Nonthermal heavy dark matter from a first-order phase transition

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Ref. G. Giudice, HML, A. Pomarol, B. Shakya, JHEP12 (2024) 190.

Focus workshop on cosmological phase transitions
IBS CTPU, Daejeon, Nov 22, 2025

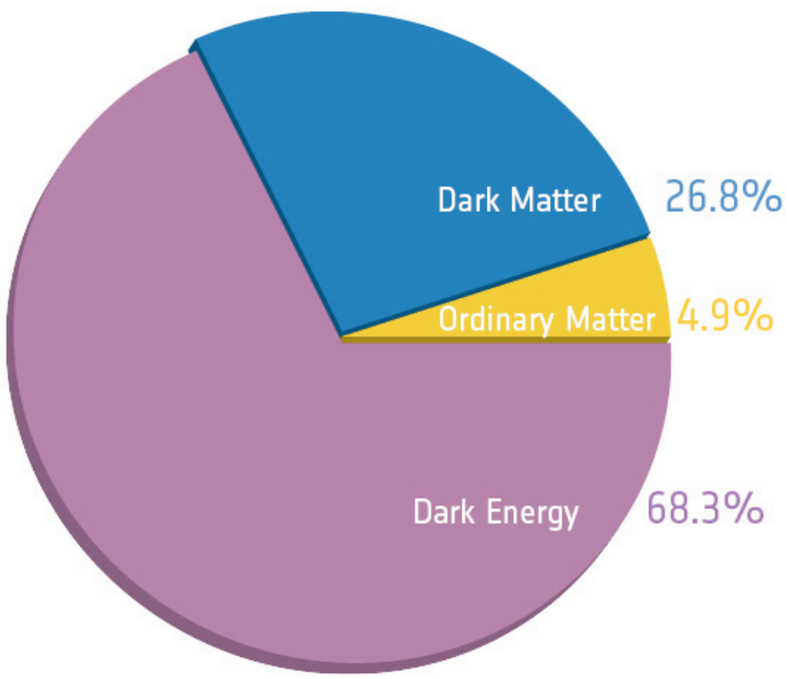
Outline

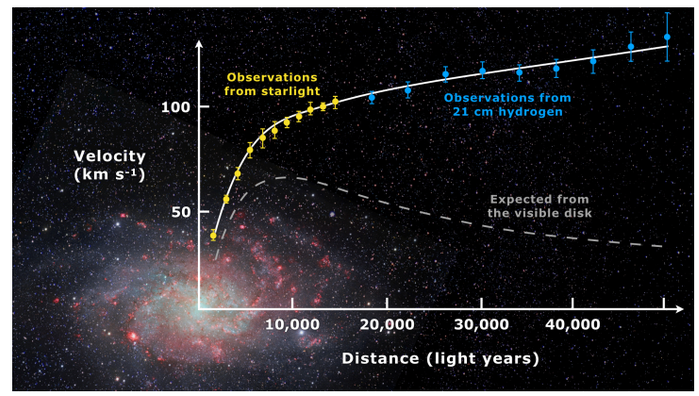
- Introduction
- Dark matter from bubble collisions
- Vector dark matter and gauge choices
- Conclusions

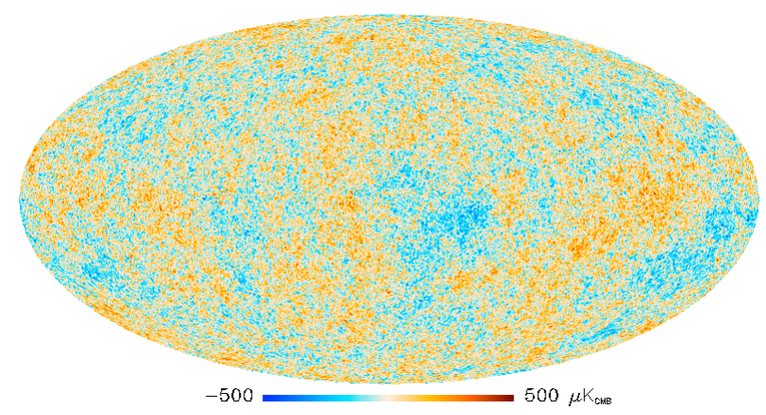
Introduction

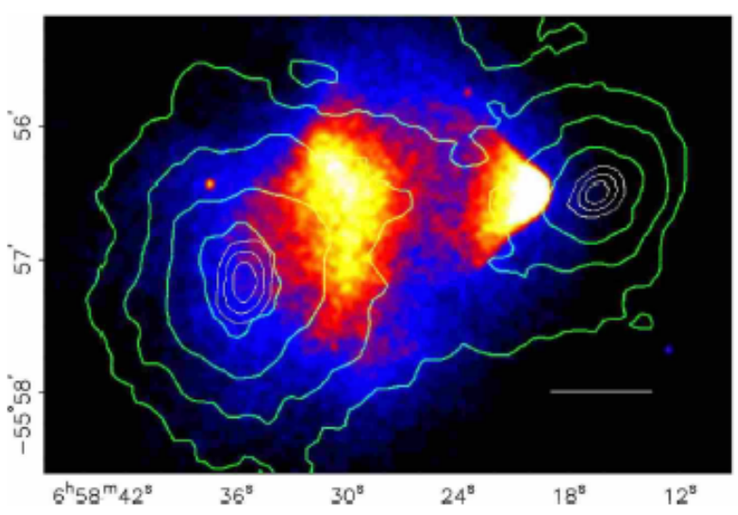
Origin of dark matter

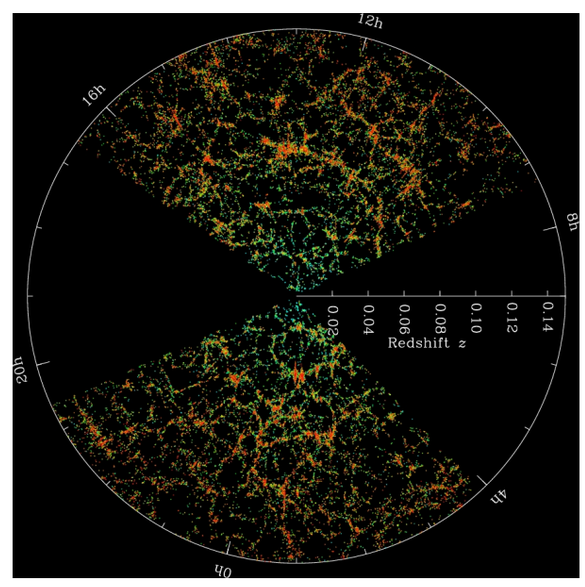
We don't have a clue to the nature of 27% dark matter and 68% dark energy.



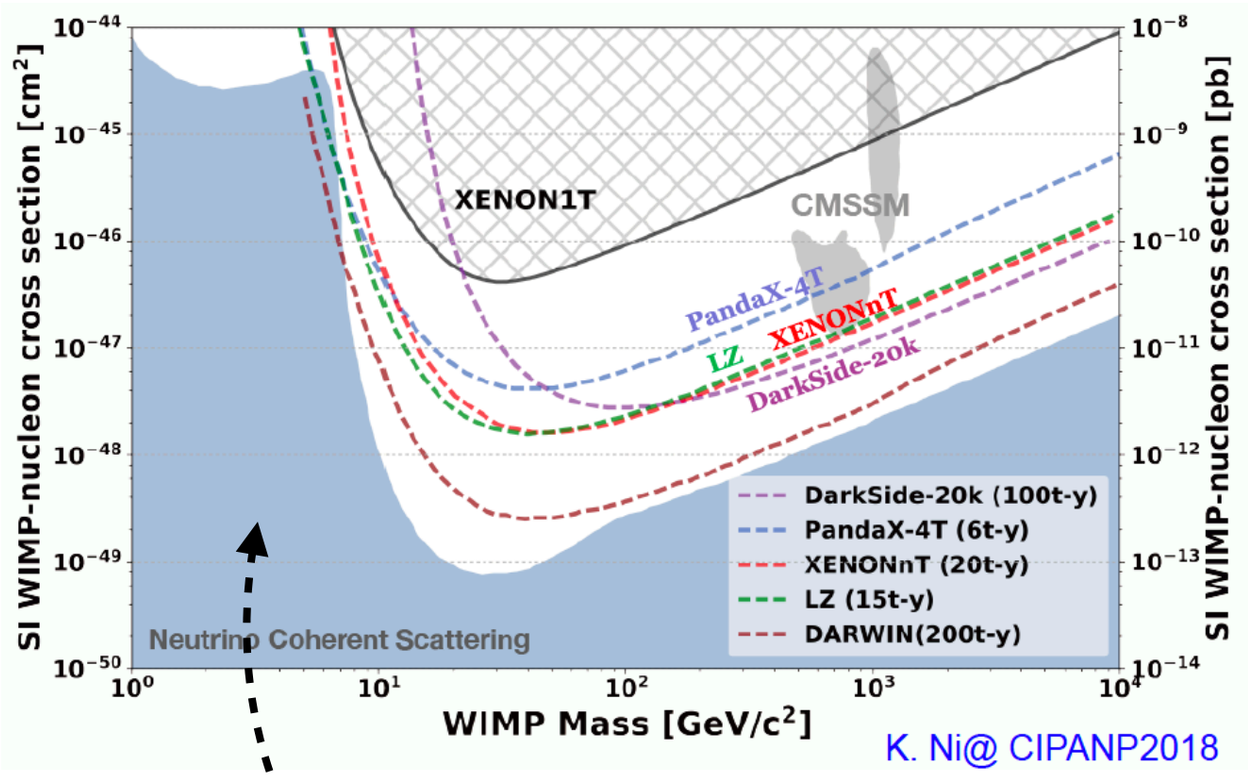


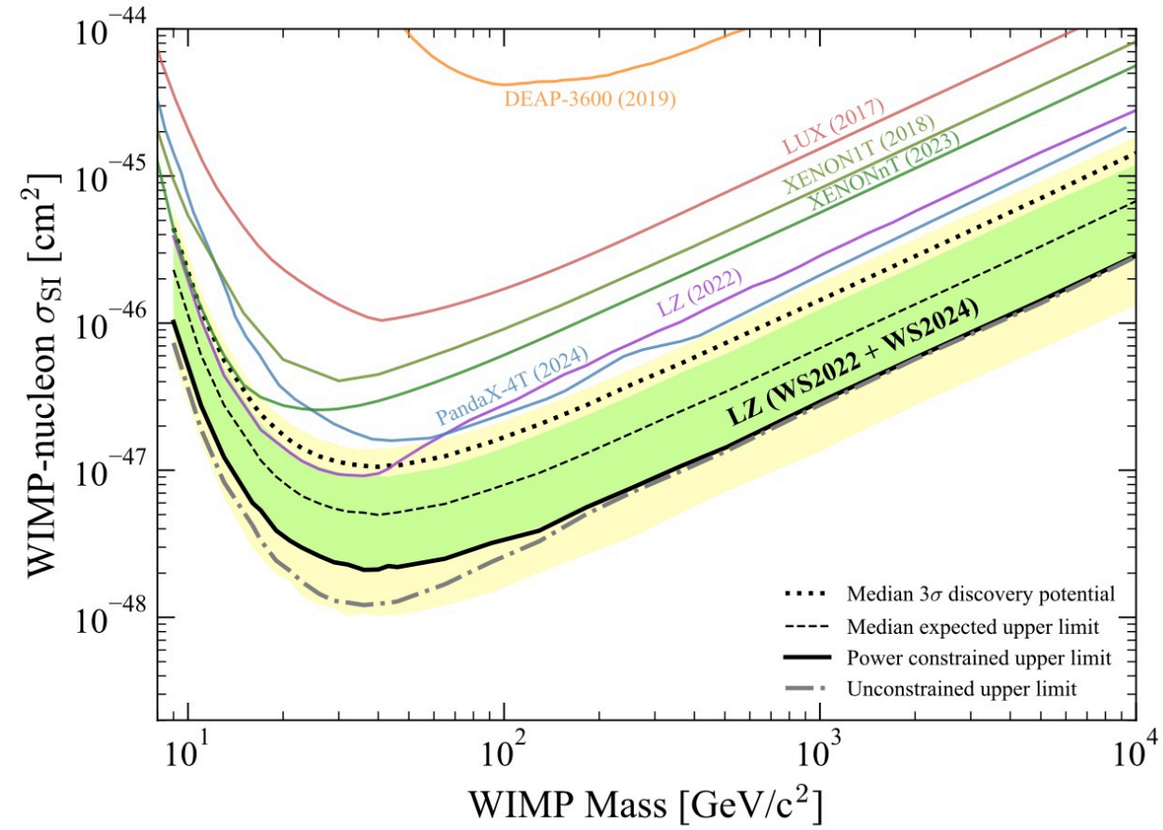


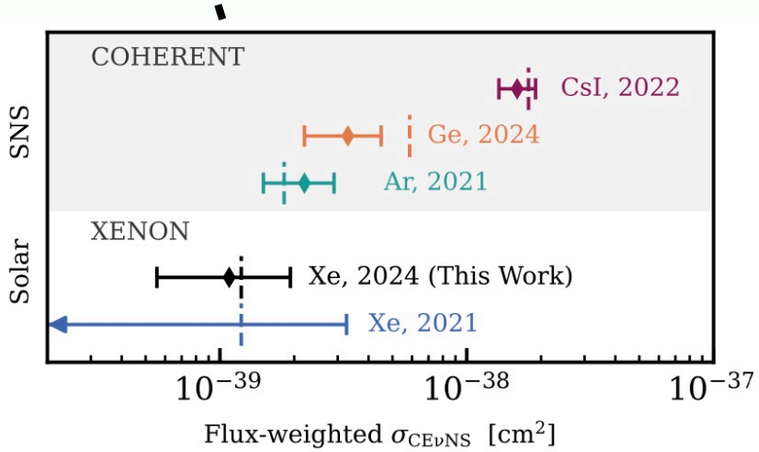




Bounds on WIMP







Solar (^{8}B) neutrinos (2.6-2.7σ),
e.g. 2408.02877

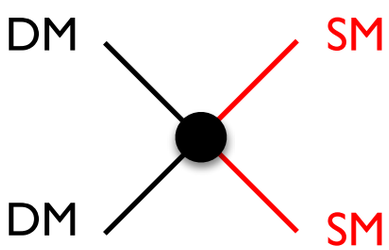
- WIMP-nucleon cross section strongly constrained by direct detection, such as XENON experiments.
- New DM: Axion-like DM, leptophilic DM, resonant DM, etc.

Dark matter production

DM in thermal equilibrium

- **Freeze-out mechanism**

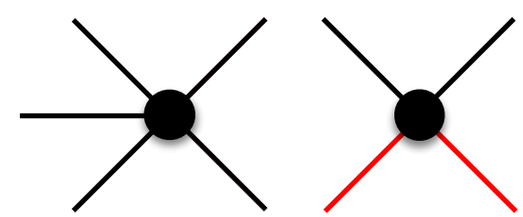
DM annihilation to thermal bath



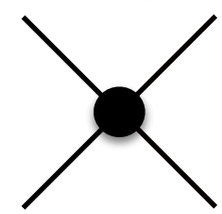
$$\Omega_{\rm DM} \sim \frac{1\,{\rm pb}}{\langle \sigma v \rangle}$$

- **DM self-interactions**

N(≥3)→2 annihilation but small DM-SM interaction



$$\Omega_{\rm DM} \sim \frac{1\,{\rm pb}}{(\alpha_{\rm self}\sigma_{\rm self}/(m_{\rm DM}M_P))^{1/2}}$$

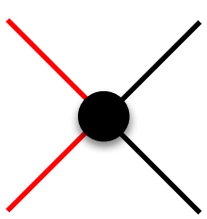


$$\sigma_{\rm self} = \frac{\alpha_{\rm self}^2}{m_{\rm DM}^2} \sim 1\,{\rm cm}^2/{\rm g}$$

DM out of equilibrium

- **Freeze-in mechanism**

Tiny DM-SM interaction



$$\langle \sigma v \rangle = \sigma_0 (T/m_{\rm DM})^n,\ T > m_{\rm DM}$$

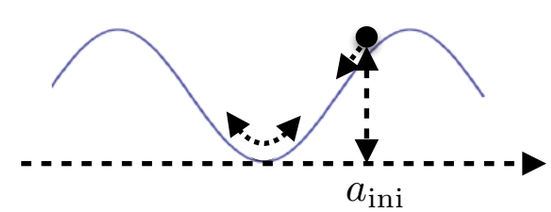
$$\Omega_{\rm DM} \propto \sigma_0 (T_{\rm RH})^{n+1},\ n > -1$$

$n < -1$: $T_{\rm RH}$-independent

- **Misalignment (boson DM)**

DM initial potential density

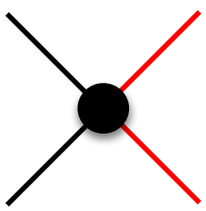
Ultra light DM $\quad m_a \sim H \sim \frac{T_{\rm osc}^2}{M_P}$



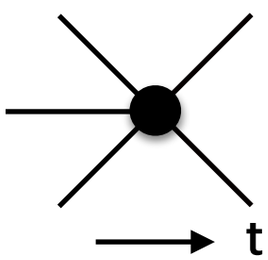
$$\Omega_a h^2 \simeq 0.12 \left(\frac{f_a}{9\times 10^{11}\,{\rm GeV}}\right)^{1.165} \left(\frac{a_{\rm ini}}{f_a}\right)^2$$

Bounds on DM masses

- WIMP/SIMP: Unitarity bounds

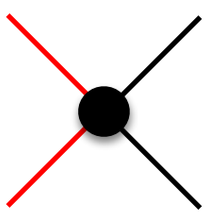


$$(\sigma v)_{\rm th} \simeq 3\times 10^{-26}\,{\rm cm}^3/s < \frac{16\pi}{m_{\rm DM}^2 v} \longrightarrow m_{\rm DM} \lesssim 100\,{\rm TeV}$$



$$\frac{\alpha_{\rm self}\sigma_{\rm self}}{m_{\rm DM}} \simeq 10^{-2}\,{\rm cm}^2/{\rm g} < \frac{16\pi\alpha_{\rm self}}{m_{\rm DM}^3 v^2} \longrightarrow m_{\rm DM} \lesssim 1\,{\rm GeV}$$

- FIMP: Temperature bound



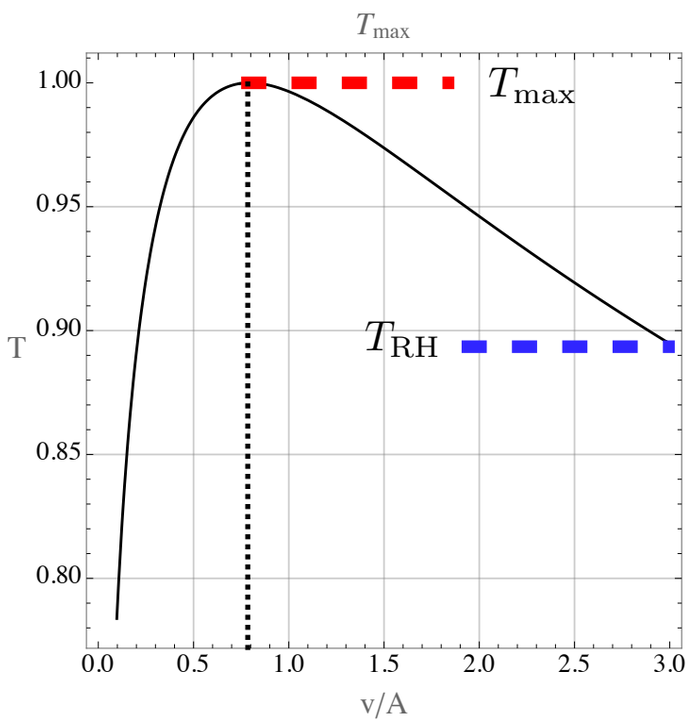
$$\langle\sigma v\rangle \propto (\sigma v)\Big|_{T=m_{\rm DM}} \times e^{-m_{\rm DM}/T} \longrightarrow m_{\rm DM} \lesssim T_{\rm RH}$$

- DM mass beyond unitarity or temperature bound?

- Evolution history (reheating, phase transitions, axion rotation)?

Bounds on DM masses

- Reheating period or low reheating temperature

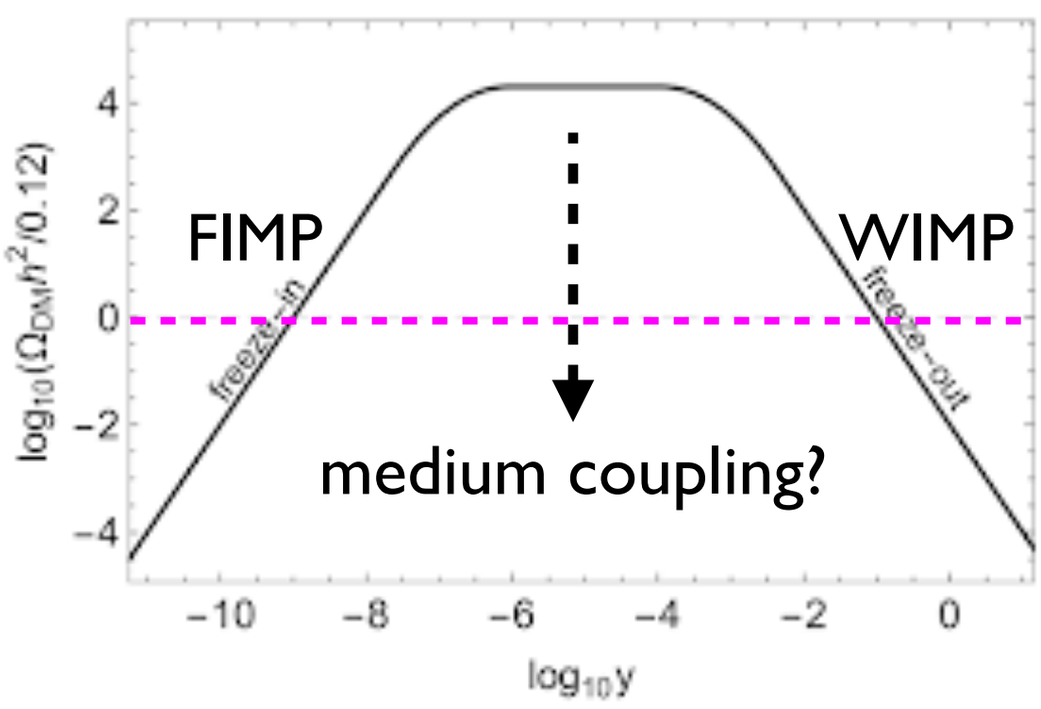


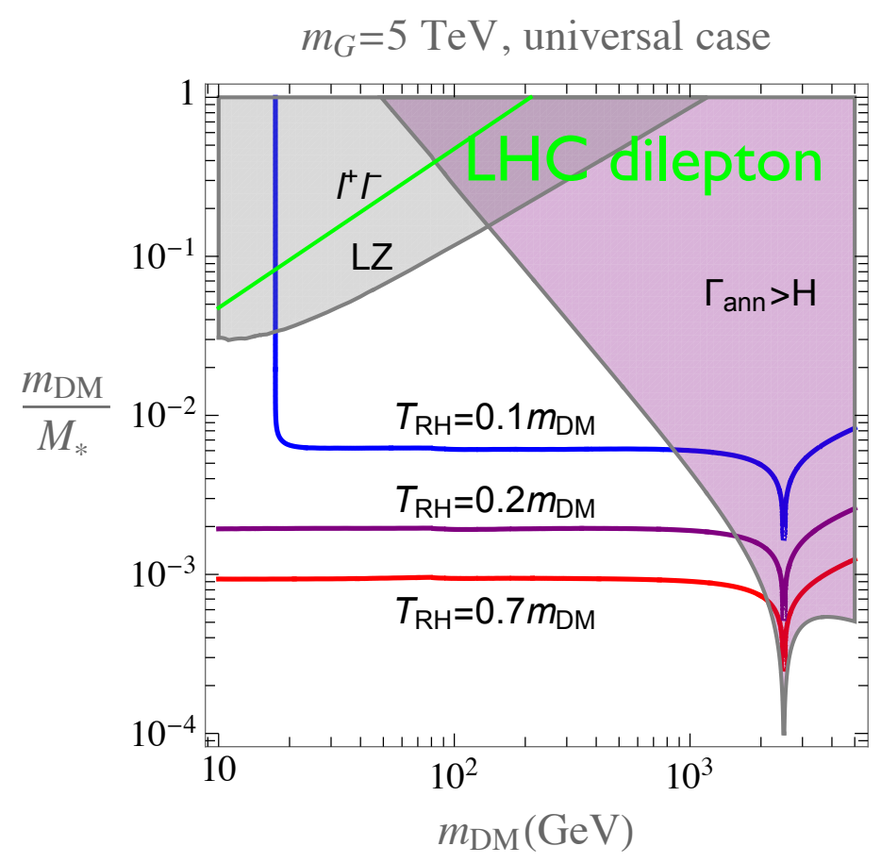
$$T_{\rm RH} \lesssim m_{\rm DM} \lesssim T_{\rm max}, \quad \langle \sigma v \rangle \propto (\sigma v)\Big|_{T=m_{\rm DM}} \times e^{-m_{\rm DM}/T}$$

"Larger cross section"

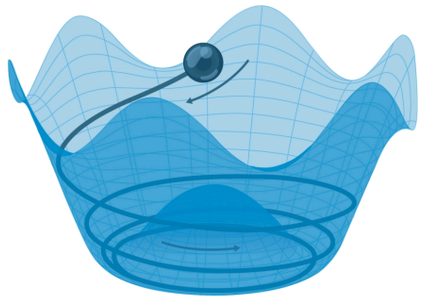
Stronger couplings than FIMP ("Strong freeze-in")

[Cosme et al (2023); Arcadi et al (2024); Bernal et al (2024); Sanz, HML, Park (2024)]

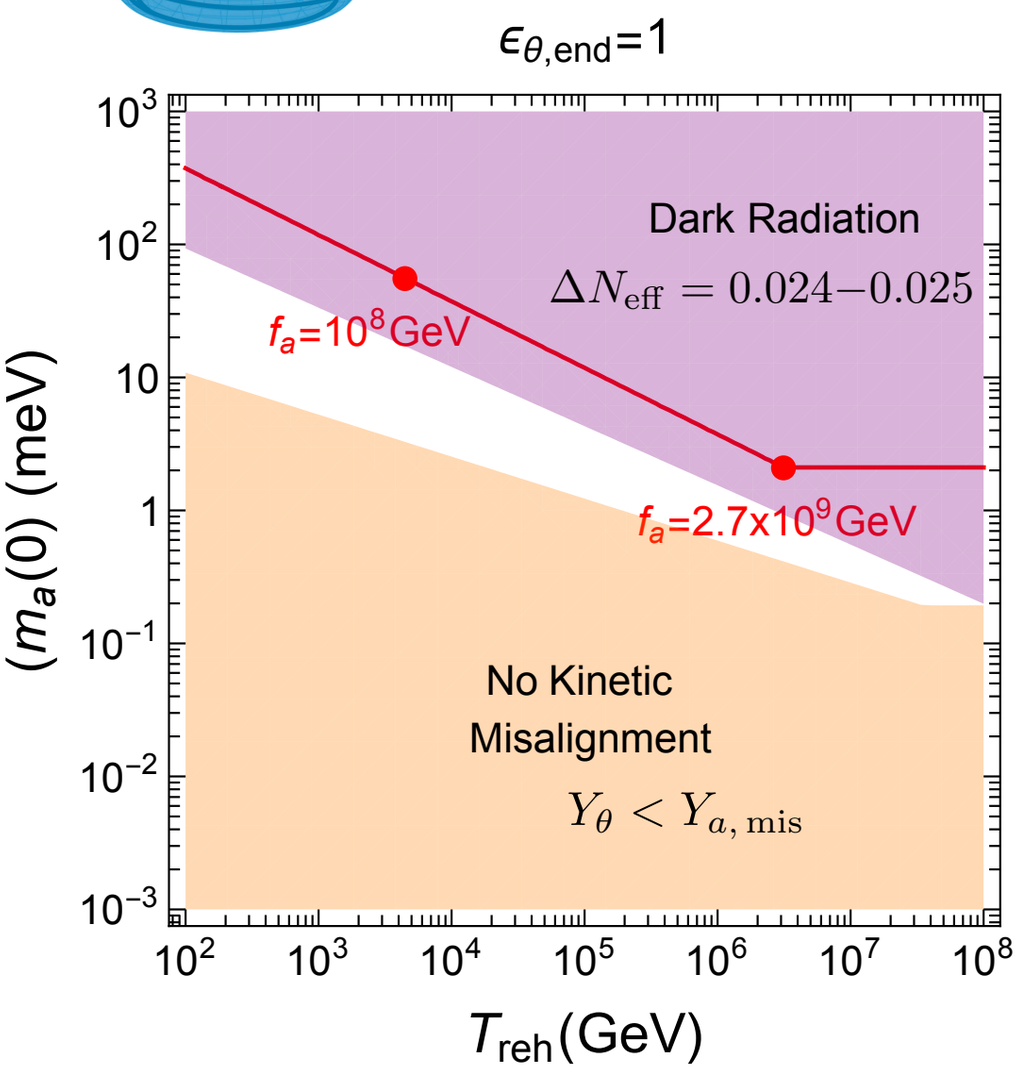


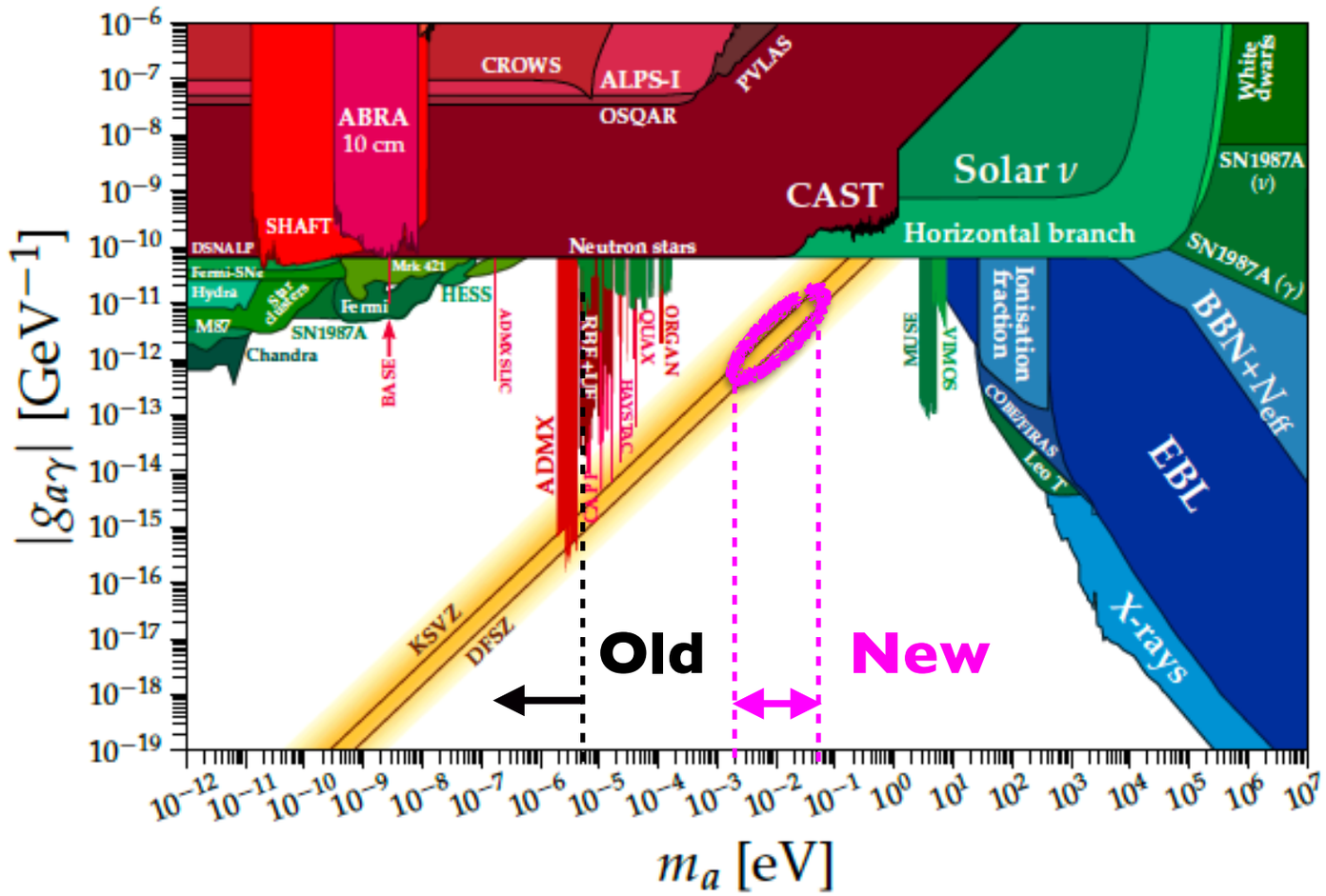


DM relic



QCD axion rotation





[Co, Hall, Harigaya (2019); HML, A. Menkara, M-J, Seung, J-H, Song (2023, 2024)]

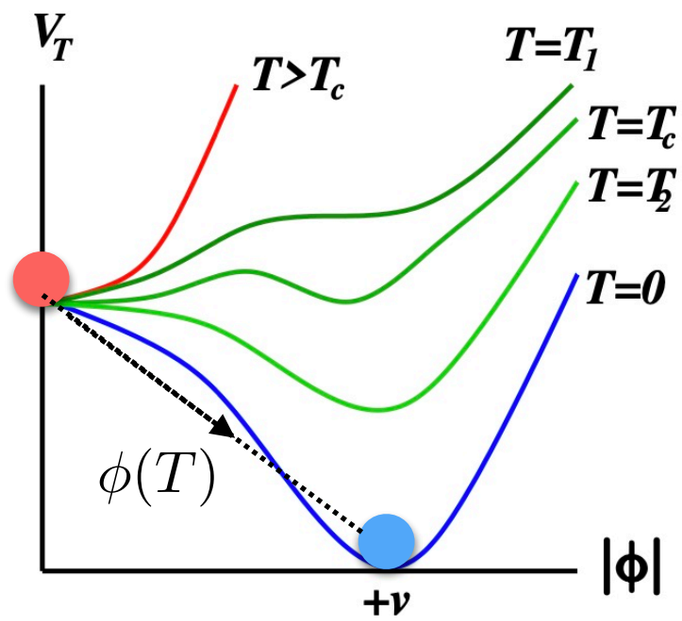
$$\epsilon_\theta \equiv \frac{M_P^2}{2V_E^2}\left(\frac{\partial V_E}{\partial \theta}\right)^2 \longrightarrow Y_\theta = \frac{n_\theta(T_{\rm RH})}{s(T_{\rm RH})} \longrightarrow \Omega_a h^2 = 0.12\left(\frac{10^9\,\mathrm{GeV}}{f_a}\right)\left(\frac{Y_\theta}{40}\right)$$

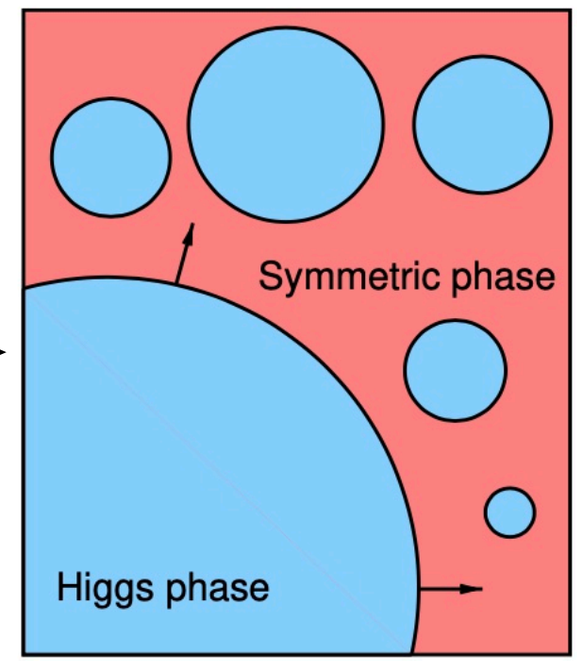
Axion velocity from inflation $n_\theta = f_a^2\dot{\theta}$

Smaller axion decay or larger axion mass

Phase transition and DM

- DM can receive mass via 1st-order phase transitions.

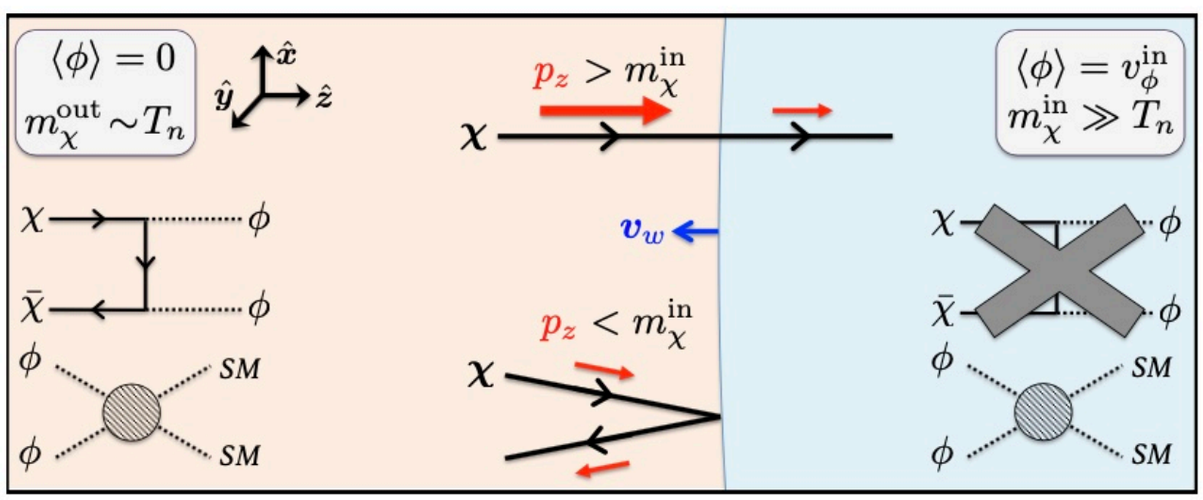




Inside bubbles: true vacuum

Outside bubbles: false vacuum

[M. Hindmarsh et al, 2020]



Outside bubbles Inside bubbles

[M. Baker et al ; D. Chway et al, 2019]

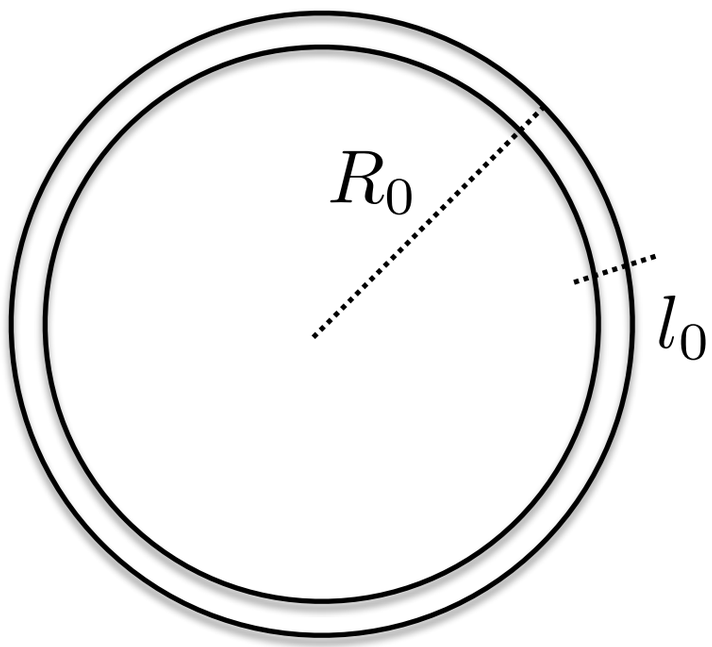
DM abundance is filtered for a non-relativistic bubble wall, $v_w \ll 1$.

DM interactions of order one:

$$\Omega_{\rm DM} h^2 \sim 0.1 \left(\frac{T_n}{1\,{\rm TeV}}\right)\left(\frac{x}{30}\right)^{5/2} e^{-(x-30)}, \quad x = \frac{m_{\rm DM}}{T_n}$$

$m_{\rm DM} \sim 30 T_n$, DM mass beyond T.

Bubble nucleation



$T \gg R_0^{-1}$: O(3) symmetric bubble

Bounce action: $S_4 = \int d\tau d^3x \mathcal{L}_E = \frac{S_3}{T},$

$$S_3 = 4\pi \int_0^\infty s^2 ds \left[\frac{1}{2} \left(\frac{d\phi}{ds} \right)^2 + V_T(\phi) \right], \; s = |\vec{x}|$$

Bubble profile: $\frac{d^2\phi}{ds^2} + \frac{2}{s}\frac{d\phi}{ds} - V_T'(\phi) = 0,$ w/ b.c. $\frac{d\phi}{dr} = 0, \quad \text{at} \quad r = 0,$

$\phi = 0, \quad \text{at} \quad r = \infty.$

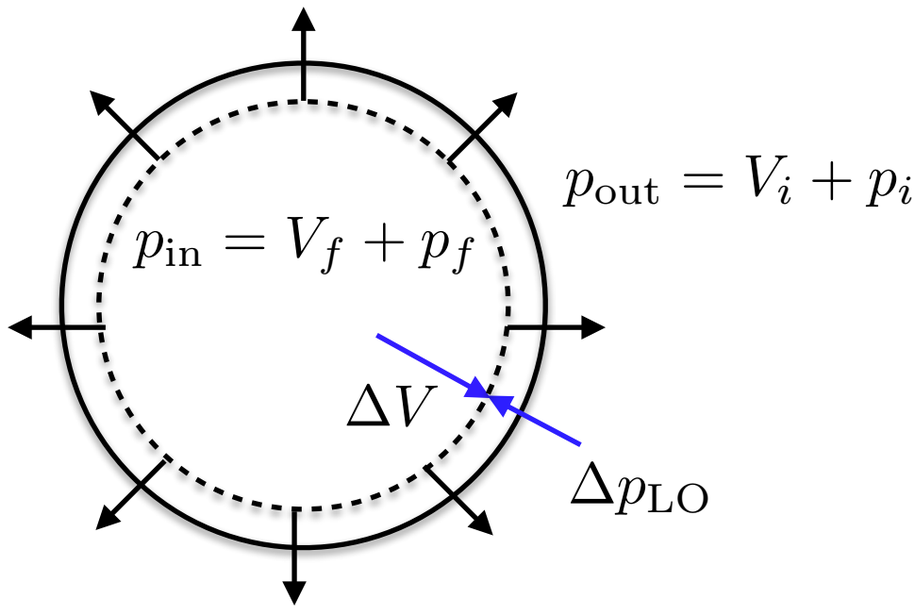
Thin-wall approximation: $l_0 \ll R_0$

$$S_3 \simeq 4\pi\sigma R^2 - \frac{4\pi}{3}\Delta V R^3 = \frac{16\pi}{3}\frac{\sigma^3}{(\Delta V)^2} \longrightarrow R_0 = \frac{2\sigma}{\Delta V}$$ "critical radius"

Bubble nucleation rate & inverse duration time:

$$\Gamma_n \sim T_n^4 e^{-S_4(T_n)} \sim H(T_n), \qquad \frac{\beta}{H_*} = T \frac{dS_4}{dT}\bigg|_{T_*} - 4 \gtrsim 1$$

Running bubbles



Bubble dynamics:

$$\mathcal{L} = -2\pi\sigma R^2\sqrt{1-\dot{R}^2} + \frac{4\pi}{3}R^3 p,$$

$$p = \Delta V - \Delta p_{\rm LO} - \gamma\Delta p_{\rm NLO}$$

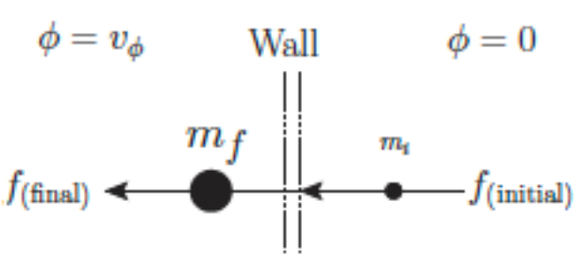
$\longrightarrow$ e.o.m: $\dfrac{d\gamma}{dR} + \dfrac{2\gamma}{R} = \dfrac{p}{\sigma}$

1) Difference in FT potential: $\Delta p_{\rm LO} = \Delta V_{\rm FT} = \dfrac{1}{24}(\Delta m^2)T^2$

[Boedecker, Moore, 2009]

See also I. Baldes, 2024.

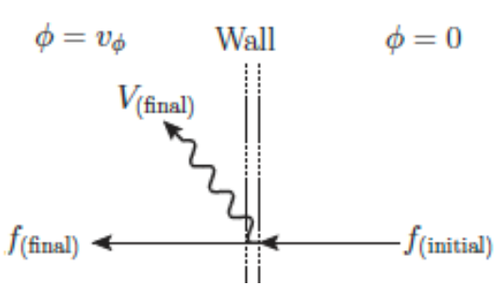
$$\gamma_{\rm max} = \frac{2R_{\rm max}}{3R_0} \quad , \quad R_{\rm max} \sim v_w \beta^{-1}$$



$$\Delta V = c_V v_\phi^4,\ T \sim v_\phi \longrightarrow \gamma_{\rm max} \sim \frac{1}{(\beta/H_*)}\frac{M_P}{v_\phi} \gg 1$$

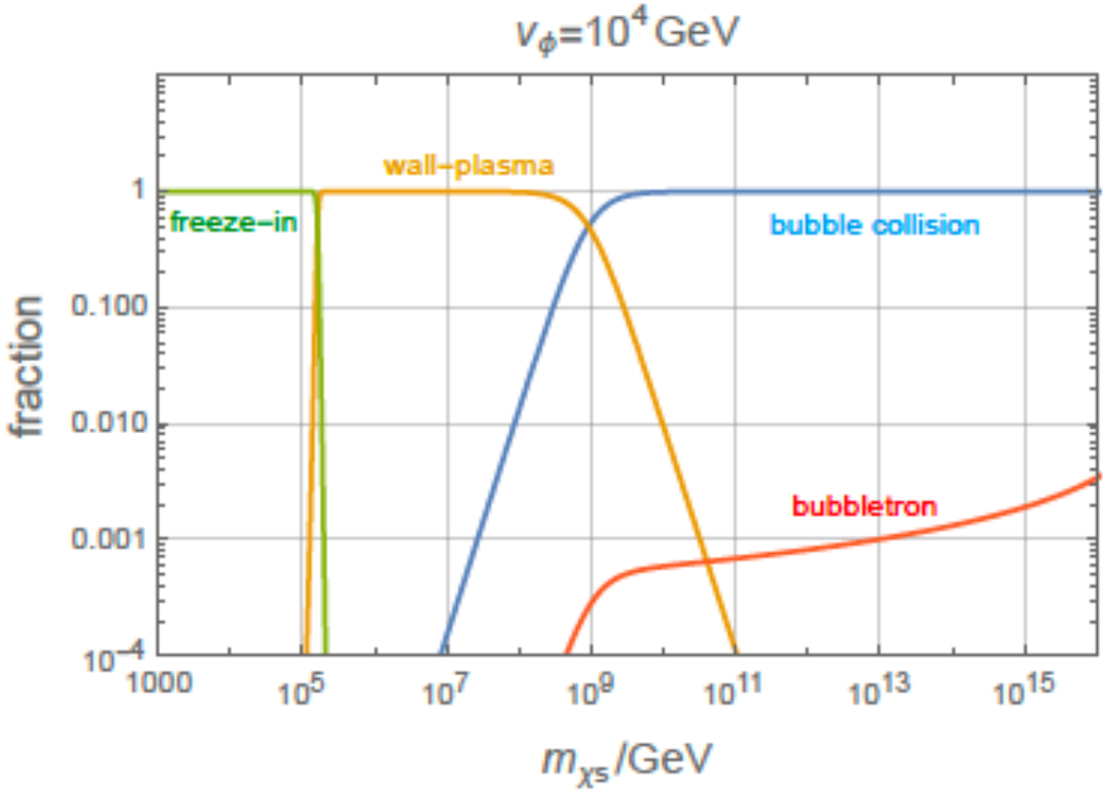
2) Pressure due to radiation of light particles:

[Boedecker, Moore, 2017; Gouttennoire et al, 2021] $\gamma\Delta p_{\rm NLO} = \gamma g^2 \Delta m_V T^3$

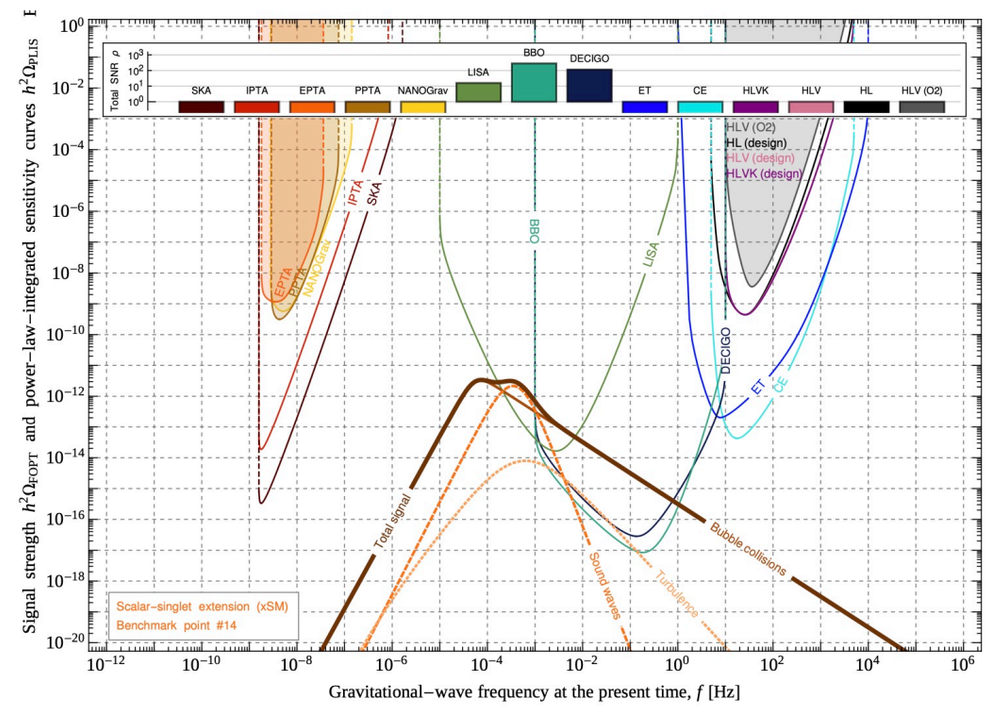


$$p = 0 \longrightarrow \gamma_{\rm eq} = \frac{\Delta V - \Delta p_{\rm LO}}{\Delta p_{\rm NLO}} \simeq \frac{\Delta V}{g^3 v_\phi T^3} \sim \frac{c_V}{g^3} \gg 1, \quad g \ll 1$$

Bubble collisions: DM & GW



[G. Giudice et al, 2024]



[K. Schmitz, 2020]

DM density: $$\Omega_\chi h^2 \approx 0.1 \left(\frac{\beta/H}{10}\right)\left(\frac{\alpha}{(1+\alpha)g_* c_V}\right)^{1/4} \frac{m_\chi v_\phi}{(2.5\ \mathrm{TeV})^2} \int_{p^2_{\min}}^{p^2_{\max}} \frac{dp^2}{p^4}\, \mathrm{Im}[\tilde{\Gamma}^{(2)}(p^2)]$$

GW spectrum: $$\Omega_{\rm coll}(f)\, h^2 = 1.66\times 10^{-5}\left(\frac{H_*}{\beta}\right)^2 \frac{\kappa^2_{\rm coll}\alpha^2}{(1+\alpha)^2}\left(\frac{100}{g_*(T_*)}\right)^{1/3} \frac{v_w^3}{1+2.4v_w^2}\cdot\frac{(f/f_{\rm coll})^{2.8}}{1+2.8(f/f_{\rm coll})^{3.8}}$$

$$\alpha = \frac{\Delta V}{\rho_R},\ v_\phi \sim T$$

Experiment	$f_{optimal}$/Hz	v_ϕ/GeV	$m_{\rm DM}$/GeV
Pulsar Timing Arrays (PTAs) [107]	10^{-8}	0.1	$10^{13}-10^{16}$
LISA [108]	0.001	10^4	10^6-10^{15}
BBO [109], DECIGO [110]	0.1	10^6	10^5-10^{13}
Einstein Telescope (ET) [111], Cosmic Explorer (CE) [112]	10	10^8	10^6-10^{10}

Dark matter from bubble collisions

Green function formalism

Probability for particle production: $\mathcal{P} = 2\,\mathrm{Im}\,(\,\Gamma[\phi]\,)$

$\Gamma[\phi]$, Effective action or generating function of 1PI Green functions

Quadratic in ϕ: $\Gamma[\phi] = \sum_{n=2}^{\infty} \frac{1}{n!} \int d^4x_1 ... d^4x_n \Gamma^{(n)}(x_1, ..., x_n)\phi(x_1)...\phi(x_n)$

$$\longrightarrow \quad \mathcal{P} = \int \frac{d^4p}{(2\pi)^4} |\tilde{\phi}(p)|^2 \,\mathrm{Im}\Big(\widetilde{\Gamma}^{(2)}(p^2)\Big), \qquad \tilde{\phi}(p) = \int d^4x\, \phi(x)\, e^{ip\cdot x}, \text{ etc.}$$

Bubble profile in momentum space: $\tilde{\phi}(p) = (2\pi)^2 \delta(p_x)\delta(p_y)\tilde{\phi}(p_z, \omega)$

Number density of particles per area: [R. Watkins, L.M. Widrow, 1992]

$$\frac{N}{A} = 2 \int \frac{dp_z\, d\omega}{(2\pi)^2} |\tilde{\phi}(p_z, \omega)|^2 \,\mathrm{Im}[\tilde{\Gamma}^{(2)}(\omega^2 - p_z^2)]$$

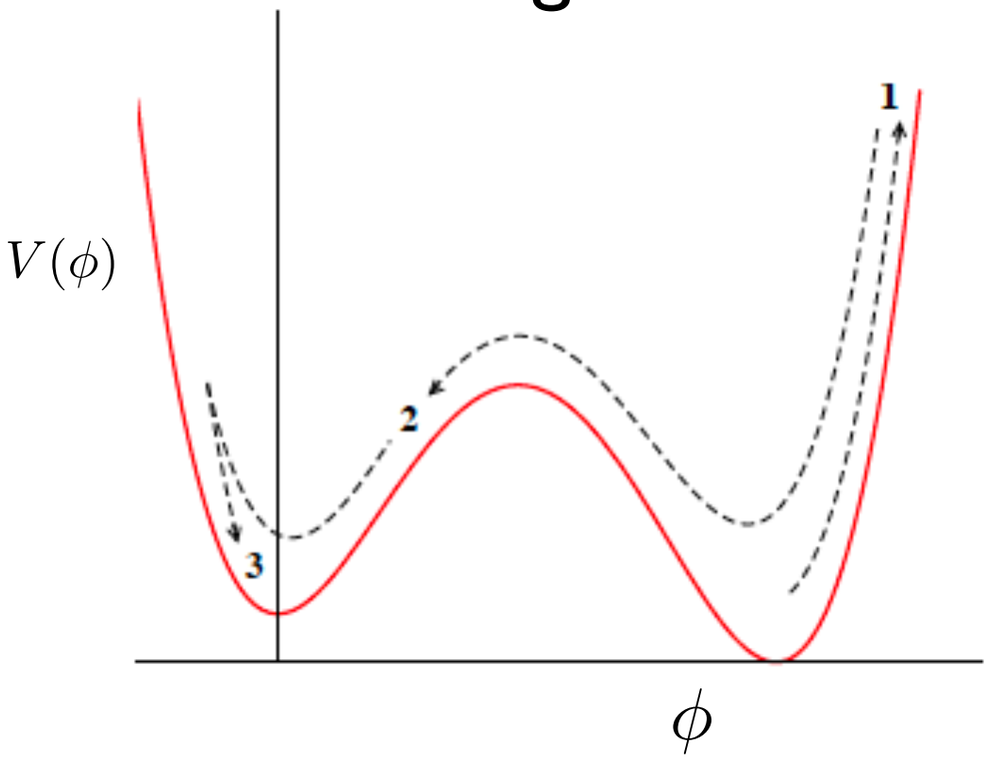
Background dynamics

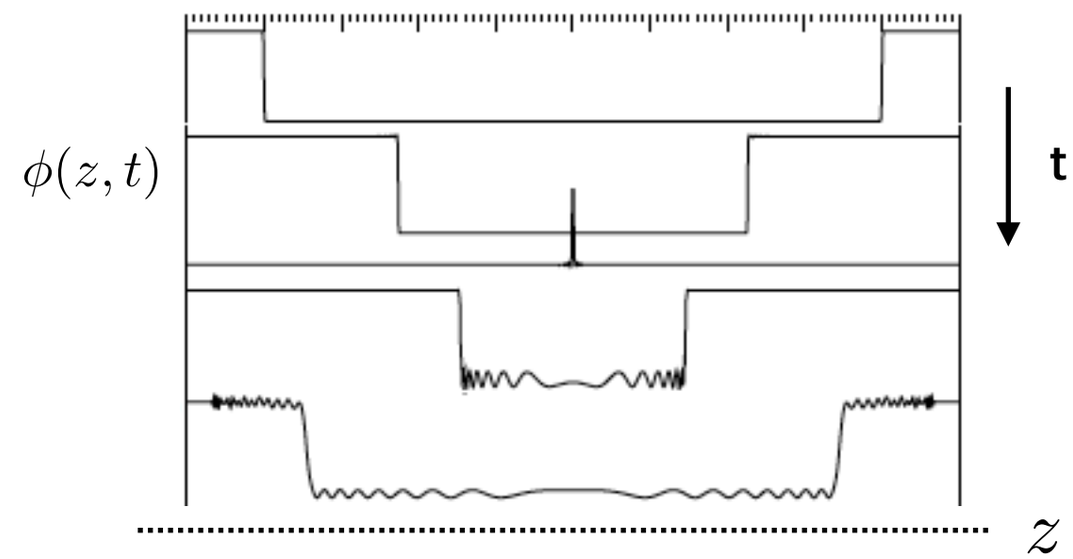
Particle physics info.

Bubble collisions: elastic

Almost degenerate minima: elastic bubble collision

=> scalar waves, particle production





[A. Falkowski, J.M. No, 2012]

Thin wall: $\phi(z,t) = \begin{cases} 0 \quad , & z^2 < v_w^2 t^2, \\ v_\phi \quad , & z^2 > v_w^2 t^2. \end{cases} \longrightarrow \tilde{\phi}(p_z,\omega) = \dfrac{4 v_w v_\phi}{\omega^2 - v_w^2 p_z^2}$

Running bubbles, $v_w \simeq 1 \longrightarrow \tilde{\phi}(p_z,\omega) \simeq \dfrac{4 v_\phi}{p^2}$, IR dominant

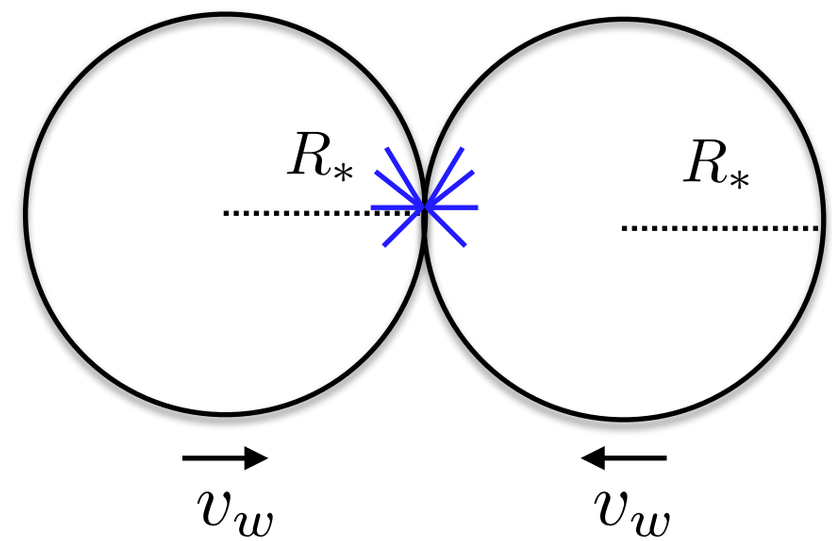
Thick wall: $\phi(z,t) = \dfrac{v_\phi}{2}\left(2 + \tanh\left(\dfrac{z-|t|}{l_w}\right) - \tanh\left(\dfrac{z+|t|}{l_w}\right)\right), \; \underline{l_w = \dfrac{l_0}{\gamma_w}}.$

$$\longrightarrow \tilde{\phi}(p,\omega) \simeq \frac{2\pi l_w v_\phi \omega}{\sinh(\frac{1}{2}\pi l_w \omega)} \frac{1}{p^2} \propto e^{-\pi l_w \omega/2}, \qquad \omega \gg 1/l_w.$$

Bubble collisions: elastic

Produced particles on surface diffuse over the bubble volume.

$$n = \frac{N}{V} = \frac{3}{4\pi^2 R_*} \int_{p^2_{\rm min}}^{p^2_{\rm max}} dp^2\, f(p^2) {\rm Im}\left[\widetilde{\Gamma}^{(2)}(p^2)\right],$$



$$p_{\rm max} = 2/l_w,\ \ p_{\rm min} = \min(2m, (2R_*)^{-1})$$

"Efficiency" factor for particle production

$$f(p^2) = \frac{1}{4}\int_{p^2}^{\infty} \frac{d\Psi}{\sqrt{\Psi^2 - p^4}}\, |\tilde{\phi}(p_z, \omega)|^2, \quad p^2 = \omega^2 - p_z^2$$

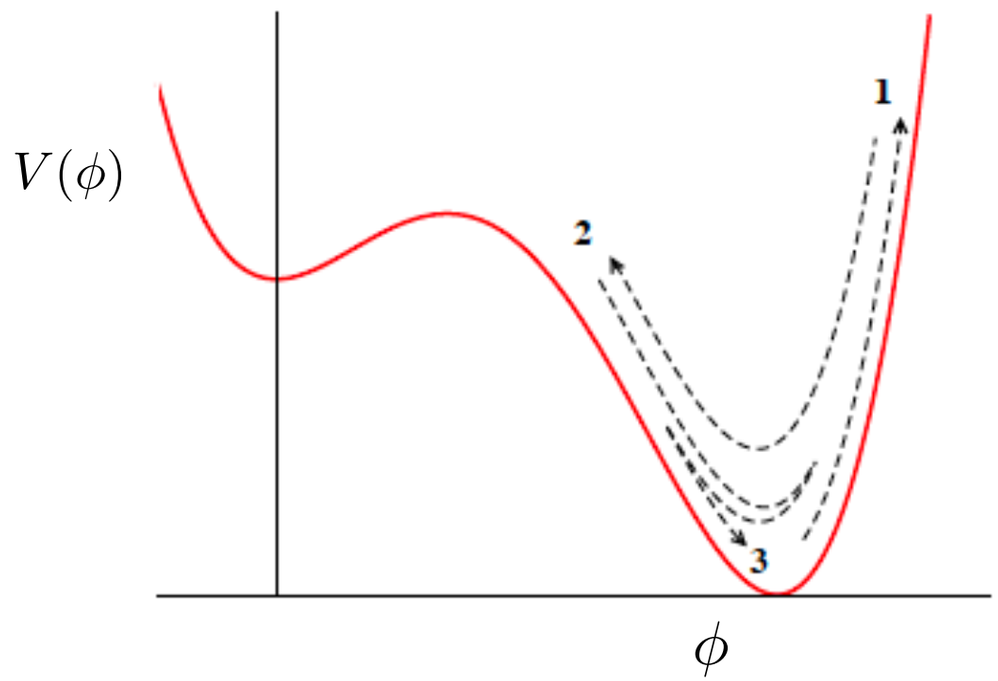
Elastic bubbles: $\tilde{\phi}(p,\omega) \simeq \dfrac{2\pi l_w v_\phi \omega}{\sinh(\frac{1}{2}\pi l_w \omega)}\, \dfrac{1}{p^2} \propto e^{-\pi l_w \omega/2}, \quad \omega \gg 1/l_w.$

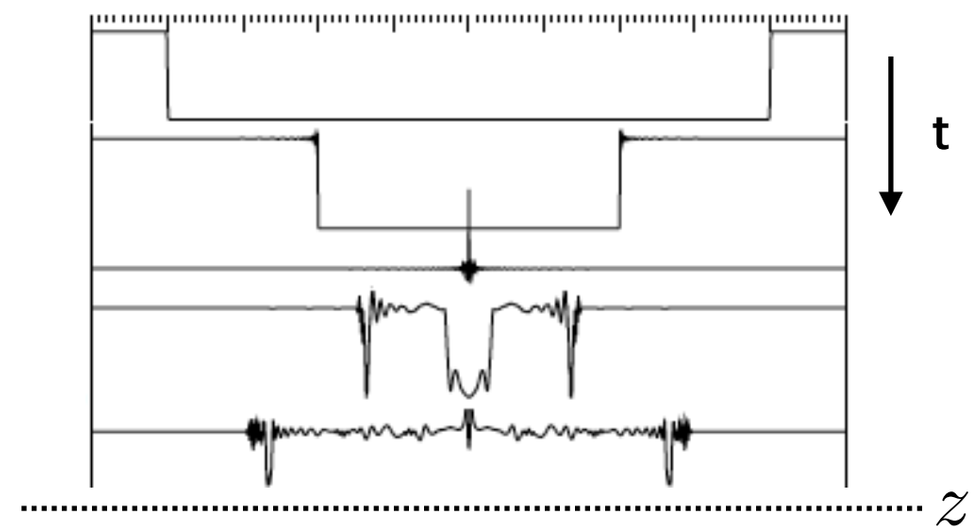
$$\omega \sim m \sim \frac{\gamma_w}{l_0} \sim \gamma_w v_\phi \gg T$$ => unsuppressed production

Bubble collisions: inelastic

Non-degenerate minima: inelastic bubble collision

=> scalar waves, mainly





[A. Falkowski, J.M. No, 2012]

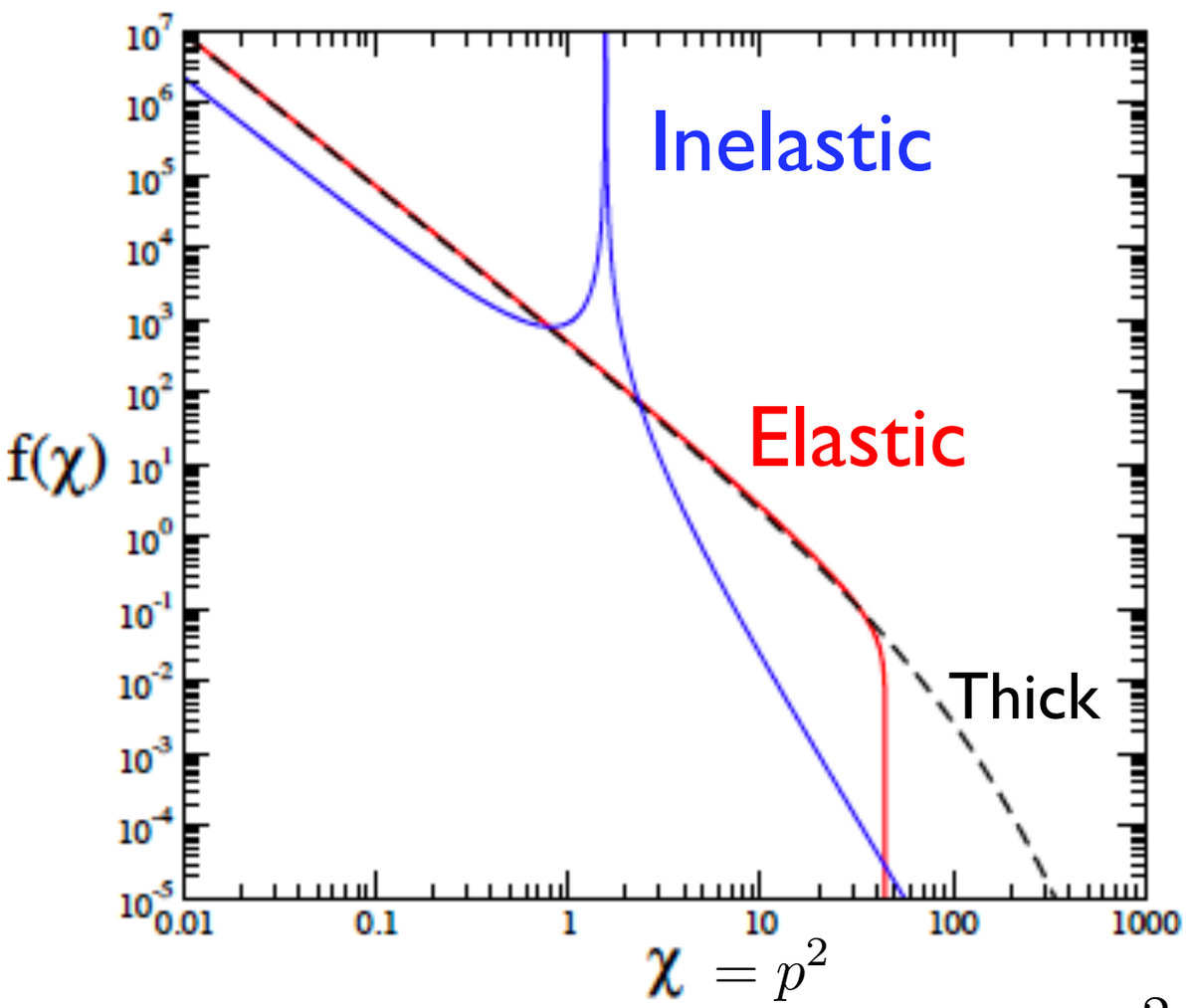
Equation for bubble profile: $(\partial_t^2 - \partial_z^2)\phi = -\frac{\partial V}{\partial \phi}\,, \quad t>0: \; V(\phi) \simeq \frac{1}{2} m_\phi^2 (\phi - v_\phi)^2,$

$$\longrightarrow \quad \phi(z,t) = v_\phi + v_\phi l_w \int_0^\infty dp_z \frac{p_z}{\sqrt{p_z^2 + m_\phi^2}} \frac{\cos(p_z z)}{\sinh(\frac{1}{2}\pi l_w p_z)} \sin(\sqrt{p_z^2 + m_\phi^2}\, t), \; t>0.$$

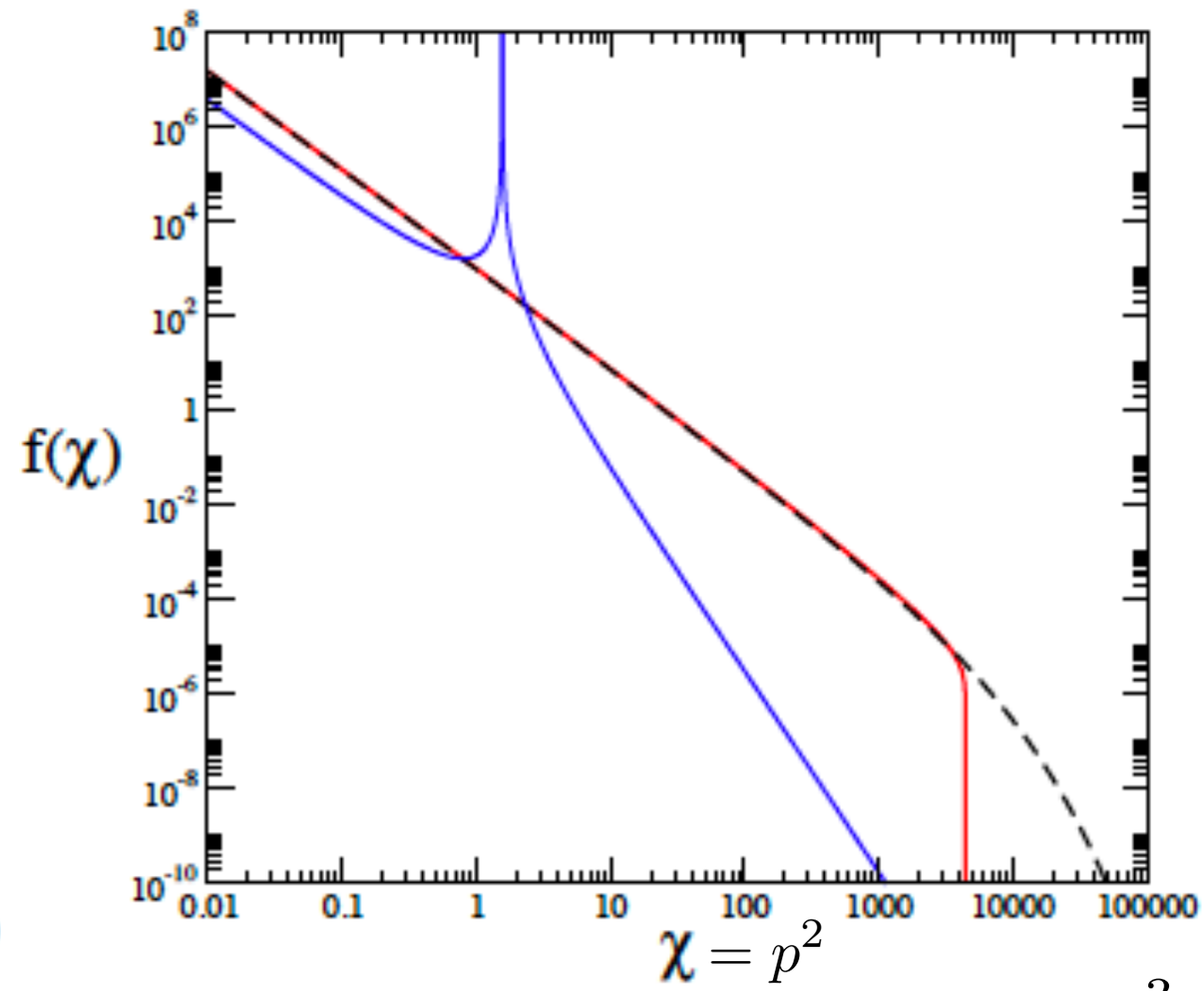
$$\longrightarrow \quad \tilde{\phi}(p_z,\omega) = \frac{\pi l_w v_\phi p_z}{\sinh(\frac{1}{2}\pi l_w p_z)} \left(\frac{1}{\omega^2 - p_z^2} - \frac{1}{\omega^2 - p_z^2 - m_h^2} \right),$$

$$m_{\rm DM} \gg m_h : \qquad f(p^2) \simeq f(p^2)\Big|_{\rm elastic}$$

Elastic vs inelastic collisions



$\gamma_w = 10^2$



$\gamma_w = 10^3$

Perfectly Elastic: $f_{\rm PE}(p^2) = \frac{16 v_\phi^2}{p^4} \mathrm{Log}\left[\frac{2(1/l_w)^2 - p^2 + 2(1/l_w)\sqrt{(1/l_w)^2 - p^2}}{p^2}\right]$

[A. Falkowski, J.M. No, 2012]

Elastic collision describes well beyond the resonance.

2-body vs 3-body decays

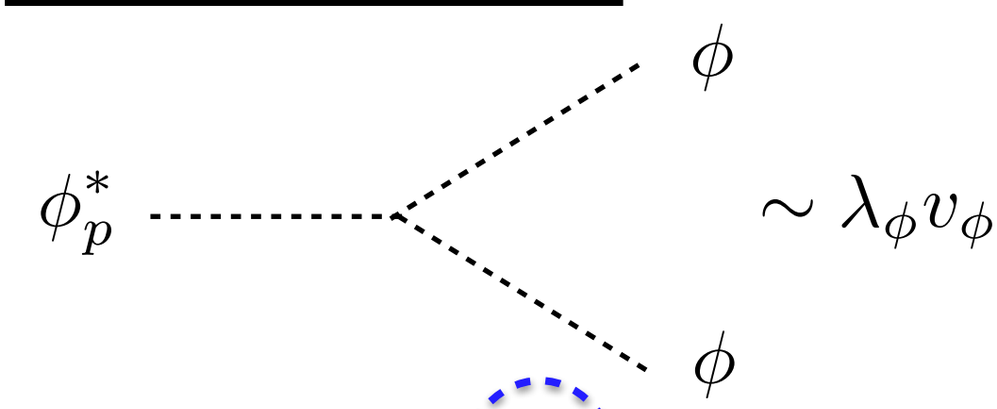
$\widetilde{\Gamma}^{(2)}(p^2)$: 2-point 1PI Green function (or self-energy)

Optical theorem: $\mathrm{Im}[\tilde{\Gamma}^{(2)}(p^2)] = \frac{1}{2}\sum_k \int d\Pi_k |\bar{\mathcal{M}}(\phi_p^* \to \underline{k})|^2$

sum over all decays of off-shell scalar

Scalar self-interaction: $\frac{\lambda_\phi}{4!}\phi^4 \longrightarrow \frac{\lambda_\phi}{3!} v_\phi \bar{\phi}^3 + \frac{\lambda_\phi}{4!}\bar{\phi}^4$

2-body decay

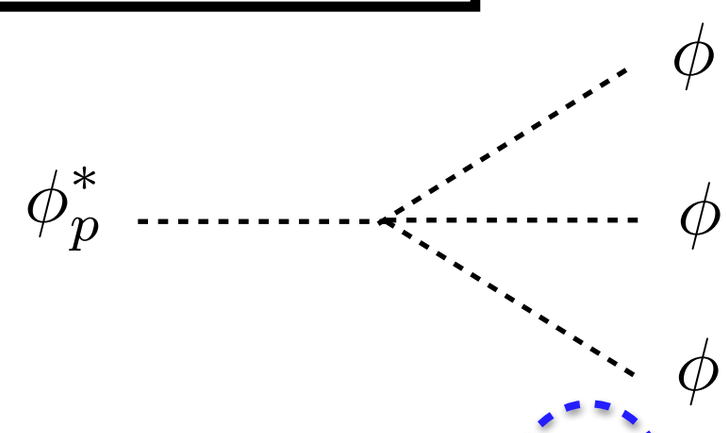


$\sim \lambda_\phi v_\phi$

$$\mathrm{Im}[\tilde{\Gamma}^{(2)}(p^2)]_{\phi_p^* \to \phi\phi} = \frac{\lambda_\phi^2 v_\phi^2}{8\pi}(1 - 4m_\phi^2/p^2)\,\Theta(p - 2m_\phi)$$

3-body decay

[G. Giudice et al, 2024]



$\sim \lambda_\phi$

$$\mathrm{Im}[\tilde{\Gamma}^{(2)}(p^2)]_{\phi_p^* \to 3\phi} = \frac{\lambda_\phi^2 p^2}{3072\,\pi^3}(1 - 9m_\phi^2/p^2)\,\Theta(p - 3m_\phi)$$

$p^2 \gg v_\phi^2$: highly off-shell => 3-body process dominates!

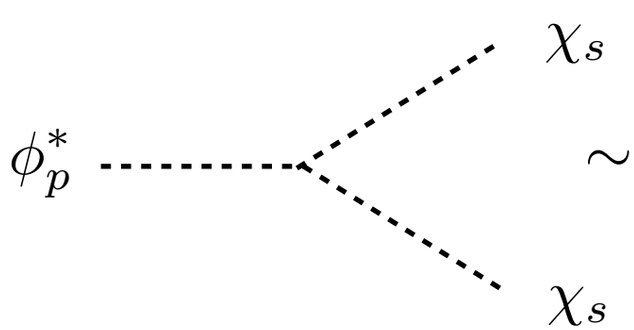
Scalar vs fermion DM

Assume that DM mass receives minor corrections from SSB.

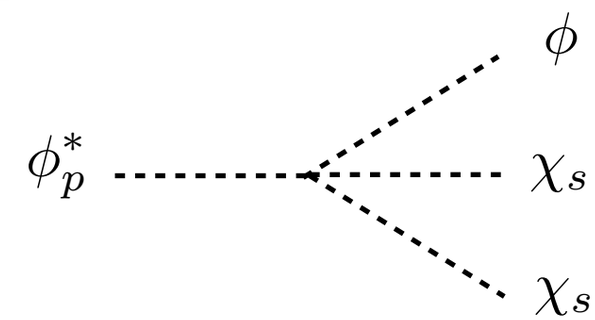
Scalar DM χ_s

[G. Giudice et al, 2024]

$$\mathcal{L}_{\chi_s} = -\frac{\lambda_s}{4}\phi^2\chi_s^2 \longrightarrow \Delta m_{\chi_s}^2 = \frac{1}{2}\lambda_s v_\phi^2 \lesssim m_{\chi_s}, \quad \mathcal{L}_{\rm int} = -\frac{\lambda_s}{2} v_\phi \bar{\phi}\chi_s^2$$



$\sim \lambda_s v_\phi$



$\sim \lambda_s$

$p^2 \sim m_{\chi_s}^2 \gg v_\phi^2$: 3-body process dominant for SDM production.

Fermion DM χ_f

$$\mathcal{L}_{\chi_f} = -y_f \phi \bar{\chi}_f \chi_f \longrightarrow \Delta m_{\chi_f} = y_f v_\phi \lesssim m_{\chi_f}$$

Fermion DM self-energy: 2-body process is sufficient.

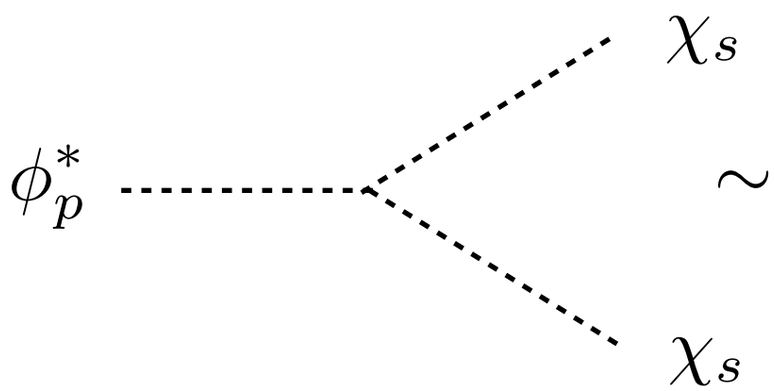
$$\mathrm{Im}[\tilde{\Gamma}^{(2)}(p^2)]_{\phi_p^* \to \chi_f \bar{\chi}_f} = \frac{y_f^2}{8\pi} p^2 (1 - 4m_{\chi_f}^2/p^2)^{3/2}\, \Theta(p^2 - 4m_{\chi_f}^2) \sim p^2$$

Scalar DM relic density

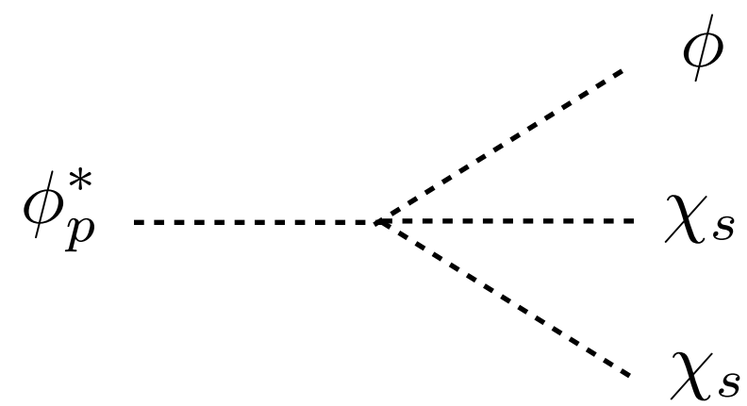
Scalar DM can be produced through 2- & 3-body decays.

Scalar DM χ_s

$$\frac{\lambda_s}{4}\phi^2\chi_s^2 \longrightarrow \frac{\lambda_s}{4}(v_\phi+\bar{\phi})^2\chi_s^2 \to \frac{\lambda_s}{2}v_\phi\bar{\phi}\chi_s^2$$



$\sim \lambda_s v_\phi$



$\sim \lambda_s$

DM relic density: [G. Giudice et al, 2024]

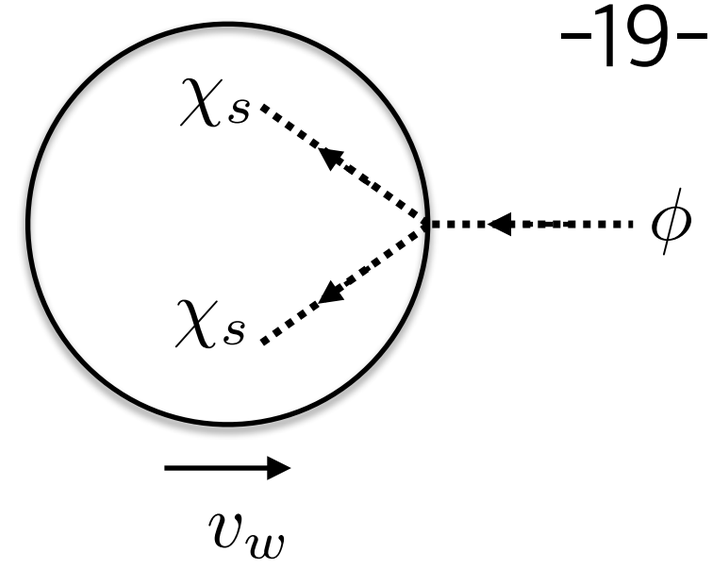
$$\Omega_\chi h^2 \approx 0.1\,\frac{\beta/H}{10}\left(\frac{\alpha}{(1+\alpha)g_* c_V}\right)^{1/4}\frac{\lambda_s^2 m_{\chi_s} v_\phi}{(24\ \mathrm{TeV})^2}\left[\underbrace{\frac{v_\phi^2}{m_{\chi_s}^2}}_{\text{2-body}}+\underbrace{\frac{1}{16\pi^2}\ln\left(\frac{2\gamma_w/l_{w0}}{(2m_{\chi_s}+m_\phi)}\right)}_{\text{3-body}}\right]$$

$$\alpha=\frac{\Delta V}{\rho_R},\ \Delta V = c_V v_\phi^4$$

$m_{\chi_s} \gg v_\phi \sim T$ ⟶ dominant 3-body process!

Wall-plasma interactions

[Azakov, Vanvlasselaer, 2020; + W. Yin, 2021]



A wall scalar with boost can split into DM.

1. kinematic condition: $p_\phi = \gamma T_* > 2m_{\chi_s}$

2. non-adiabatic condition: $(\Delta p_z) l_0 \lesssim 1, \;\; \Delta p_z = \dfrac{m_{\chi_s}^2}{2p_\phi}$

$$\gamma T_* \gtrsim m_{\chi_s}^2 l_0 \sim \frac{m_{\chi_s}^2}{v_\phi} : \quad m_{\chi_s} \gg v_\phi \longrightarrow \gamma T_* \gg m_{\chi_s}$$

Heavy DM

much stronger than kinematic condition.

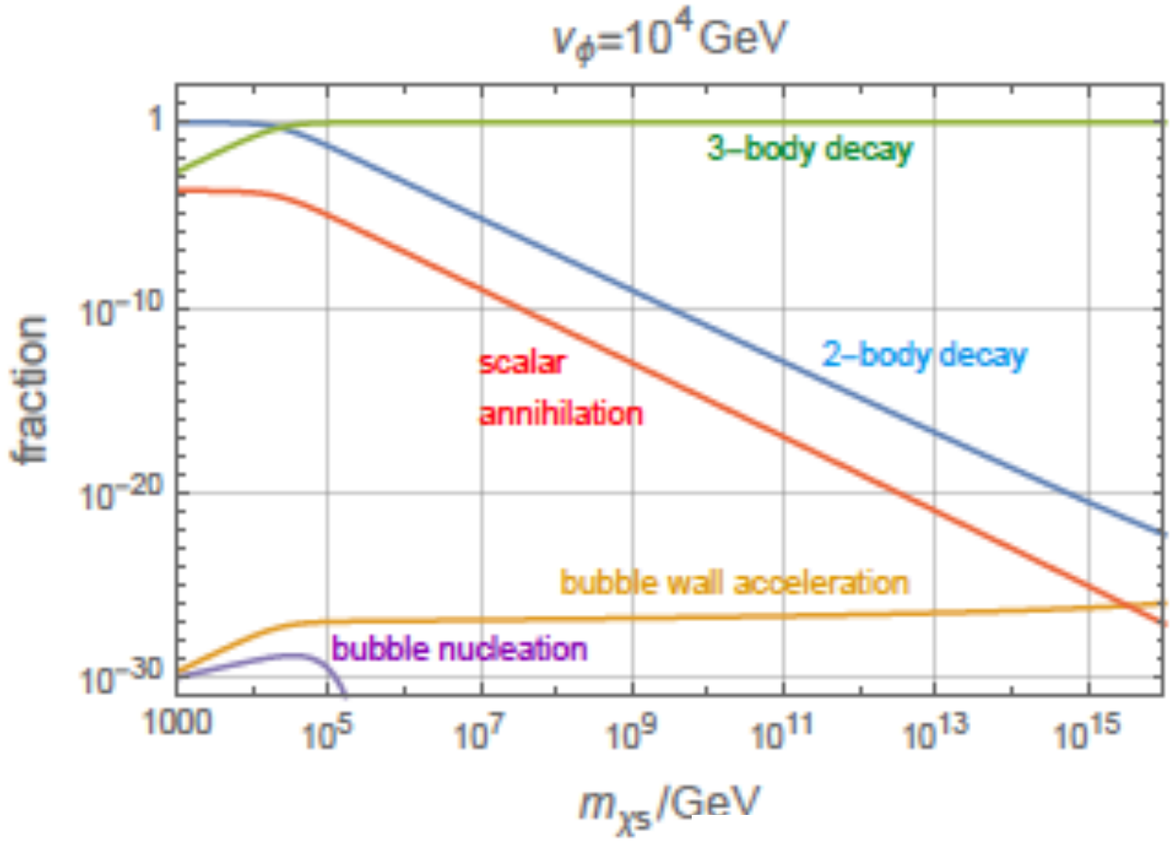
Thus, wall-plasma effects suppressed for mildly boosted wall.

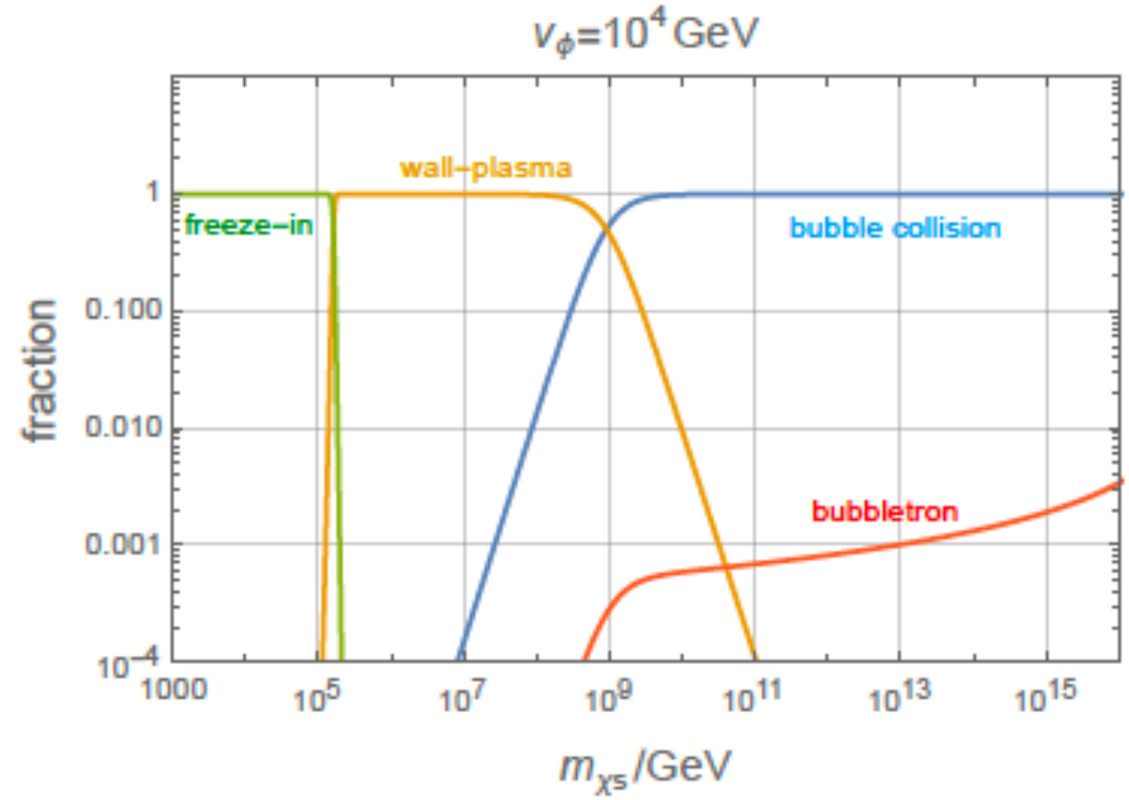
Extra relic abundance: $$\Omega_\chi h^2 \approx \frac{1.35 \times 10^5 \lambda_s^2}{g_*} \frac{v_\phi^2}{m_\chi^2} \frac{m_\chi}{\mathrm{GeV}} \left(\frac{T_n}{T_*}\right)^3$$

[G. Giudice et al, 2024]

[See also J. Cembranos et al, 2024.]

Bubble collisions for DM





$$c_V = 0.1,\ \beta/H = 10,\ \alpha = 1,\ R_0 = 10/v_\phi,\ l_{w0} = 1/v_\phi$$

[G. Giudice et al, 2024]

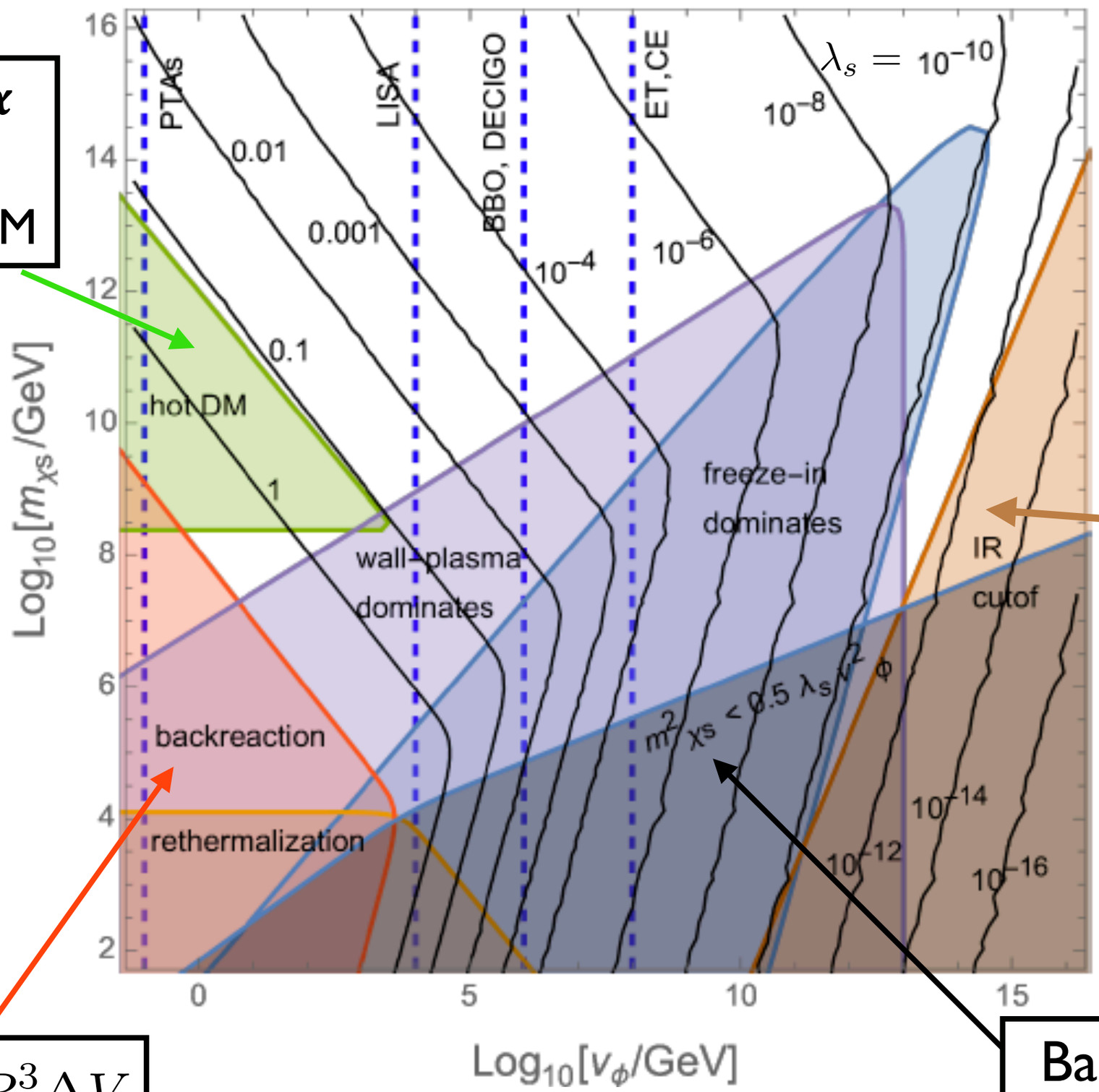
DM density: $$\Omega_\chi h^2 \approx 0.1 \left(\frac{\beta/H}{10}\right)\left(\frac{\alpha}{(1+\alpha)g_* c_V}\right)^{1/4} \frac{m_\chi\, v_\phi}{(2.5\ \mathrm{TeV})^2} \int_{p^2_{\min}}^{p^2_{\max}} \frac{dp^2}{p^4}\, \mathrm{Im}[\tilde{\Gamma}^{(2)}(p^2)]$$

Heavy dark matter: $m_{\chi_s} \gg v_\phi \sim T_*$ $\qquad \alpha = \frac{\Delta V}{\rho_R},\ v_\phi \sim T$

⟶ 3-body decay process from bubble collision.

Scalar dark matter

Lyman-α bounds on hot DM



Portal for scalar DM:

$$\mathcal{L}_{\chi_s} = \frac{\lambda_s}{4}\phi^2\chi_s^2$$

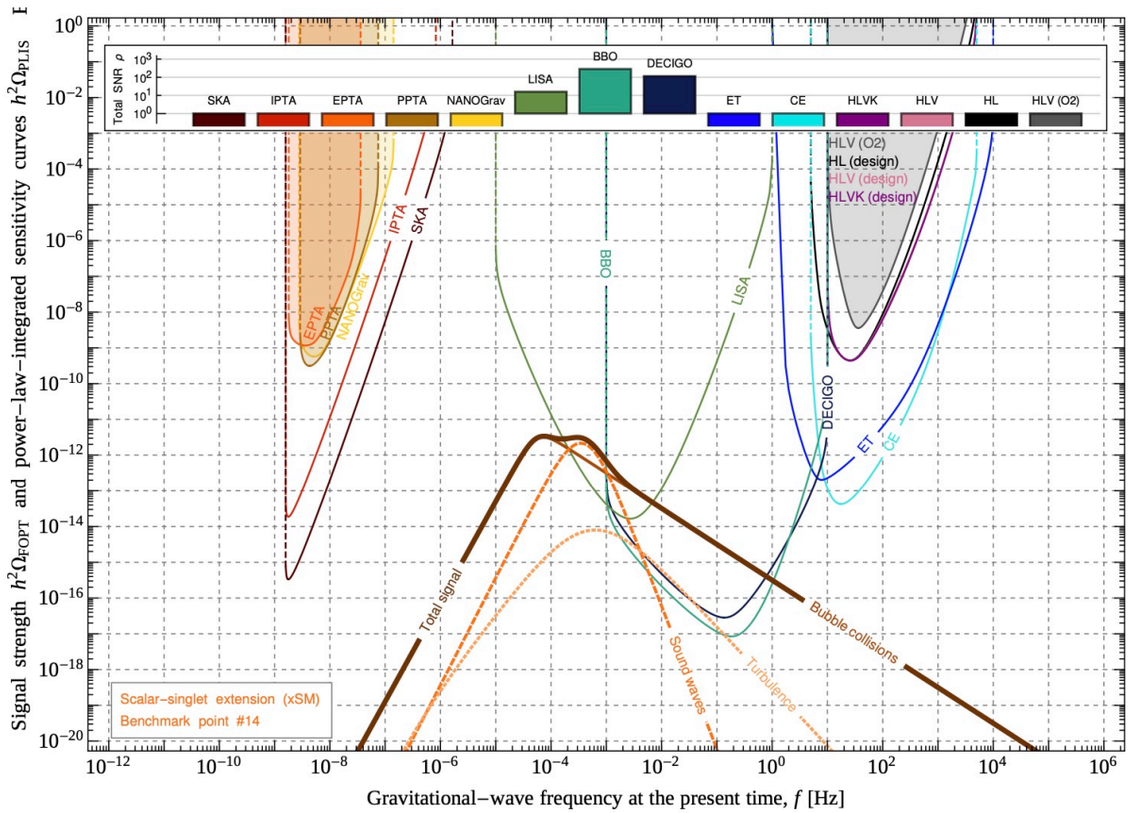
$m_{\chi_s} < R_*^{-1}$

Multiple bubbles in the Fourier analysis needed.

$E_{\chi_s} \gtrsim R_*^3 \Delta V$

Bare DM mass is small: higher order corrections

GWs from bubble collisions



[K. Schmitz, 2020]

Bubble wall energy is released to GWs.

$\boxed{\text{i) Bubble collision}}$

ii) Plasma friction

$\left\{ \begin{array}{l} \text{sound waves} \\ \text{turbulences} \end{array} \right.$

$\kappa_{\rm coll}$: efficiency parameters

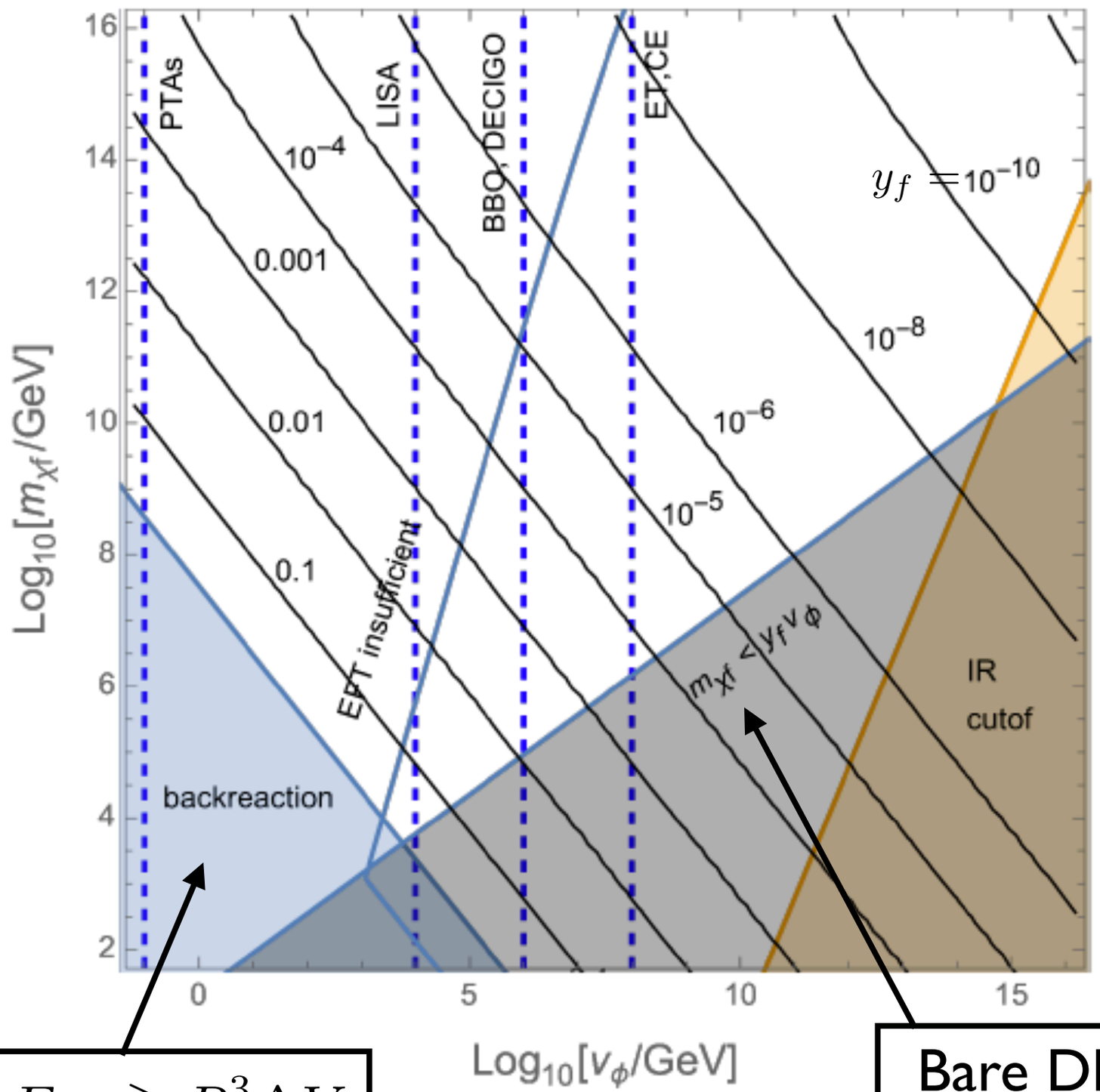
Peak frequency: $$f_{\rm coll} = 56.8{\rm Hz}\left(\frac{\beta/H_*}{10}\right)\left(\frac{T_*}{10^8\,{\rm GeV}}\right)\left(\frac{1}{1-0.05v_w+0.55v_w^2}\right)\left(\frac{g_*(T_*)}{100}\right)^{1/6}$$

GW spectrum: $$\Omega_{\rm coll}(f)\,h^2 = 1.66\times10^{-5}\left(\frac{H_*}{\beta}\right)^2\frac{\kappa_{\rm coll}^2\alpha^2}{(1+\alpha)^2}\left(\frac{100}{g_*(T_*)}\right)^{1/3}\frac{v_w^3}{1+2.4v_w^2}\cdot\frac{(f/f_{\rm coll})^{2.8}}{1+2.8(f/f_{\rm coll})^{3.8}}$$

DM vs GW

Experiment	$f_{optimal}$/Hz	v_ϕ/GeV	$m_{\rm DM}$/GeV
Pulsar Timing Arrays (PTAs) [107]	10^{-8}	0.1	$10^{13}-10^{16}$
LISA [108]	0.001	10^4	10^6-10^{15}
BBO [109], DECIGO [110]	0.1	10^6	10^5-10^{13}
Einstein Telescope (ET) [111], Cosmic Explorer (CE) [112]	10	10^8	10^6-10^{10}

Fermion dark matter



EFT for fermion DM:

$$\mathcal{L}_{\chi_f} = \frac{1}{\Lambda_f}\phi^2\bar{\chi}_f\chi_f$$

$$\longrightarrow \mathcal{L}_{\rm eff} = y_f\phi\,\bar{\chi}_f\chi_f,$$

$$y_f = \frac{v_\phi}{\Lambda_f},\ \Lambda_f < m_{\chi_f}, v_\phi$$

Hot DM, rethermalization, freeze-in bounds are not shown: UV-dependent.

$$E_{\chi_f} \gtrsim R_*^3\Delta V$$

Bare DM mass is small: higher order corrections

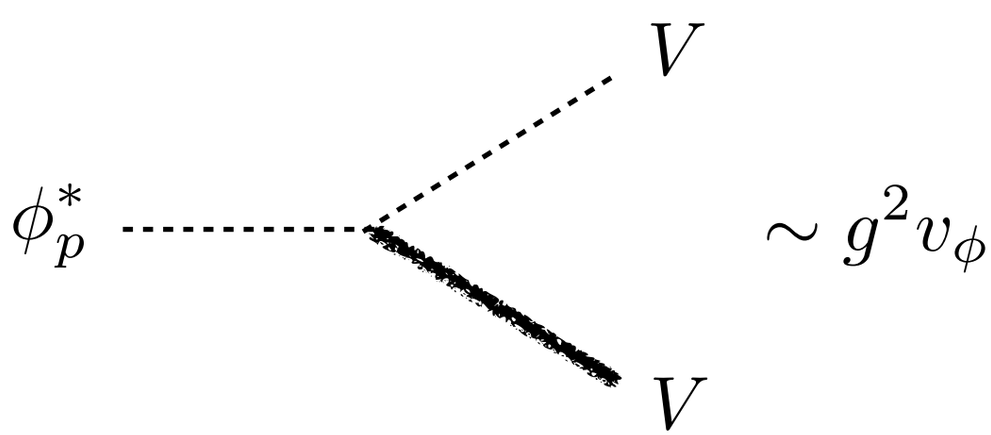
Vector dark matter and gauge choices

Vector dark matter

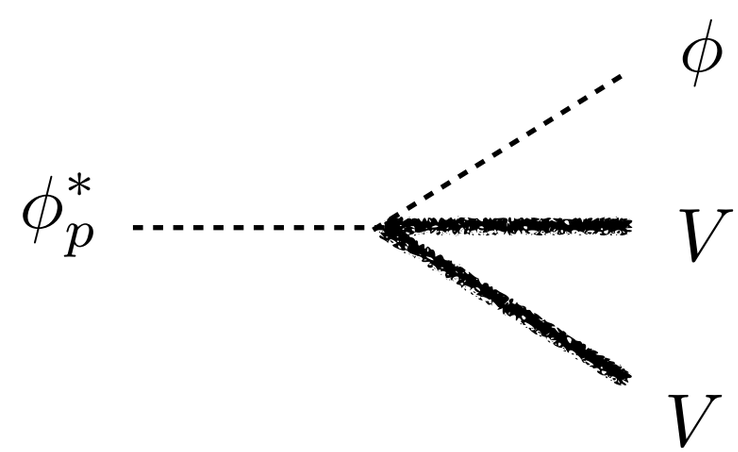
Vector DM

$$g^2\phi^2 V_\mu V^\mu \longrightarrow g^2(v_\phi + \bar{\phi})^2 V_\mu V^\mu$$

e.g. in unitary gauge



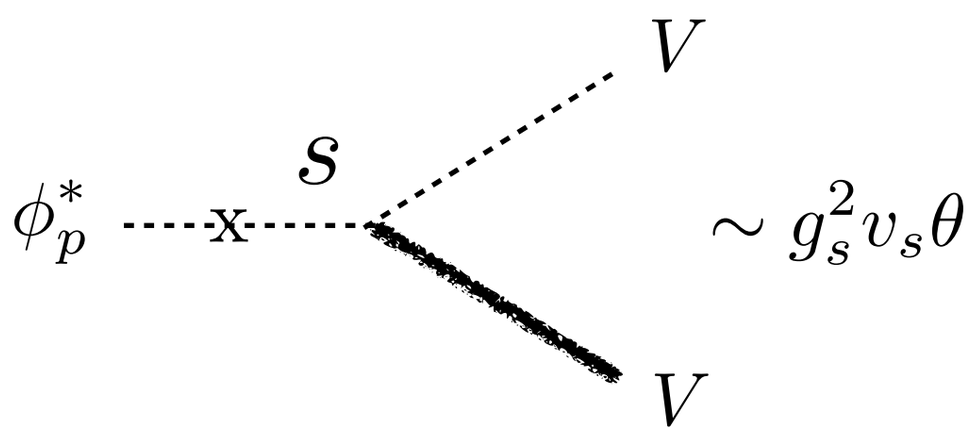
$\sim g^2 v_\phi$



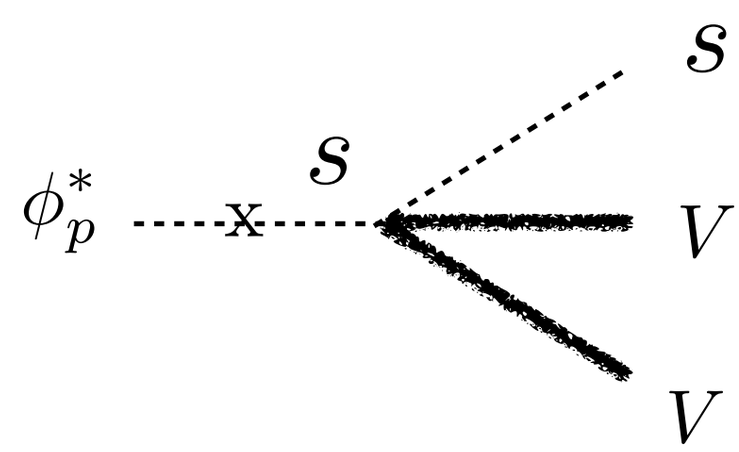
$\sim g^2$

$m_V > g v_\phi \longrightarrow$ Another mechanism for vector DM mass

e.g. mixing with extra Higgs s induces effective interactions.



$\sim g_s^2 v_s \theta$



$\sim g_s^2 \theta$

Gauges for vector DM

Production of on-shell gauge bosons:

Polarization sum, $\sum \epsilon^\mu \epsilon^\nu \to -g^{\mu\nu} + (1-\xi)\frac{p^\mu p^\nu}{p^2 - \xi m_V^2}$

+ ghosts, Goldstone fields with $m^2 = \xi m_V^2$

1) unitary gauge: $\xi \to \infty$, $\sum \epsilon^\mu \epsilon^\nu \to -g^{\mu\nu} + \frac{p^\mu p^\nu}{m_V^2}$

$$|\bar{\mathcal{M}}(\phi_p^* \to VV)|^2 = g^2 m_V^2 \left(3 - \frac{p^2}{m_V^2} + \frac{p^4}{4m_V^4}\right)$$ [A. Falkowski, J.M. No, 2012]

$$p^2 \gg m_\phi^2, m_V^2, \quad |\bar{\mathcal{M}}(\phi_p^* \to VV)|^2 \sim \frac{g^2}{4m_V^2}\, p^4$$

2) Feynman-'t Hooft gauge: $\xi = 1$, $\sum \epsilon^\mu \epsilon^\nu \to -g^{\mu\nu}$

+ ghosts, Goldstone fields with $m_c^2 = m_G^2 = m_V^2$

$$|\bar{\mathcal{M}}(\phi_p^* \to VV)|^2 = g^2 m_V^2 \left(3 - \frac{p^2}{m_V^2} + \frac{\lambda_\phi^2}{g^4}\right) \longrightarrow$$ still negative p^2 ?

Vector DM in R_ξ gauge

3) R_ξ gauge: $m_c^2 = m_G^2 = \xi m_V^2 \neq m_V^2$

$$|\bar{\mathcal{M}}(\phi_p^* \to VV)|^2 = (g m_V)^2 \left(2 + \frac{1}{4m_V^4}(p^2 - 2m_V^2)^2\right)\sqrt{1 - \frac{4m_V^2}{p^2}}$$

[G. Giudice et al, 2024]

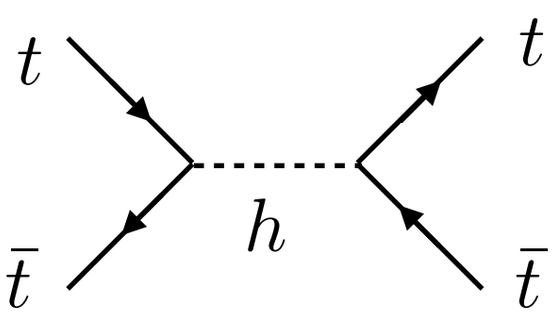
"gauge-independent"

$$+ \frac{g^2}{4m_V^2}(-p^2 + m_\phi^2)(p^2 + m_\phi^2)\sqrt{1 - \frac{4\xi m_V^2}{p^2}}$$

"gauge-dependent": nonzero of off-shell scalar

$$|\bar{\mathcal{M}}(\phi_p^* \to VV)|^2 = g^2 m_V^2 \times \begin{cases} \frac{(\xi-3)p^2}{2m_V^2} + \frac{\lambda_\phi^2}{g^4} + 3 & \text{for } \frac{p^2}{m_V^2} \gg \xi, 1 \\ \frac{p^4}{4m_V^4} - \frac{p^2}{m_V^2} + 3 & \text{for } \xi \gg \frac{p^2}{m_V^2}, 1 \end{cases}$$

Fried-Yennie gauge: $\xi = 3$ ⟶ no p^2 or p^4 terms!



off-shell Higgs self-energy: gauge-dependent

⟶ + vertex, box diagrams

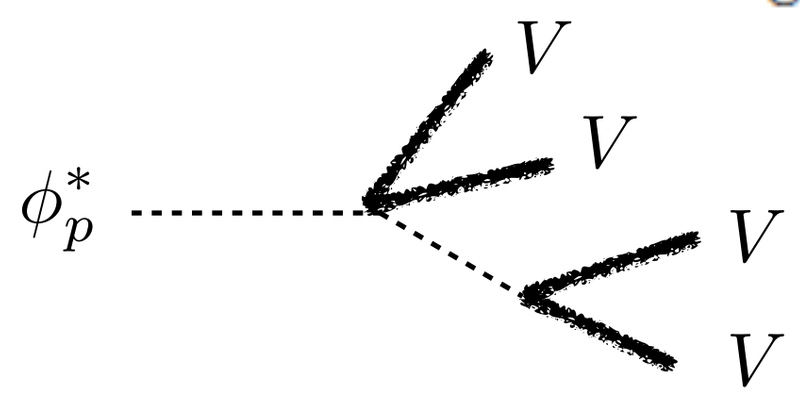
cf. [J. Papavasilliou, A. Pilfatsis et al, 1997]

Problem in unitary gauge

$p^2 \gg m_\phi^2, m_V^2, \quad |\bar{\mathcal{M}}(\phi_p^* \to VV)|^2 \sim \dfrac{g^2}{4m_V^2}\, p^4$ in unitary gauge.

2-body $\phi_p^* \to 2V$: $\mathrm{Im}[\tilde{\Gamma}^{(2)}(p^2)]_{\phi_p^* \to 2V} \sim \dfrac{g^2 m_V^2}{8\pi}\left(\dfrac{p^4}{4m_V^4}\right)$

4-body $\phi_p^* \to 4V$:



$$\mathrm{Im}[\tilde{\Gamma}^{(2)}(p^2)]_{\phi_p^* \to 4V} \sim \frac{g^6 m_V^2}{8\pi(4\pi^2)^2}\left(\frac{p^4}{4m_V^4}\right)^2$$

4-body process "grows faster" with large momentum.

Processes with higher vector boson multiplicity grows even faster with large momentum.

$\Rightarrow$ Breakdown of perturbative expansion!

Goldstone equivalence

Goldstone Equivalence Theorem: $\epsilon_L^\mu \frac{p_\mu}{m_V} = G$ (Goldstone)

Physical processes with longitudinal modes at high energy:

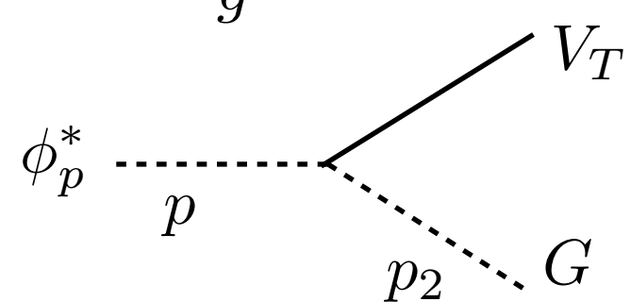
$$\bar{\mathcal{M}}(\phi; V_L) = \bar{\mathcal{M}}(\phi; G) + \mathcal{O}\Big(\frac{m_V^2}{p^2}\Big)$$

[B.W. Lee et al, 1977]

(i) TT: $|\bar{\mathcal{M}}|^2 \sim 2m_V^2,$

(ii) LL: $\phi_p^* \to GG, \qquad |\bar{\mathcal{M}}|^2 \sim \lambda_\phi^2 v_\phi^2 = \frac{\lambda_\phi^2}{g^2} m_V^2$

(iii) TL: $\phi_p^* \to V_T(p_1)G(p_2),$



$\sim ig(p^\mu + p_2^\mu)\epsilon_{\mu,T}$

In rest frame for ϕ_p^*, $\quad p = (E, 0, 0, 0)$

V_T, G back-to-back $\longrightarrow$ $|\bar{\mathcal{M}}|^2 \sim 0$

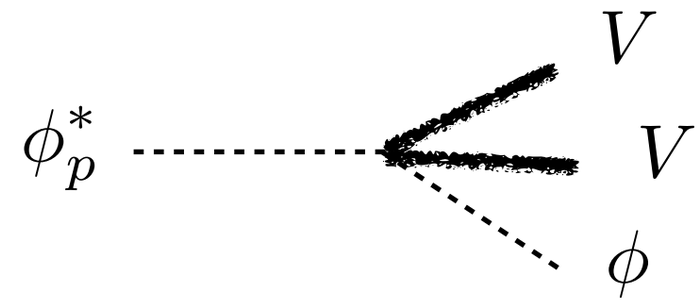
$$|\bar{\mathcal{M}}|^2 \sim m_V^2\left(2g^2 + \frac{\lambda_\phi^2}{g^2}\right)\left(1 + \mathcal{O}\Big(\frac{m_V^2}{p^2}\Big)\right), \quad p^2 \gg m_V^2$$

: finite at large p^2

Other processes

3-body process for vector DM:

$$\phi_p^* \to \phi V V$$



VDM part: $|\bar{\mathcal{M}}|^2_{p^2<v_\phi^2} \sim v_\phi^2 g^4 \left(3 - \frac{p^2}{m_V^2} + \frac{p^4}{4m_V^4}\right), \quad |\bar{\mathcal{M}}|^2_{p^2>v_\phi^2} \sim v_\phi^2(\lambda_\phi^2 + 2g^4)$

$$\longrightarrow \quad \mathrm{Im}\,\widetilde{\Gamma}^{(2)}(p^2)\Big|_{\phi^*\to\phi VV} \sim p^2|\bar{\mathcal{M}}|^2 \sim p^2(\lambda_\phi^2 + 2g^4), \qquad p^2 > v_\phi^2$$

cf. 2-body process: $\mathrm{Im}\,\widetilde{\Gamma}^{(2)}(p^2)\Big|_{\phi^*\to VV} \sim |\bar{\mathcal{M}}|^2\Big|_{\phi^*\to VV}$

$p^2 > v_\phi^2$: 3-body process dominant for vector DM!

Higher order processes for DM: $\phi^*_{p_1}\phi^*_{p_2} \to \chi_s\chi_s, \bar{\chi}_f\chi_f, VV$

$\phi^*_{p_1} \sim \left(\frac{v_\phi}{p^2}\right)^2 \longrightarrow$ Suppressed for $p^2 \sim m_{\mathrm{DM}}^2 \gg v_\phi^2$

Back-reaction issue

Energy density of particles can be comparable to latent heat:

$$E_{\rm DM} = 4\pi R_*^2 \int \frac{dp^2}{2\pi^2}\, p f(p^2) {\rm Im}\, \widetilde{\Gamma}^{(2)}(p^2) \sim \frac{4\pi^2}{3} R_*^3 \Delta V$$

$\longrightarrow$ Bubble dynamics & DM production subject to corrections.

$$f(p^2) \sim \frac{1}{p^4} \qquad \text{(Elastic collisions)}$$

$$|\bar{\mathcal{M}}|^2 \sim \begin{cases} (p^2)^0, \ \text{(scalar, 2-body)} \ ; \ p^2, \ \text{(scalar, 3-body)} \\ p^2, \ \text{(fermion, 2-body)} \\ \cancel{p^4, \ \text{(Vector, 2-body)}} \quad \cancel{\text{Back-reaction issue!}} \\ (p^2)^0, \ \text{(vector, 2-body)} \ ; \ p^2, \ \text{(vector, 3-body)} \end{cases}$$

Important to treat the growth of vector DM properly for back-reaction issue.

Conclusions

- Bubble dynamics in the 1st-order phase transitions is an interesting testing ground for DM and GWs.
- Non-relativistic bubbles: if DM receives mass from SSB,

  ⟶ "Filtering mechanism" DM with masses parametrically larger than T.
- Relativistic bubbles: if DM mass is not related to SSB,

  ⟶ "Bubble collisions" produce DM with masses much larger that T & v_ϕ.

$$m_{\rm DM} \lesssim \frac{1}{l_w} \sim \gamma_w v_\phi \sim \gamma_w T, \quad \gamma_w \gg 1$$

cf. wall-plasma: $m_{\rm DM} \ll \gamma_w T$

- Vector dark matter is subject to gauge dependence: a practical solution with GET is provided.