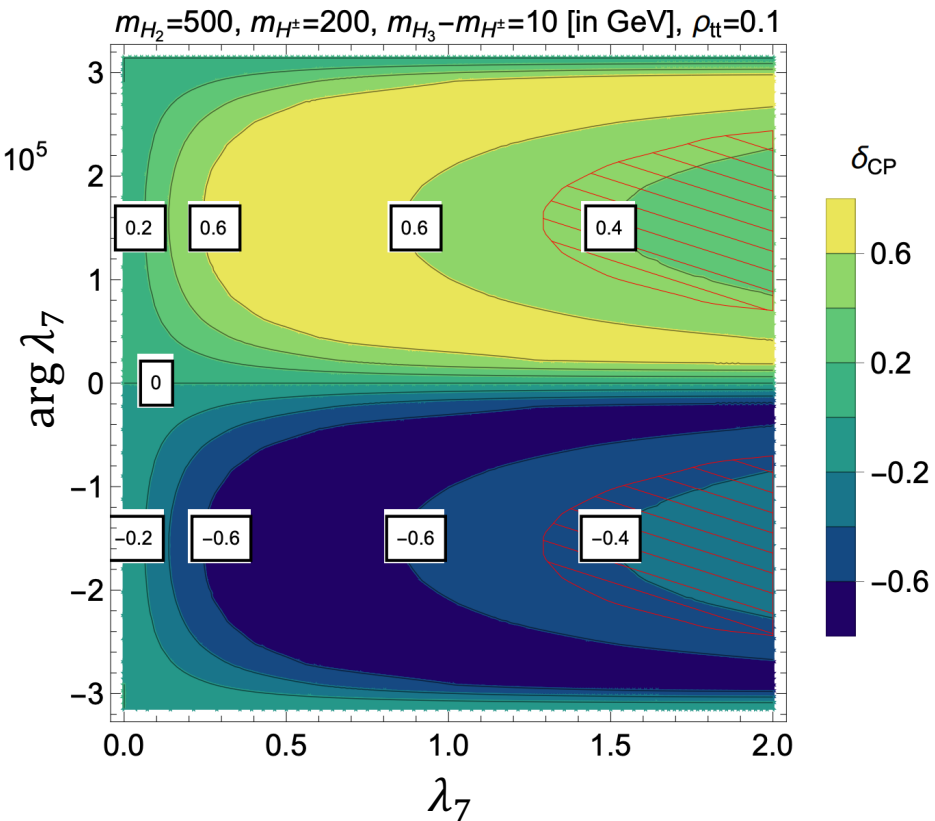
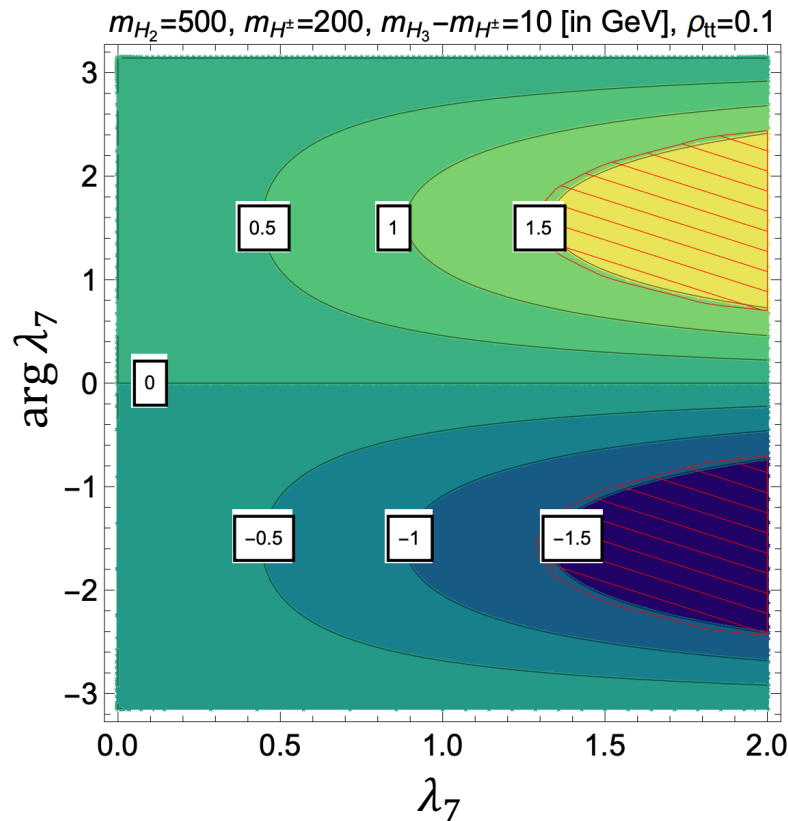


- At $\delta_{CP} \simeq 0.6$, $\Gamma(H^- \rightarrow W^- Z) \simeq \Gamma(H^+ \rightarrow W^+ Z)/4$ is shown by definition

Testing CP violation

Left: $\Delta \equiv \Gamma(H^+ \rightarrow W^+ Z) - \Gamma(H^- \rightarrow W^- Z)$

Right: $\delta_{CP} \equiv \frac{\Delta}{\Gamma(H^+ \rightarrow W^+ Z) + \Gamma(H^- \rightarrow W^- Z)}$



- Can we test direct CPV in future colliders? Humphrey, Kanemura and YM, work in progress

Summary

◆ New physics is necessary for EWBG

- New sources of CPV and FOPT can be introduced in 2HDM.

◆ Minimal setup for EWBG in general 2HDM

- EDMs and BAU correlated by the CP phase $\text{Im}[\lambda_7 \rho_{tt}]$
- It is viable under current bounds but would be tested in the future.

◆ $H^\pm \rightarrow W^\pm Z$ decay in general 2HDM

- We calculated $H^\pm \rightarrow W^\pm Z$ with the most general setup in 2HDM.
- We find the decay asymmetry is sensitive to the CP phases.

Back up

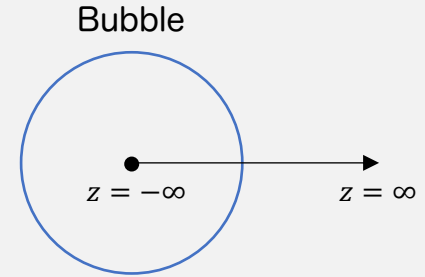
WKB method

Cline, Joyce, and Kainulainen (2000);
Fromme and Huber (2007); and more

- Transport equation for chemical potential

$$C_1 \mu''(z) + C_2 \mu'(z) + C_3 \mu(z) = S_{\text{CPV}}$$

where, $\mu(z) = \mu_\psi - \mu_{\bar{\psi}}$



- By solving Dirac eq. for ψ with WKB approximation, we have

$$S_{\text{CPV}} = C_4 (m_\psi^2 \theta'_\psi)' + C_5 m_\psi^2 \theta'_\psi (m_\psi^2)'. \quad C_i \text{ are functions of } z, T, m, \text{ and } v_w.$$

- Final BAU: $\eta_B \simeq (\text{const.}) \Gamma_{\text{sph}}^{\text{sym}} \int_0^\infty dz \mu_{BL} e^{-(\text{const.}) \Gamma_{\text{sph}}^{\text{sym}} z}$

washout

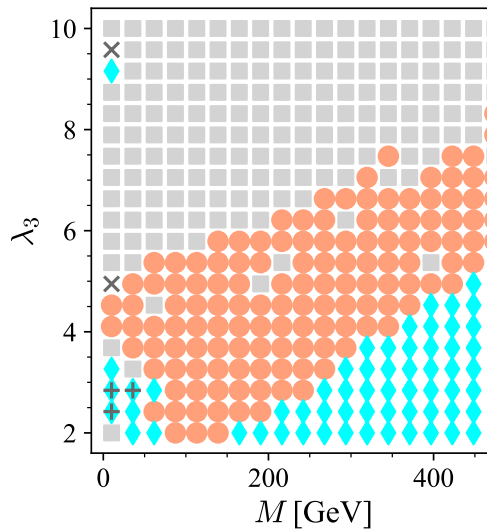
- Comments on VEV-Insertion-Approximation (VIA) method Riotto (1996) (1998);

- BAU evaluated by VIA method tends to be larger than that by WKB method.
- Main difference: CPV source is derived by SK formalism. Cline and Laurent (2021); Basler (2023); and more
- Long-term used CPV source (considered as LO) vanishes by correct resummation.

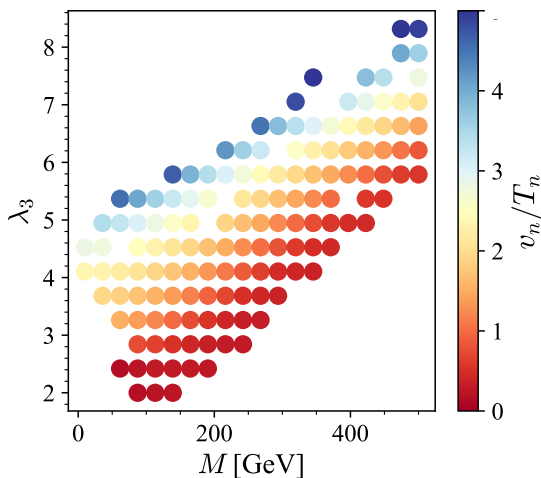
Kainulainen (2021); Postma et al. (2022); 32

Electroweak phase transition

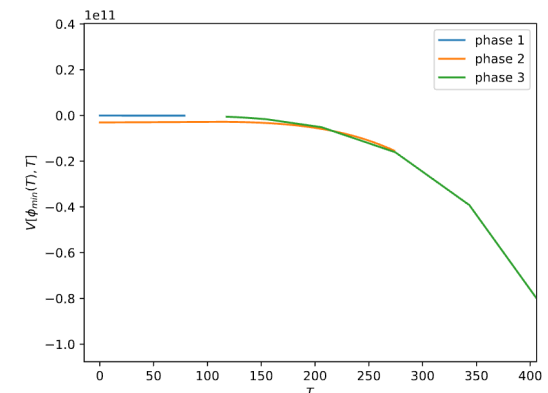
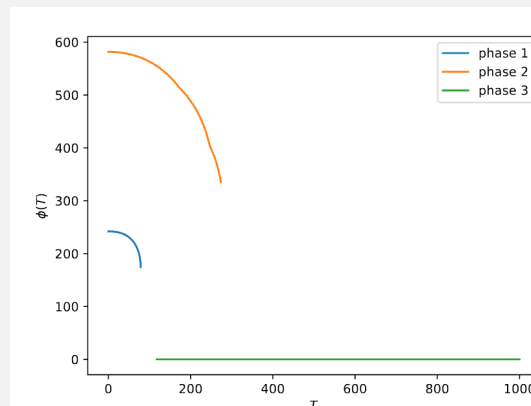
- Several fates of the vacuum (We used CosmoTransitions) [Wainwright, Comput. Phys. Commun. 183 \(2011\)](#)



- (a) **Pink filled circle:** The phase transition from phase A to phase B is first order, B is the vacuum at $T = 0$, B is the electroweak vacuum, and the VEV in A is *below* than 1 GeV.
- (b) **Gray +:** The phase transition $A \rightarrow B$ is first order, B is the vacuum at $T = 0$, B is the electroweak vacuum, and the VEV in A is *larger* than 1 GeV.
- (c) **Gray ×:** The phase transition $A \rightarrow B$ is first order, B is the vacuum at $T = 0$, and B is *not* the electroweak vacuum.
- (d) **Black dot:** The phase transition $A \rightarrow B$ is first order, and B is *not* the vacuum at $T = 0$.
- (e) **Blue diamond:** The phase transition $A \rightarrow B$ is second order.
- (f) **Gray box:** No phase transition is returned, including the case of $\Gamma/H^4 < 1$.



• Example of unrealistic phase transition



Electron EDM

- Effective Lagrangian $\mathcal{L}_{\text{eff}} = -\frac{1}{\Lambda^2} \left(\frac{g'}{\sqrt{2}} C_{tB} \overline{Q}_L \sigma^{\mu\nu} t_R \tilde{\Phi}_1 B_{\mu\nu} + \frac{g}{\sqrt{2}} C_{tW} \overline{Q}_L \sigma^{\mu\nu} t_R \tau^a \tilde{\Phi}_1 W_{\mu\nu}^a + \text{h.c.} \right),$

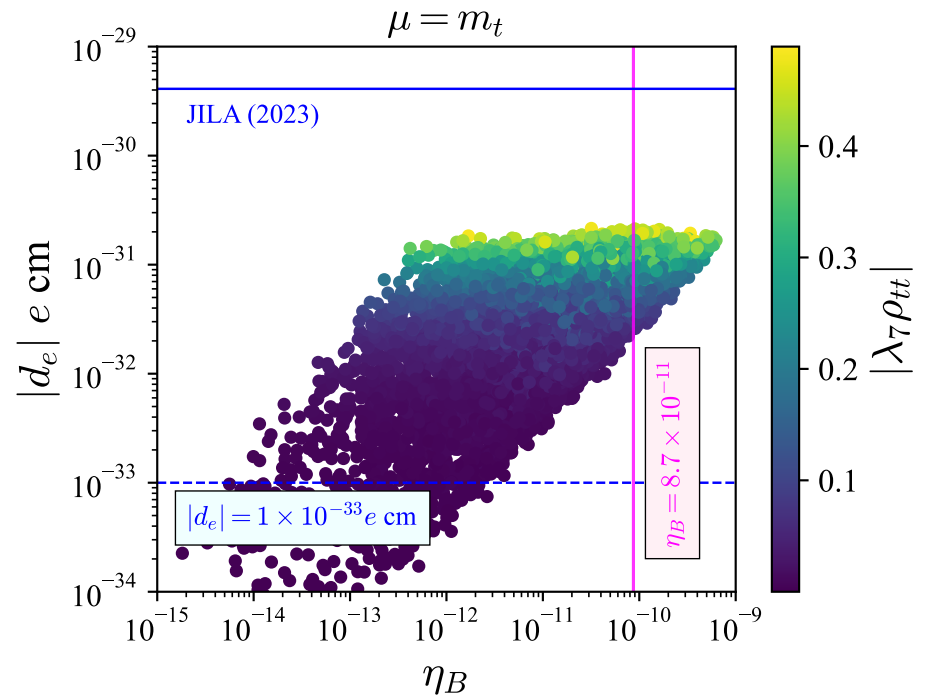
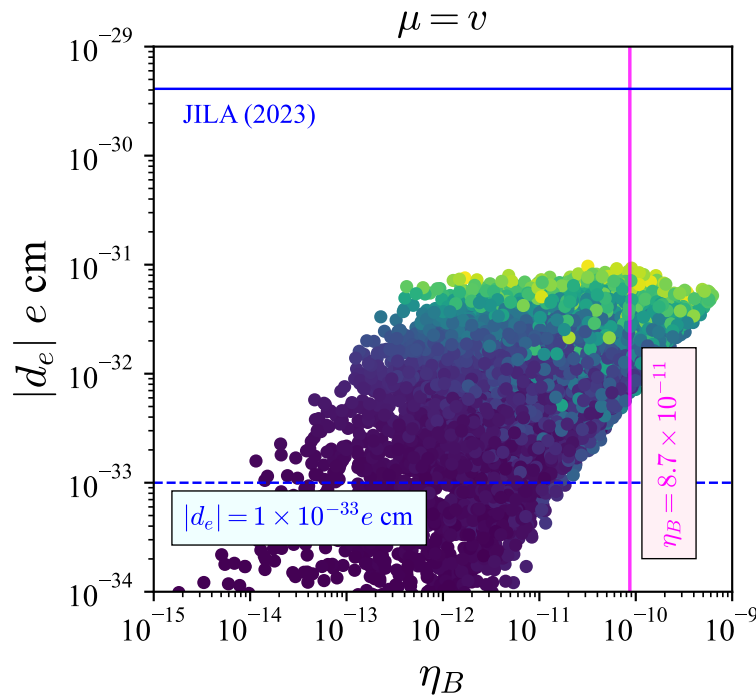
- Electron EDM induced by top EDM

- Matching at Λ

$$d_t^{B\mu} = \frac{g_1 v}{\Lambda^2} \text{Im}[C_{tB}], \quad d_t^{W^3\mu} = \frac{g_2 v}{\Lambda^2} \text{Im}[C_{tW}].$$

$$d_e = -\frac{e}{2v} \left(\frac{v}{\Lambda} \right)^2 \left(\log \frac{\Lambda}{\mu} \right)^2 \left[(A_e - D_e) \text{Im}[C_{tB}] + (B_e - E_e) \text{Im}[C_{tW}] \right],$$

Fuyuto and Ramsey-Musolf (2017)



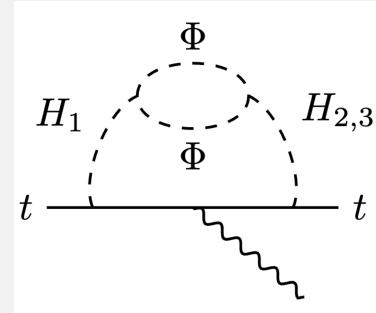
Scheme dependence

- Renormalization scheme for effective potential (EP scheme)

$$\left. \frac{\partial V_{T=0}^{\text{eff}}}{\partial \varphi_i} \right|_{\varphi=\varphi_{vac}} = 0, \quad (i = 1, 2, 3),$$

$$\left. \frac{\partial^2 V_{T=0}^{\text{eff}}}{\partial \varphi_i \partial \varphi_j} \right|_{\varphi=\varphi_{vac}} = \mathcal{M}_{ij}^2, \quad (i, j = 1, 2, 3),$$

make



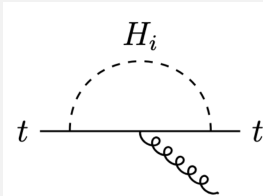
UV finite.

- Scheme difference (MS bar and EP scheme)

$$\tilde{d}_t^{(2),\overline{\text{MS}}} = \tilde{d}_t^{(2)} + \Delta \tilde{d}_t^{(2),\overline{\text{MS}}},$$

$$\Delta \tilde{d}_t^{(2),\overline{\text{MS}}} = \frac{\text{Im}[\lambda_7 \rho_{tt}]}{\sqrt{2}} \frac{3\lambda_3 v}{(16\pi^2)^2} \frac{2m_t^2}{m_\Phi^2 - m_{H_1}^2} \left(C_{11}[\Phi, t, t] - C_{11}[H_1, t, t] \right) \log \frac{\mu^2}{m_\Phi^2} \Big|_{\overline{\text{MS}}},$$

- Scheme conversion and one-loop EDM



$$\lambda_6^{\overline{\text{MS}}} = \lambda_6 - \frac{3}{16\pi^2} \lambda_3 \lambda_7 \log \frac{\mu^2}{m_\Phi^2} + \dots \Big|_{\text{EP}}$$

causes

$$\tilde{d}_t^{(1),\overline{\text{MS}}} = \frac{\text{Im}[\lambda_6 \rho_{tt}]}{\sqrt{2}} \frac{v}{16\pi^2} \frac{2m_t^2}{m_\Phi^2 - m_{H_1}^2} \left(C_{11}[\Phi, t, t] - C_{11}[H_1, t, t] \right) \Big|_{\overline{\text{MS}}}$$

- Consequently, we have

$$\tilde{d}_t^{(1),\overline{\text{MS}}} + \tilde{d}_t^{(2),\overline{\text{MS}}} = \tilde{d}_t^{(2)} + \mathcal{O}(\hbar^3) \Big|_{\text{EP}}$$

and there are no scale dependence.

Estimation of baryon asymmetry

- Boltzmann equation with perturbations from thermal equilibrium

$$(\partial_t + \mathbf{v}_g \cdot \partial_{\mathbf{x}} + \mathbf{F} \cdot \partial_{\mathbf{p}}) f_i = C[f_i, f_j, \dots] \quad f_i = \frac{1}{e^{\beta[\gamma_w(E_i + v_w p_z) - \mu_i]} \pm 1} + \delta f_i$$

- Chemical potentials
(particle – anti-particle)

- μ_{B_L} affects rate of $\Delta B \neq 0$ process

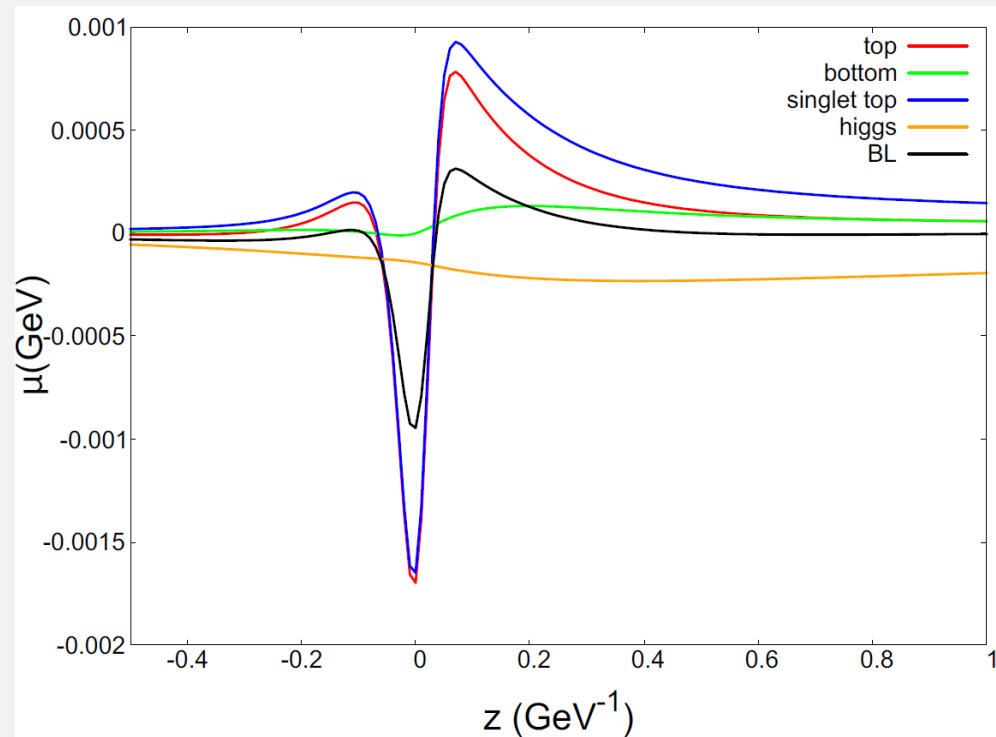
Cline, Joyce and Kainulainen, JHEP 07 (2000)

In plasma flame

$$\frac{\partial n_B}{\partial t} = \frac{3}{2} \Gamma_{\text{sph}} \left(\frac{3\mu_{B_L}}{T} - \frac{A}{T^3} n_B \right)$$

Integrated in wall flame

$$\eta_B \equiv \frac{n_B - n_{\bar{B}}}{s} = \frac{405 \Gamma_{\text{sph}}}{4\pi^2 v_w g_* T} \int_0^\infty dz \mu_{B_L} e^{-(\text{const.}) \Gamma_{\text{sph}} z}$$



CP violating bubble

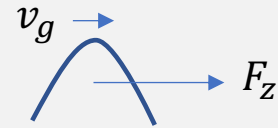
- “Semi classical force approach” (WKB method)

Cline, Joyce and Kainulainen, JHEP 07 (2000);
Cline and Kainulainen Phys. Rev. D 101 (2020)

$$(i\partial_\mu\gamma^\mu - m(z)P_L - m^*(z)P_R)\psi = 0$$

$$v_g = \frac{p_z}{E} \pm (\partial_z\theta \text{ corrections})$$

$$F_z = -\frac{\partial_z|m^2|}{2E} \pm (\partial_z\theta \text{ corrections})$$



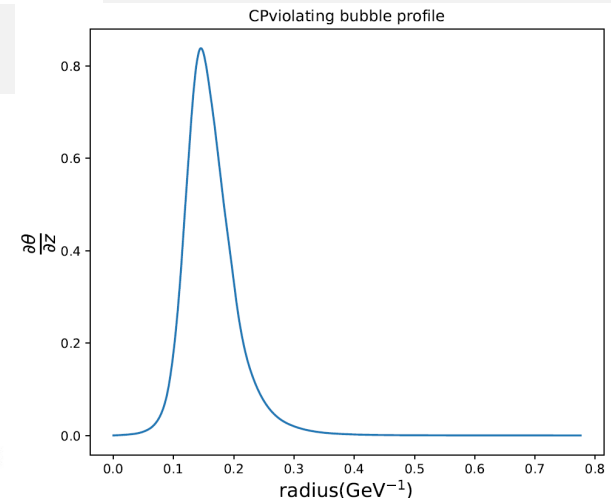
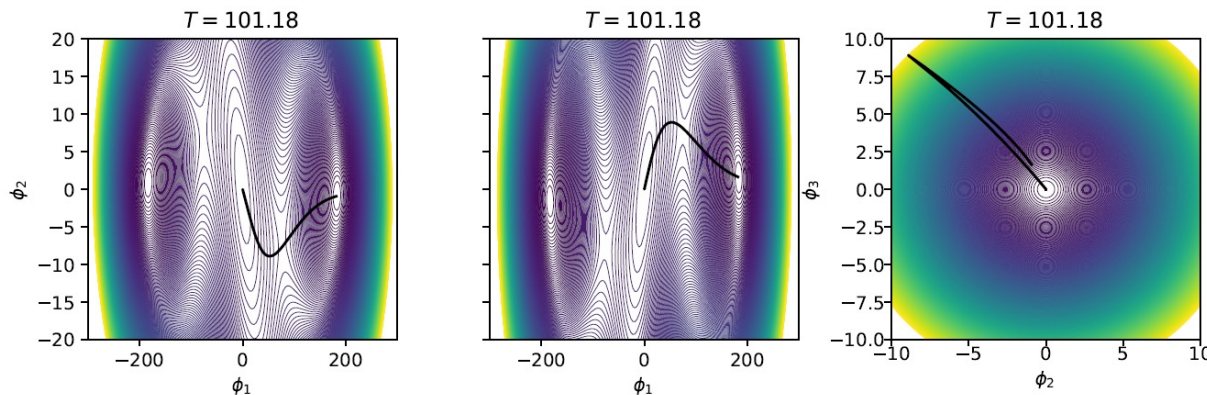
WKB wave packet

$$m_t^2 = \frac{1}{2} \left(y_t^2 \varphi_1^2 + |\rho_{tt}|^2 (\varphi_2^2 + \varphi_3^2) + 2y_t |\rho_{tt}| \varphi_1 (\varphi_2 \cos \theta_{tt} + \varphi_3 \sin \theta_{tt}) \right),$$

$$\theta'_t = \frac{1}{2m_t^2} \left\{ y_t |\rho_{tt}| \left((\varphi_3 \varphi'_1 - \varphi_1 \varphi'_3) \cos \theta_{tt} + (\varphi_1 \varphi'_2 - \varphi_2 \varphi'_1) \sin \theta_{tt} \right) + |\rho_{tt}|^2 (\varphi_3 \varphi'_2 - \varphi_2 \varphi'_3) \right\}$$

$$+ \frac{1}{\varphi_1^2 + \varphi_2^2 + \varphi_3^2} (\varphi_3 \varphi'_2 - \varphi_2 \varphi'_3), \quad (3.10)$$

- Order parameters $\varphi_1, \varphi_2, \varphi_3$ (Black lines: path of PT)



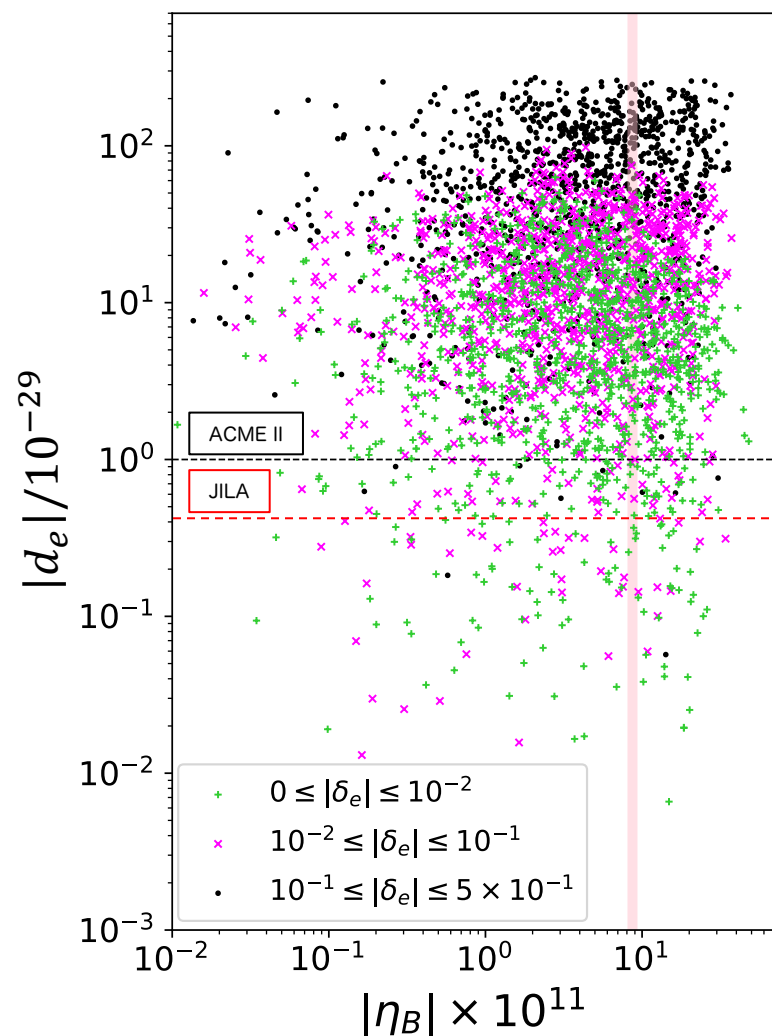
- Large φ_2, φ_3 during PT are needed to enhance BAU.

With Yukawa alignment

Enomoto, Kanemura and YM,
JHEP 09 (2022) 121

$$\lambda_2 = 0.1, m_\Phi = 350 \text{ GeV}, M = 30 \text{ GeV}, v_w = 0.1,$$

$$\theta_u = \theta_d = [0, 2\pi), |\zeta_d| = |\zeta_e| = [0, 10], |\lambda_7| = [0.5, 1.0], \theta_7 = [0, 2\pi).$$



These points are allowed from various constraints.

Fermion loop contributions
are proportional to $|\zeta_u||\zeta_e|\sin\delta_e$.
($\delta_e \equiv \theta_u - \theta_e$)

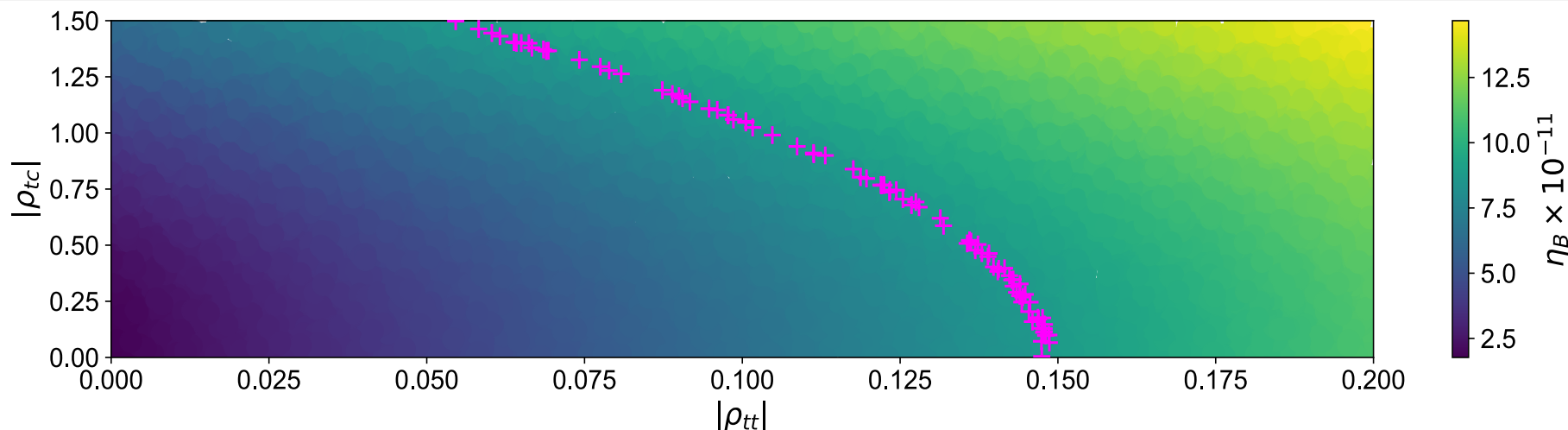
Many points are satisfied from eEDM data
and they generate sufficient BAU.

Top-charm transport scenario

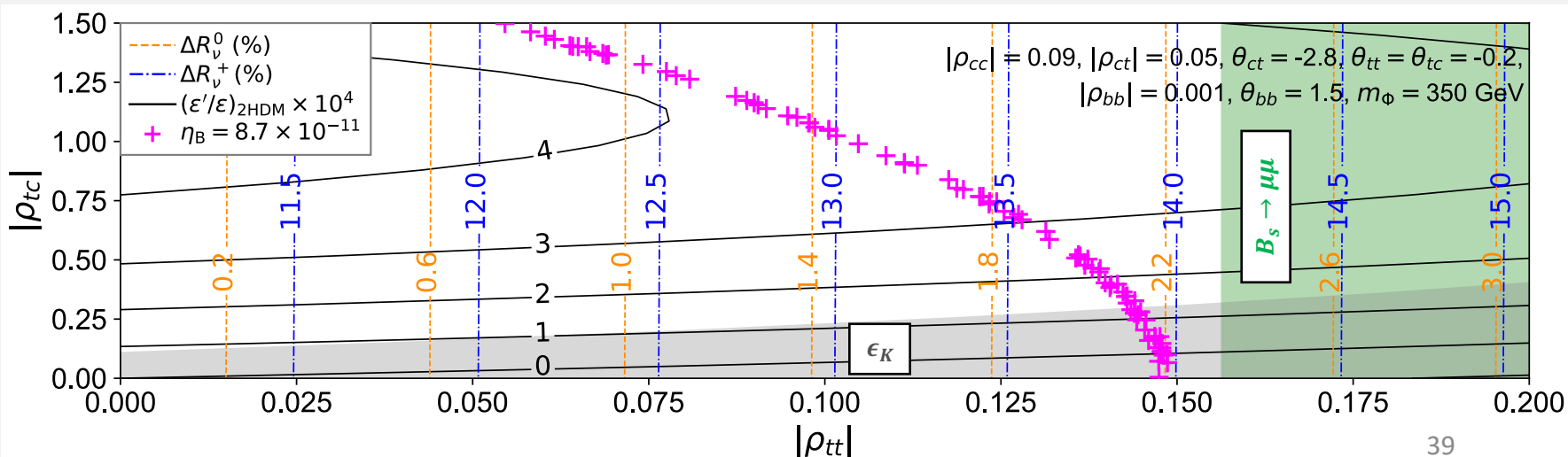
Kanemura and YM, JHEP 09 (2023) 153

magenta : observed BAU $\eta_B = 8.7 \times 10^{-11}$

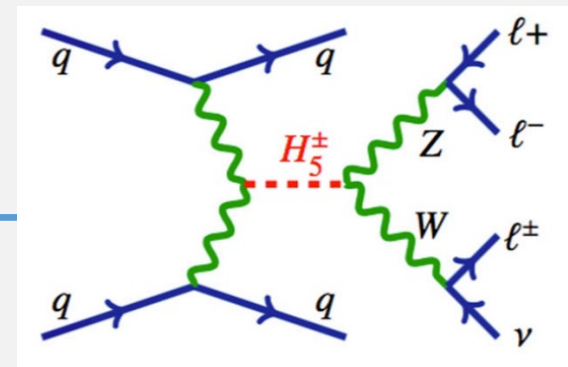
Top-charm transport



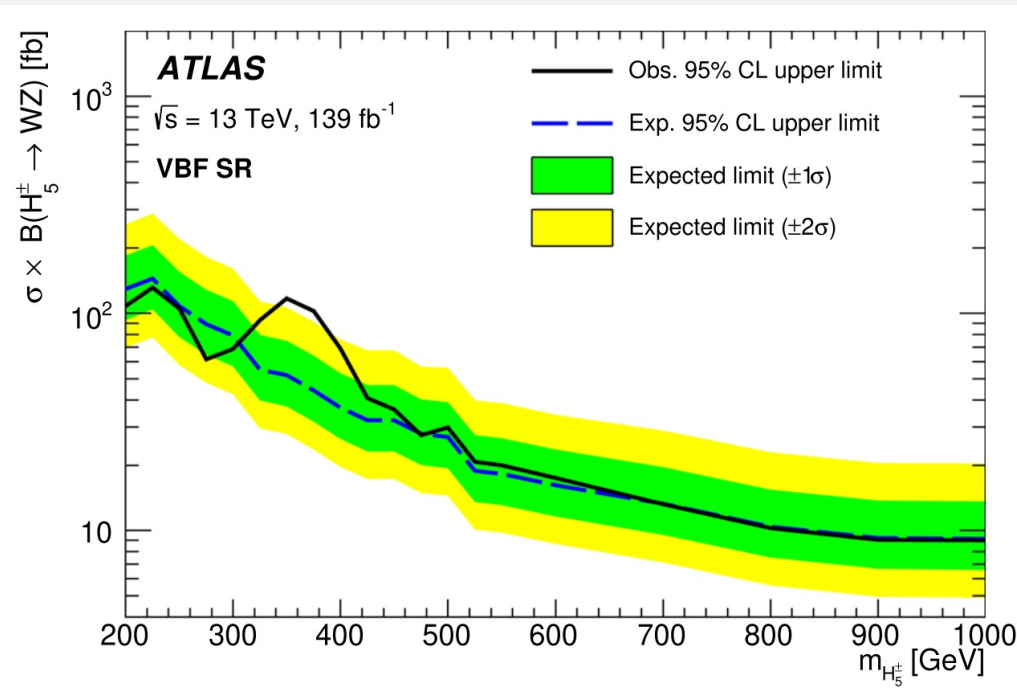
Top transport



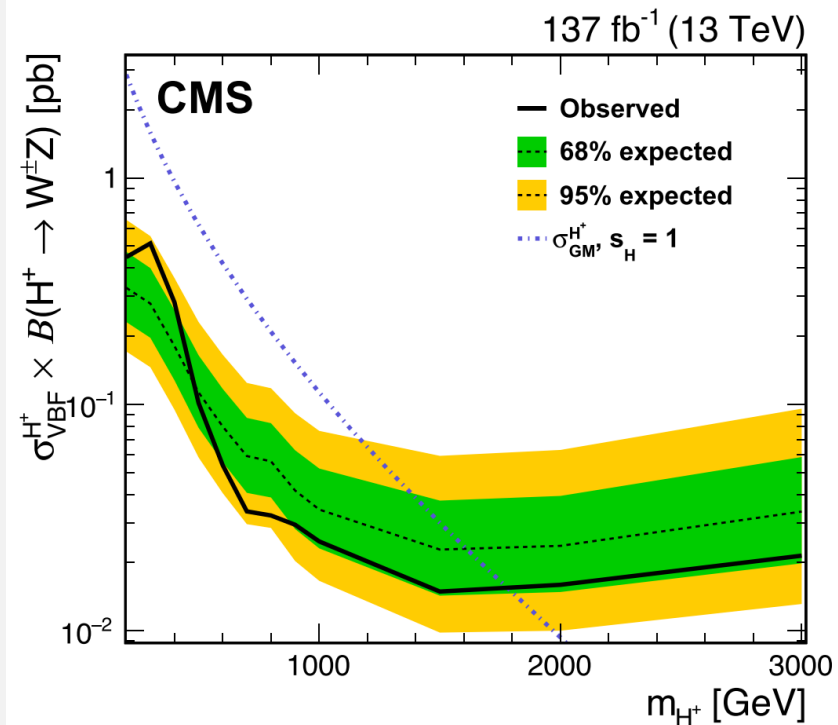
ATLAS and CMS search



ATLAS, Eur. Phys. J. C (2023) 83:633



CMS, Eur. Phys. J. C 81 (2021) 8, 723

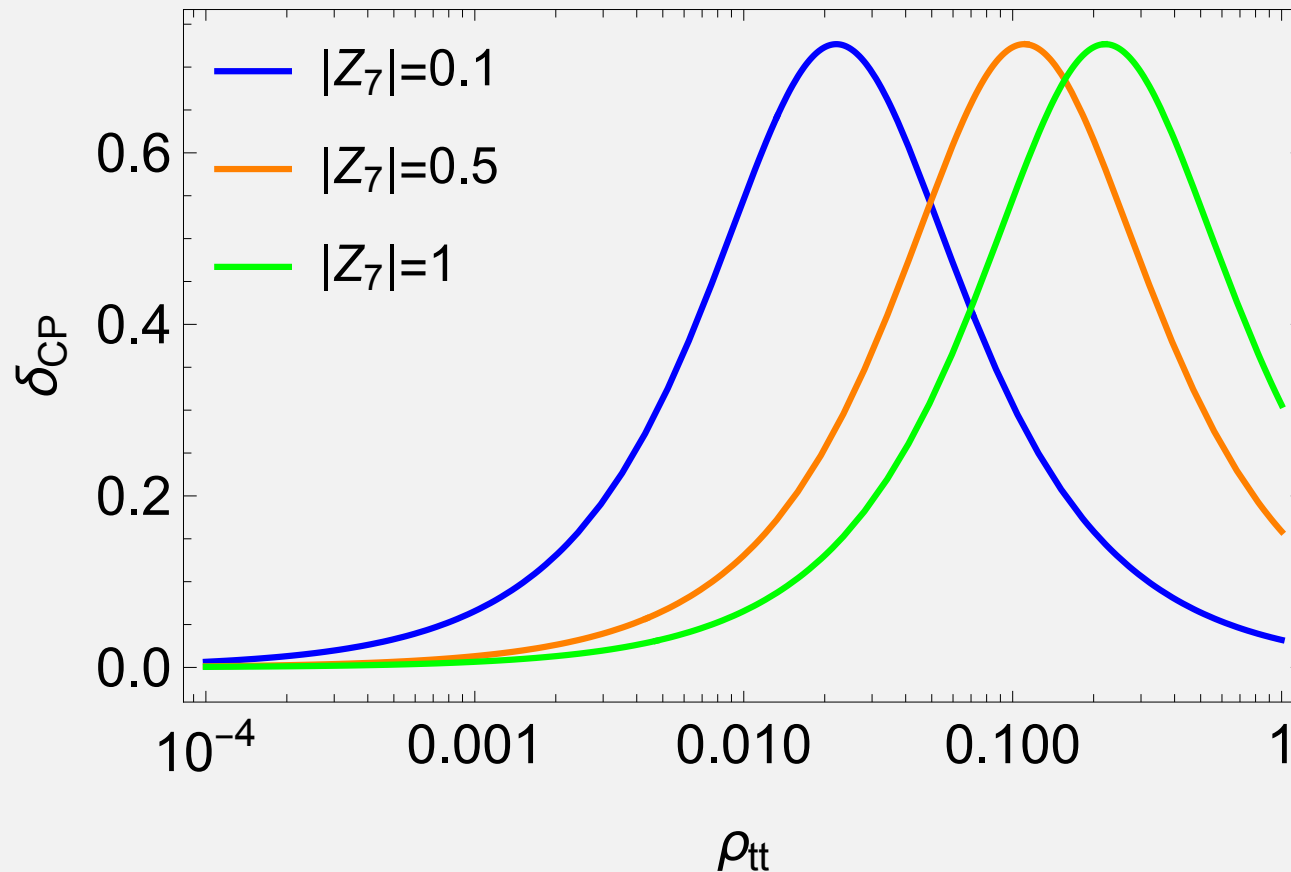


$$s_H = \frac{8\langle\chi\rangle^2}{v^2}, \quad \langle\chi\rangle: \text{triplet VEV}$$

Georgi and Machacek (1985);
 Chanowitz and Golden (1985);

δ_{CP} as a function of Z_7 and ρ_{tt}

$m_{H_2}=500, m_{H^\pm}=200, m_{H_3}-m_{H^\pm}=10$ [in GeV], $\arg[Z_7]=\pi/2$



Impacts on collider phenomenology

- Benchmark points

large ρ_{tt}

small ρ_{tt}

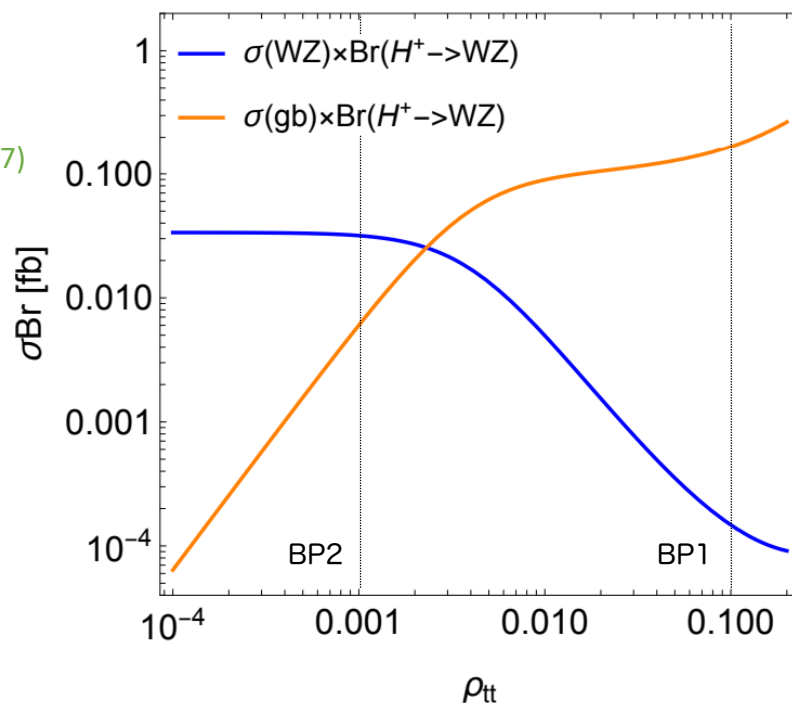
(in GeV)	$m_{H^\pm}$	m_{H_2}	m_{H_3}	Z_7	ρ_{tt}	$\alpha_1 = -\alpha_2$
BP1	200	500	210	$1.3e^{2.0i}$	0.1	0.01
BP2	200	500	210	$1.3e^{2.0i}$	0.001	0.01

- Production of $H^\pm$ in hadron collider

$$pp(WZ) \rightarrow H^\pm + X \quad \text{Asakawa, Kanemura, and Kanzaki (2007)}$$

$$gb \rightarrow \bar{t}H^\pm \quad \text{LHC Higgs Cross Section Working Group (2017)}$$

	BP1	BP2
$ F ^2$	2.8×10^{-5}	1.7×10^{-5}
$\text{Br}(H^+ \rightarrow W^+ Z)$	2.6×10^{-3}	9.4×10^{-1}
$\sigma_{WZ} [\text{fb}]$	5.6×10^{-2}	3.4×10^{-2}
$\sigma_{WZ} \times \text{Br} [\text{fb}]$	1.5×10^{-4}	3.2×10^{-2}
$\sigma_{gb} [\text{fb}]$	6.4×10	6.4×10^{-3}
$\sigma_{gb} \times \text{Br} [\text{fb}]$	1.7×10^{-1}	6.0×10^{-3}



At HL-LHC (3000 fb^{-1}), $O(100)$ events are expected.

Impacts on collider phenomenology

- **Benchmark points**

large ρ_{tt}

small ρ_{tt}

(in GeV)	$m_{H^\pm}$	m_{H_2}	m_{H_3}	Z_7	ρ_{tt}	$\alpha_1 = -\alpha_2$
BP1	200	500	210	$1.3e^{2.0i}$	0.1	0.01
BP2	200	500	210	$1.3e^{2.0i}$	0.001	0.01

- **Production of $H^\pm$ in e+e- collider**

- Pair production process $e^+e^- \rightarrow H^+H^-$ S. Komamiya, Phys. Rev. D (1988)

- $\sigma(e^+e^- \rightarrow H^+H^-) \simeq 30 \text{ fb}$ (ILC 500 GeV)

- **At BP2, almost all $H^\pm$ decay into $W^\pm Z$**

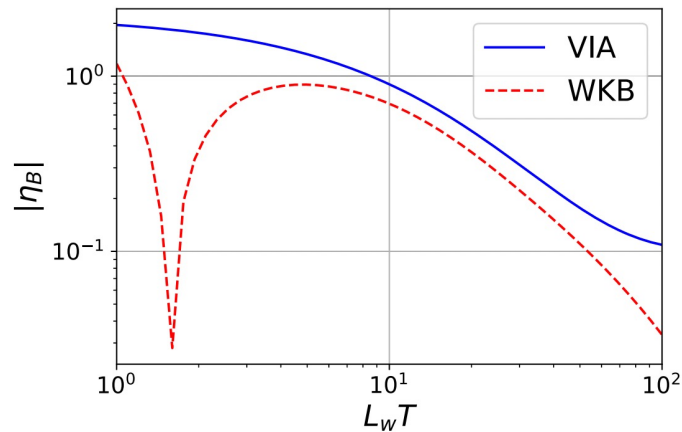
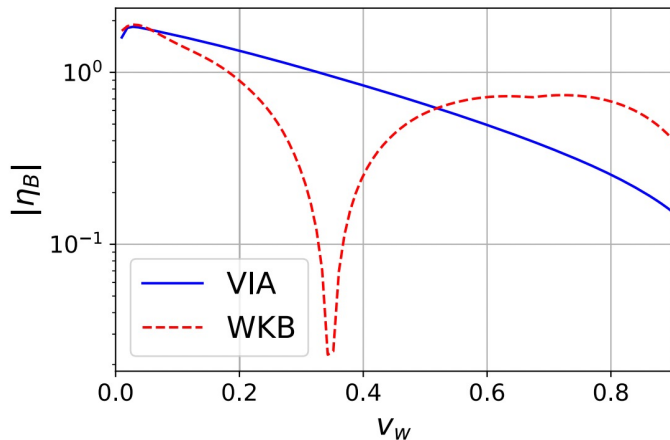
- When 3 ab^{-1} integrated luminosity is assumed, $O(10^5)$ events can be expected.

- **Testing $H^\pm W^\mp Z$ vertex in the general 2HDM motivates future colliders.**

Discrepancy between WKB and VIA

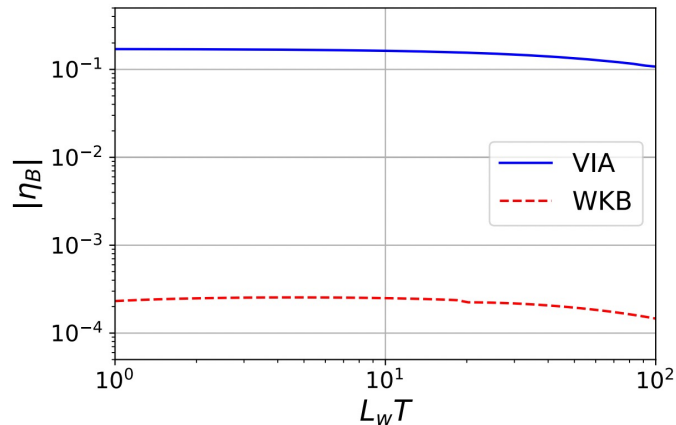
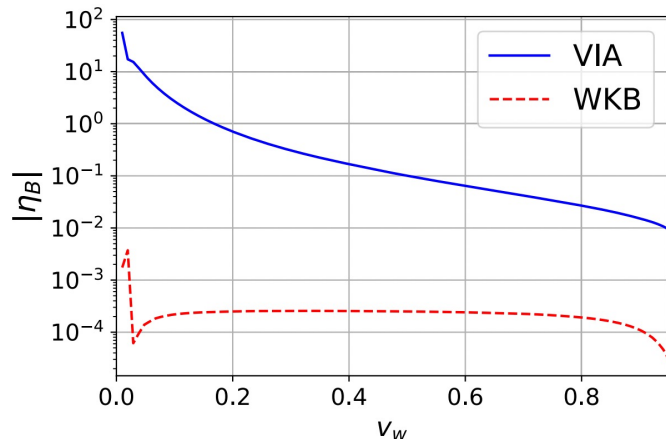
- Each method provides different results. [Cline and Laurent \(2021\)](#)

Quark (charm-top mixing) case



v_w : wall velocity
 L_w : wall width

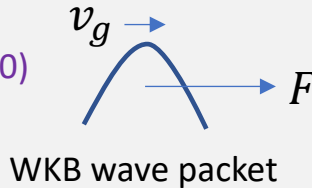
Lepton (tau) case



Discrepancy between WKB and VIA

• WKB method

Cline, Joyce and Kainulainen (2000)



Boltzmann eq.

$$(\partial_t + \mathbf{v}_g \cdot \partial_{\mathbf{x}} + \mathbf{F} \cdot \partial_{\mathbf{p}}) f_i = C[f_i, f_j, \dots]$$

$$v_g = \frac{p}{E} \pm (\text{from wall})$$

$$F = (\text{from wall}) \pm (\text{correction})$$

Group velocity v_g and force F are derived from Dirac or Klein-Gordon equation.

• VIA method A. Riotto (1995), (1997), (1998)

Schwinger-Dyson eq. $-\left(\partial_u^2 + m^2\right) i\Delta(u, v) = i\delta^4(u - v) + \int d^4w \Pi(u, w) i\Delta(w, v)$

Self energy contains CPV with VEV insertion approx.



Quantum diffusion eq. $\partial_\mu j^\mu = \partial_t n + \nabla \cdot \mathbf{j} = (Source)$

$$\mathbf{j} \simeq -D \nabla n \quad (\text{Fick's law})$$

Usual VEV insertion approximation

- Perturbation about space dependent off-diagonal mass

Self energy at 2nd order

$$\Pi_{LL}^{(2)} \supset \begin{array}{c} m_{RL}^2(y) \quad m_{LR}^2(x) \\ \diagdown \quad \diagup \\ \phi_R \\ \diagup \quad \diagdown \\ \phi_L^\dagger \quad \phi_L \end{array} = -m_{LR}^2 G_{RR} m_{RL}^2$$

- CP violating source term in usual VIA

$$g(x, y) = m_{LR}^2(x) m_{RL}^2(y)$$

$$S_{LL}^{(2)}(x) = \int d^4y \left(\text{Re}[g(x, y) + g(y, x)] \text{Re}[G_{RR}^<(x, y) G_{LL}^>(y, x) - G_{RR}^>(x, y) G_{LL}^<(y, x)] \right. \\ \left. - \text{Im}[g(x, y) - g(y, x)] \text{Im}[G_{RR}^<(x, y) G_{LL}^>(y, x) - G_{RR}^>(x, y) G_{LL}^<(y, x)] \right)$$

CP conserving part $\Rightarrow$ relaxation term

CP violating part $\Rightarrow$ source term

$$S_{LL}^{(2)}(x) \supset S_{\text{CPV}}^{(2)} = 2\text{Im}[m_{LR}^2 \partial_\mu m_{RL}^2] \\ \times \int d^4y (y - x)^\mu \text{Im}[G_{RR}^<(x, y) G_{LL}^>(y, x) - G_{RR}^>(x, y) G_{LL}^<(y, x)]$$

with further approximation: $m_{IJ}^2(y) = m_{IJ}^2(x) + (x - y)^\mu \partial_\mu m_{IJ}^2(x) + O(\partial^2)$

KMS relation

- M^2 $\Rightarrow$ mass matrix including background field interaction
- Π $\Rightarrow$ thermal self energy
- G $\Rightarrow$ Green functions obtained from constraint eq.

- In thermal equilibrium and single flavor system, Wightman functions and self energies satisfy

$$(1 + n)G^< = nG^>, \quad (1 + n)\Pi^< = n\Pi^>, \quad \text{where,} \quad n = \frac{1}{e^{(E-\mu)/T} - 1}.$$

(bosonic case of Kubo-Martin-Schwinger relation)

- We assume self energies satisfy this relation and are diagonal even in two flavor system.
- At the 0th order in VIA (free for the background field), Green functions can be obtained from constraint eq. such as

$$G_{(0),IJ}^> = (n_I + 1)\rho_{(0),I}\delta_{IJ}, \quad (I,J \text{ is flavor indices})$$

$$G_{(0),IJ}^< = n_I\rho_{(0),I}\delta_{IJ}, \quad \text{where,} \quad \rho_{(0),I} = \frac{\gamma_I}{(k^2 - m_I^2 - \Pi^h)^2 - \gamma_I^2/4}, \quad \gamma_I = -4ik_0\Gamma_I$$

spectral function thermal width

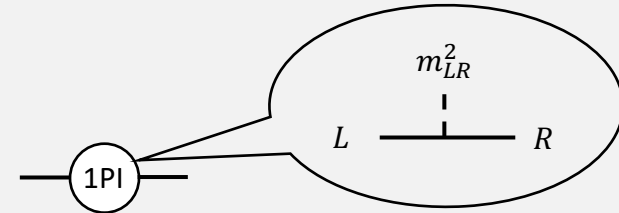
Of course, we can find these 0th order Wightman functions satisfy KMS relation.

2nd order in VIA

- Green function at 1st and 2nd order in VIA

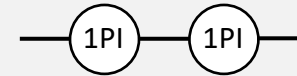
$$G_{(1),IJ}^{ab} = c \sum_c G_{(0),II}^{ac} m_{IJ}^2 G_{(0),JJ}^{cb},$$

(off diag.)



$$G_{(2),IJ}^{ab} = cd \sum_{cd} G_{(0),II}^{ac} m_{IJ}^2 G_{(0),JJ}^{cd} m_{JI}^2 G_{(0),II}^{db}. \quad (\text{diag.})$$

($a, b, c, d = \pm$)



- Source term, for example diagonal component, is

$$\begin{aligned} \bar{S}_{LL}^{(2)} = & \underbrace{[\delta M^2, G_{(1)}^> + G_{(1)}^<]}_{LL} + \underbrace{[M_d^2, G_{(2)}^> + G_{(2)}^<]}_{LL} \\ & + \underbrace{[\Pi^> + \Pi^<, G_{(2)}^h]}_{LL} + \underbrace{\left(\{\Pi^>, G_{(2)}^<\} - \{\Pi^<, G_{(2)}^>\} \right)}_{LL} \\ & = 0 \quad (\because \text{both are diagonal}) \end{aligned}$$

$$M^2 = M_d^2 + \delta M^2$$

(diag.) + (off diag.)

$$\underbrace{= 2|m_{LR}|^4 \rho_{(0),L} \rho_{(0),R} (n_L - n_R)}_{\text{blue}} - \underbrace{2|m_{LR}|^4 \rho_{(0),L} \rho_{(0),R} (n_L - n_R)}_{\text{green}} = 0.$$

- The source term of right and off diagonal components also vanish.

Full order in VIA

- We can solve constraint equations exactly at leading order in derivative expansion.

$$k^2 G^\lambda = \frac{1}{2} \left(\{M^2, G^\lambda\} + \{\Pi^\lambda, G^h\} + \frac{1}{2} ([\Pi^>, G^<] - [\Pi^<, G^>]) \right),$$

$$k^2 G^t = 1 + \frac{1}{2} \left(\{M^2 + \Pi^t - \Pi^h, G^t\} - \Pi^< G^> - G^< \Pi^> \right).$$

Solution $G_{LL}^\lambda = \frac{\gamma_R \gamma_L}{\mathcal{D}_+ \mathcal{D}_- \rho_{(0),R}} \left(g_L^\lambda + g_R^\lambda \frac{\rho_R}{\gamma_L} |m_{LR}|^4 \right),$

$$G_{LR}^\lambda = \frac{m_{LR}^2}{\mathcal{D}_+ \mathcal{D}_-} \left(\gamma_R g_R^\lambda (k^2 - m_L^2) + \gamma_L g_L^\lambda (k^2 - m_R^2) + \frac{1}{2} \gamma_R \gamma_L (g_R^\lambda - g_L^\lambda) \right).$$

- By substituting these into kinetic equation, we can find source term exactly vanishes: $\bar{\mathcal{S}} = 0$.
- In conclusion, VIA source does not appear at the leading order in derivative expansion.
($\partial_x \ll k$)

Possibility

- Thermal corrections possibly provide off diagonal element of self energy.
- We have used $e^{-i\phi} \rightarrow \mathbf{1}$, and next to leading order correction has not been calculated yet.

$\Rightarrow$ Being key to solve discrepancy between WKB and VIA ?