A positive-definite formulation of tunneling
& Curvature perturbation from first-order phase transitions

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Focus workshop@IBS, 2025/11/23

collaborators

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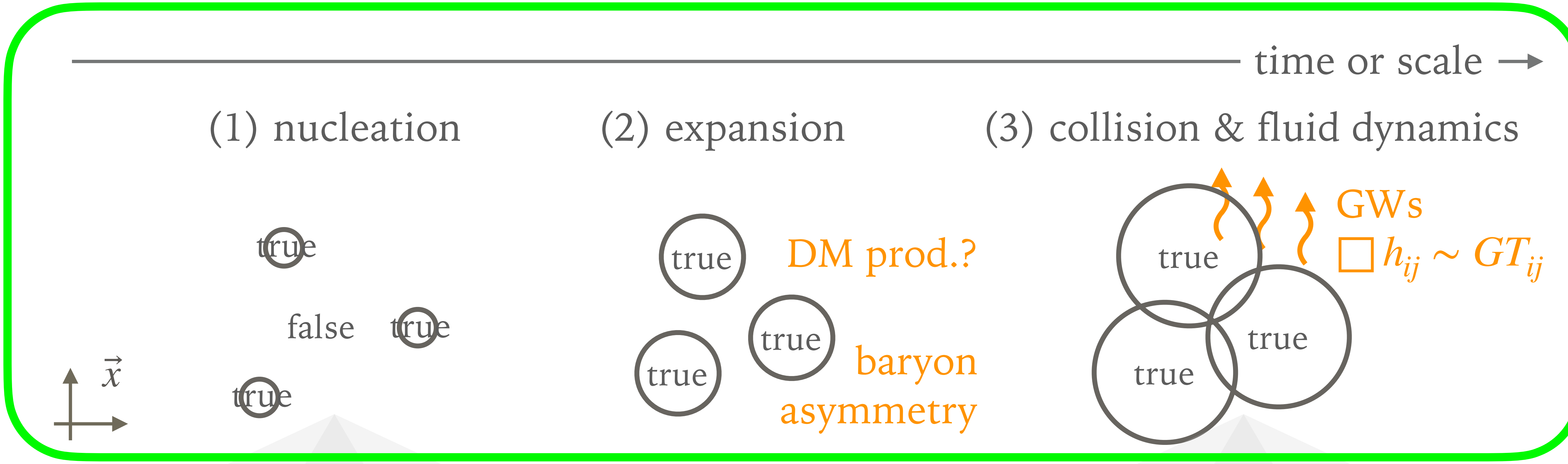
Jose Ramon Espinosa, Thomas Konstandin, Shogo Matake, Taiga Miyachi

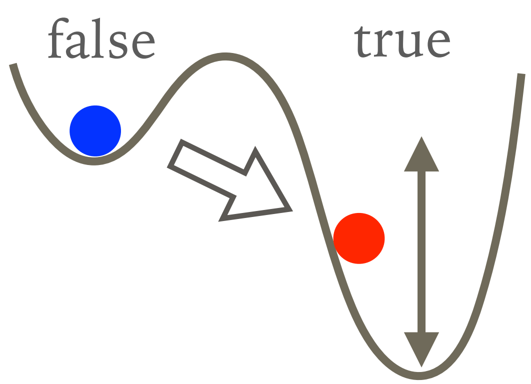
[Franciolini, RJ, Gouttenoire 2503.01962]
[Ellor, RJ, Kumar, McGehee, Tsai PRL 133 (2024) 21, 211003, 2311.16222]
[Espinosa, RJ, Konstandin JCAP 02 (2023) 021, 2209.03293] [+Matake, Miyachi in progress]

FIRST-ORDER PHASE TRANSITIONS IN THE EARLY UNIVERSE

microphysics | Dynamics of bubbles | *macrophysics*



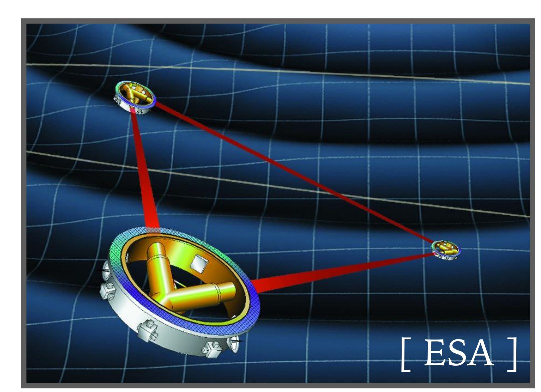
Physics of the Higgs sector



FOPTs in BSM

GWs

GW observations



OUTLINE

1. A positive definite formulation of tunneling

[Espinosa, RJ, Konstandin JCAP 02 (2023) 021, 2209.03293] [+Matake, Miyachi in progress]

2. Curvature perturbation from first-order phase transitions

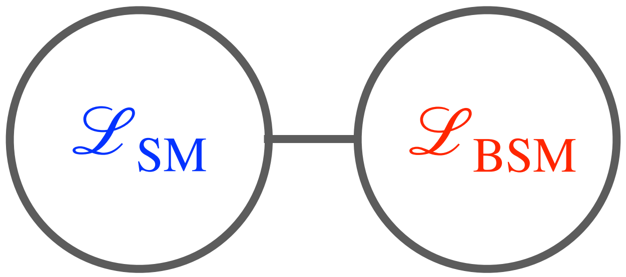
2-1) Superhorizon scales [Ellor, RJ, Kumar, McGhee, Tsai PRL 133 (2024) 21, 211003, 2311.16222]

2-2) Horizon scales [Franciolini, RJ, Gouttenoire 2503.01962]

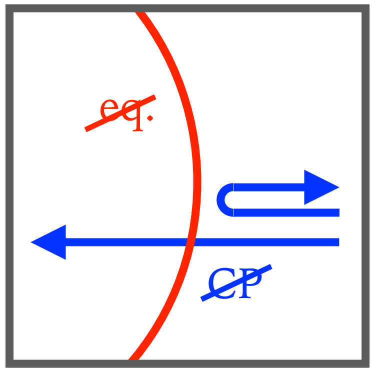
TUNNELING IN QFT

➤ Implications of tunneling in QFT

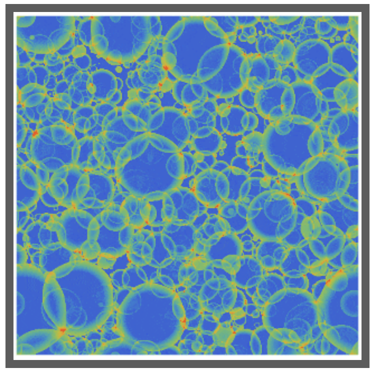
- New physics in the Higgs sector



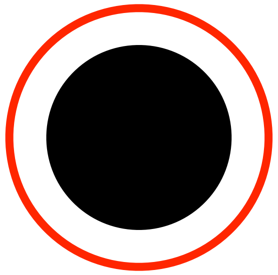
- EW baryogenesis



- Gravitational wave production



- Nucleation around compact objects



ESTIMATE ON THE TUNNELING RATE

[Coleman '77]
[Callan, Coleman '77]

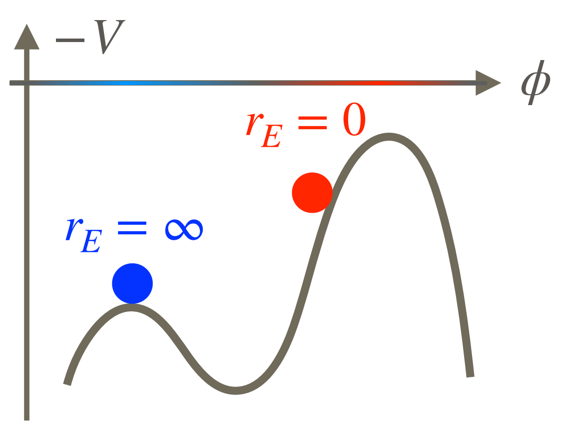
- Tunneling rate Γ is estimated from the Euclidean action $S[\phi]$

$$S[\phi] = \int d^4x \left[\frac{1}{2}(\partial\phi)^2 + V(\phi)\right] \begin{cases} \overset{\text{O(4) inv.}}{=} \int 2\pi^2 r_E^3\, dr_E \left[\frac{1}{2}(\partial_{r_E}\phi)^2 + V(\phi)\right] \\ \overset{\text{O(3) inv.}}{=} \int dt \int 4\pi r^2 dr \left[\frac{1}{2}(\partial_t\phi)^2 + \frac{1}{2}(\partial_r\phi)^2 + V(\phi)\right] \end{cases}$$

- Tunneling rate is estimated as $\Gamma \sim e^{-S[\bar{\phi}]}$,

  with "bounce" $\bar{\phi}$ being the solution for the equation of motion

O(4) case: $\partial_{r_E}^2\bar{\phi} + \frac{3}{r_E}\partial_{r_E}\bar{\phi} - \partial_{\bar{\phi}}V(\bar{\phi}) = 0$



A DIFFERENT FORMULATION?

- Can we reformulate the bounce?

Step 1: Start from the Euclidean action, and translate $\phi(t, \vec{x})$ into $t(\phi, \vec{x})$

$$S[\phi] = \int dt \int d^3x \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial t} \right)^2 + \frac{1}{2} (\nabla \phi)^2 + V \right]$$

$$= \int d\phi \int d^3x \left[\frac{1 + (\nabla t)^2}{2\dot{t}} + \dot{t} V \right] =: S[t]$$

dot is ϕ derivative

Why do I do this? Because I feel like doing so

A DIFFERENT FORMULATION?

- Can we reformulate the bounce?

Step 2: Add $(\nabla t) \cdot \dot{\boldsymbol{p}} - \dot{t}\,(\nabla \cdot \boldsymbol{p})$ (just a total derivative) and complete the square

$$S[t] = \int d\phi \int d^3x \left[\frac{1 + (\nabla t)^2}{2\dot{t}} + \underline{(\nabla t) \cdot \dot{\boldsymbol{p}}} + \dot{t}(V - \underline{\nabla \cdot \boldsymbol{p}}) \right]$$

$$= \int d\phi \int d^3x \left[\frac{1 + (\nabla t + \dot{t}\,\dot{\boldsymbol{p}})^2}{2\dot{t}} + \dot{t} \left(V - \nabla \cdot \boldsymbol{p} - \frac{\dot{\boldsymbol{p}}^2}{2} \right) \right]$$

Why do I do this? Because I feel like doing so

A DIFFERENT FORMULATION?

➤ Can we reformulate the bounce?

Step 3: Integrate $\dot{t}$ and ∇t out

drops out

$$S[t] = \int d\phi \int d^3x \left[\frac{1 + (\nabla t + \dot{t}\,\boldsymbol{p})^2}{2\dot{t}} + \dot{t}\left(V - \nabla \cdot \boldsymbol{p} - \frac{\dot{\boldsymbol{p}}^2}{2} \right) \right]$$

$$= \int d\phi \int d^3x \sqrt{2(V - \nabla \cdot \boldsymbol{p}) - \dot{\boldsymbol{p}}^2} =: S[\boldsymbol{p}]$$

Why do I do this? Because I feel like doing so

A DIFFERENT FORMULATION?

- Can we reformulate the bounce?

Step 4: If you seriously think about it, a magic factor 2 and a surface term appear

value of ϕ at which $\sqrt{\cdots}$ becomes zero

$$S[\boldsymbol{p}] = \int d^3x \int_{\phi=\phi_{\min}(\boldsymbol{x})}^{\phi=\phi_{\max}(\boldsymbol{x})} d\phi\ 2\sqrt{2\left(V - \nabla\cdot\boldsymbol{p}\right) - \dot{\boldsymbol{p}}^2}\ +\ (\text{surface})$$

ϕ at the false vacuum

relevant only to Fubini-type slowly decaying bounces

- This action reproduces O(4) Euclidean results
- This action works as a generalization of the "tunneling potential" to non-O(4) cases after the identification $\nabla\cdot\boldsymbol{p} = V_t(\phi)$ (r-independent)

TUNNELING POTENTIAL

- Originally derived by J.R.Espinosa for O(4) bounce [Espinosa '18]
- Tunneling potential $V_t(\phi)$ possesses interesting properties suggesting that it is not just a reformulation of the Euclidean method

  - Solution of the eom is ***a minimum, not a saddle point***

  - The action is ***obviously positive definite***: $S[V_t] = \int_0 d\phi \, \frac{54\pi(V - V_t)^2}{(-\dot{V}_t)^3}$

  - Once one takes gravity into account, ***both CdL and HM actions are obtained in a unified manner*** (without the annoying boundary term)

$$S[V_t] = \int_0 d\phi \, \frac{6\pi^2 M_P^2 (D + \dot{V}_t)^2}{D V_t^2} \quad \text{with} \quad D = \sqrt{\dot{V}_t^2 + \frac{6(V - V_t)V_t}{M_P^2}}$$

SUMMARY FOR PART 1

- Tunneling rate is usually estimated as $\Gamma \sim e^{-S[\bar{\phi}]}$

  with the saddle-point configuration $\bar{\phi}$ of the Euclidean action

- Recently(?) a new formulation with so-called tunneling potential V_t

  has been proposed for O(4) case

- We generalize it to less symmetric cases

  (which might be useful in calculating nucleation around impurities)

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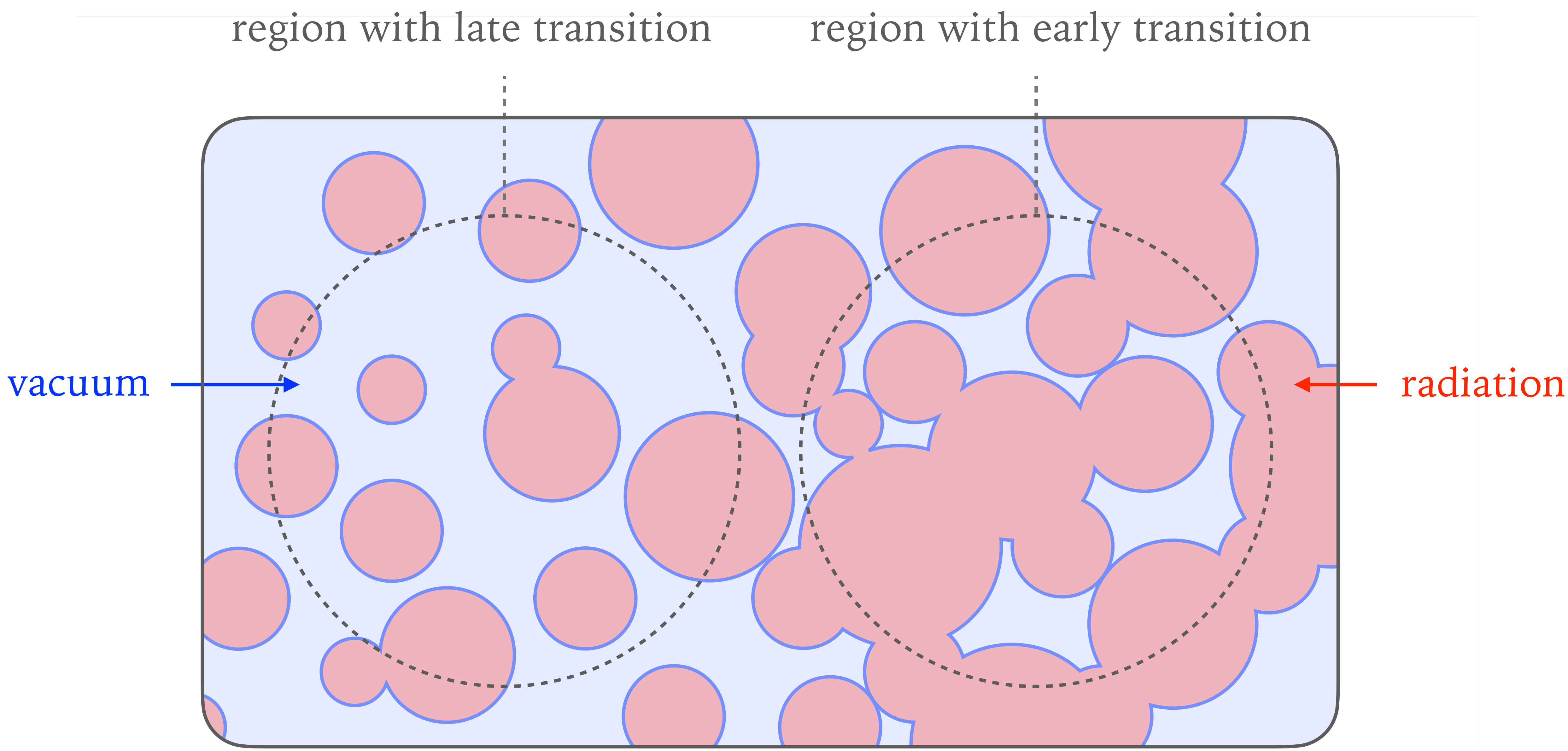
CURVATURE PERTURBATION FROM FIRST-ORDER PHASE TRANSITIONS

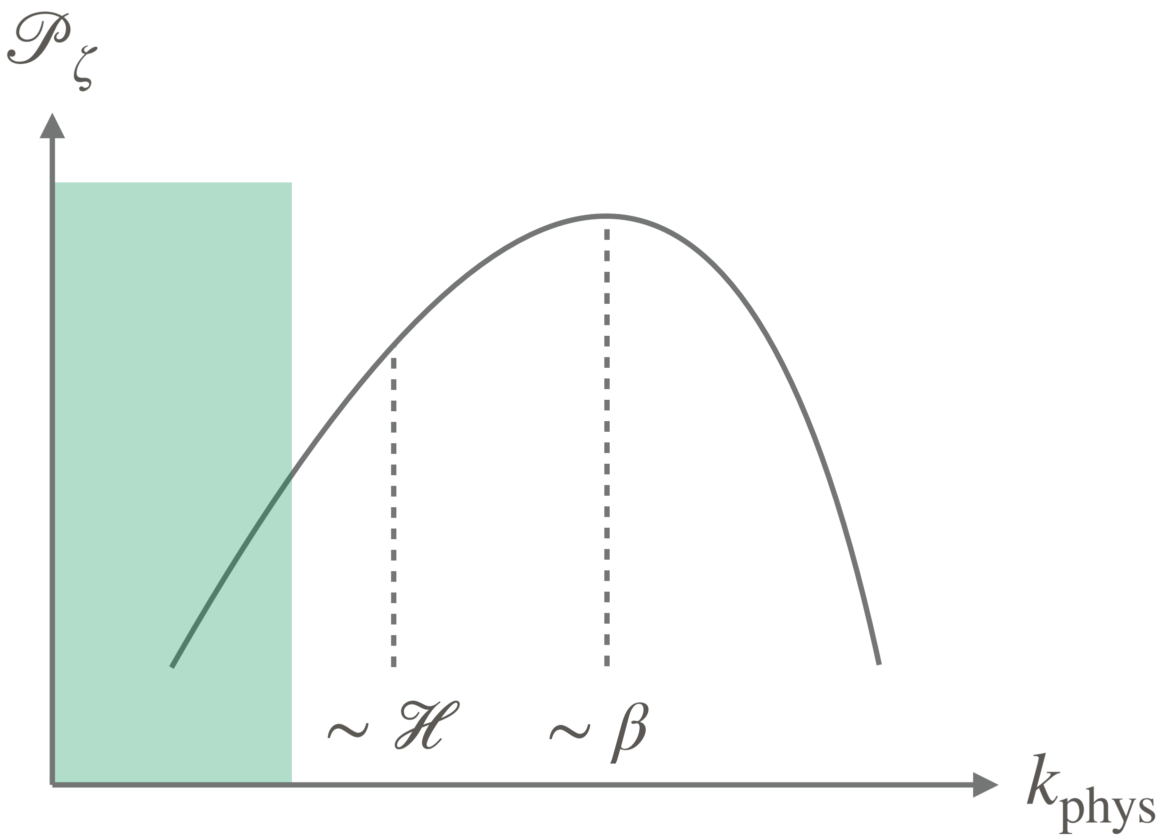
➤ How large can the curvature perturbation be? (→ PBHs? GWs?)



CURVATURE PERTURBATION FROM FIRST-ORDER PHASE TRANSITIONS

$k_{\rm phys} \ll H$

(plus, we assume $H \ll \beta$ for technical reasons)



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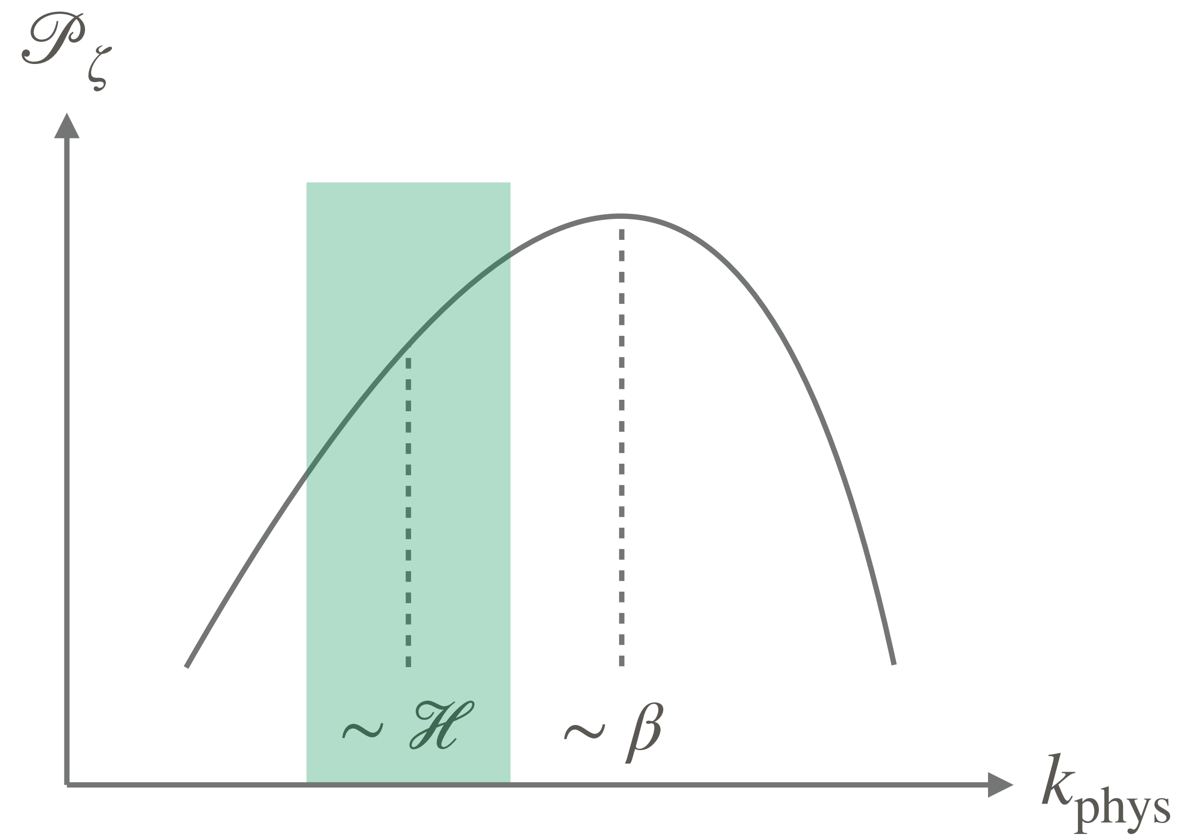

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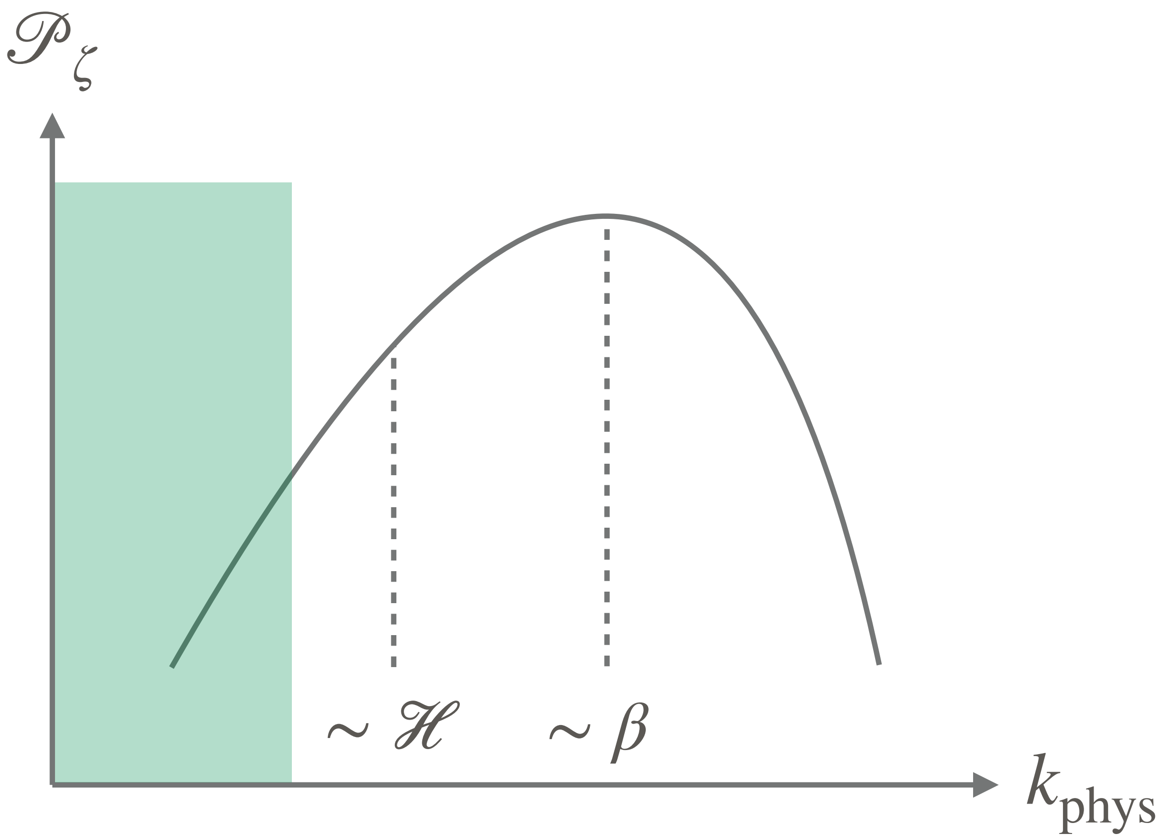

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$k_{\rm phys} \sim H$

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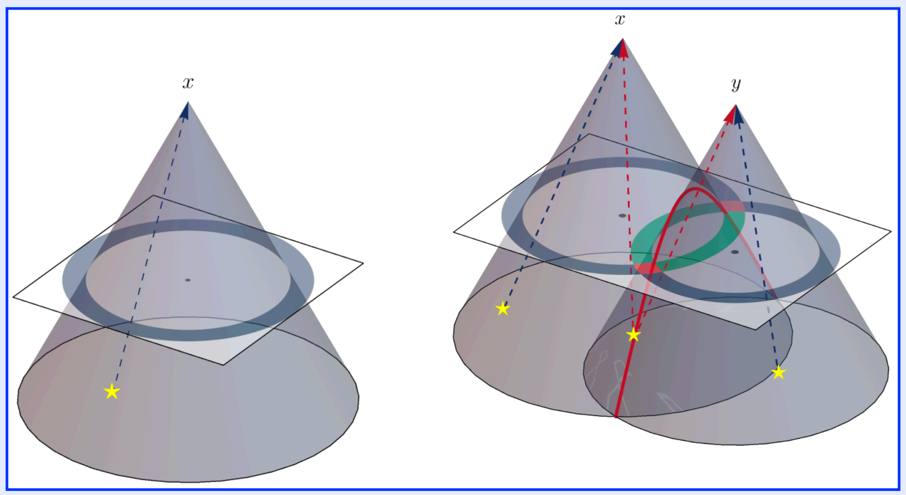
SUMMARY FOR PART 2-1

- To estimate superhorizon curvature perturbation in FOPTs, we develop "vacuum-bubble δN-formalism"

- Based on this, we report constraints on late-time FOPTs in a dark sector

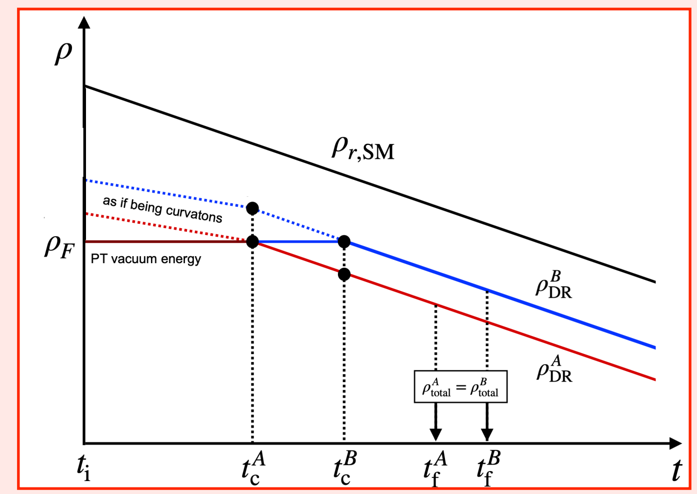
VACUUM BUBBLE δN-FORMALISM

Light-cone formalism for vac. bubbles



e.g. [RJ, Takimoto '16]

δN-formalism



[Starobinsky '85] [Salopek & Bond '90] [Sasaki & Stewart '96]
[Sasaki & Tanaka '98] [Wands, Malik, Lyth, Liddle '00] ...

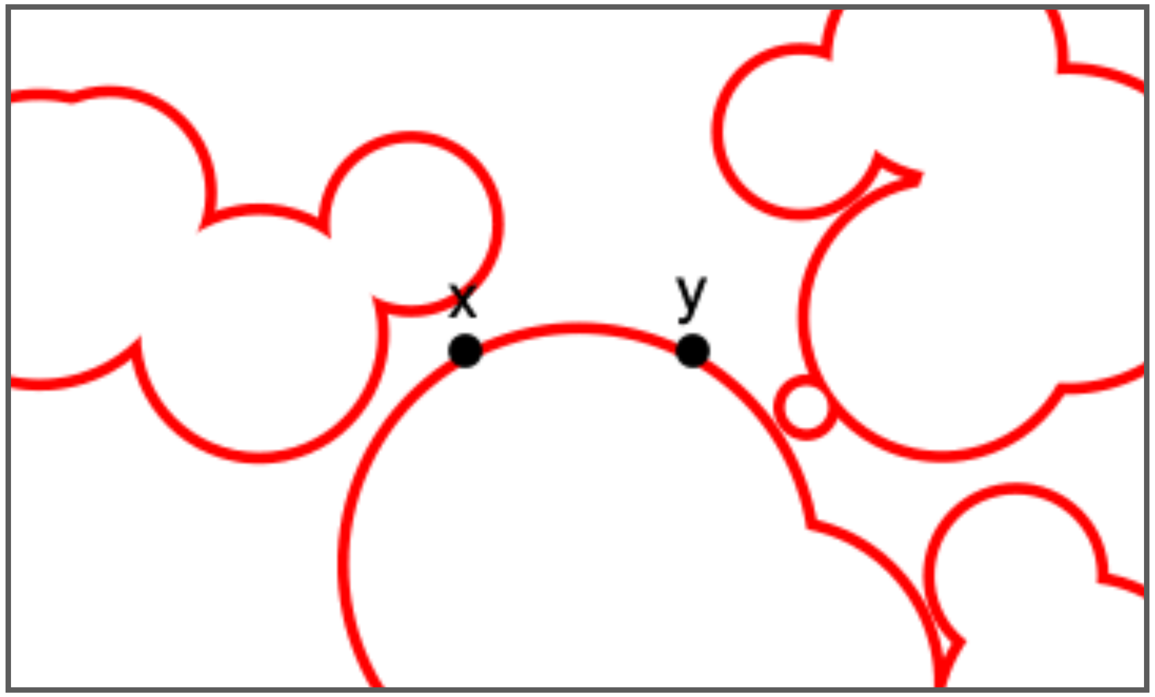
⇩

vacuum bubble δN-formalism

LIGHT-CONE FORMALISM FOR VACUUM BUBBLES

e.g. [RJ, Takimoto '16]

- A formalism to calculate multi(mostly two)-point functions of quantities determined by the spacetime distribution of vacuum bubbles
- Intuitively: Fix $(t_x, \vec{x}), (t_y, \vec{y})$ and sum up all possible configurations



- In the following we assume $\beta \gg H$ and $\Gamma(t) = \Gamma_* \, e^{\beta(t-t_*)}$

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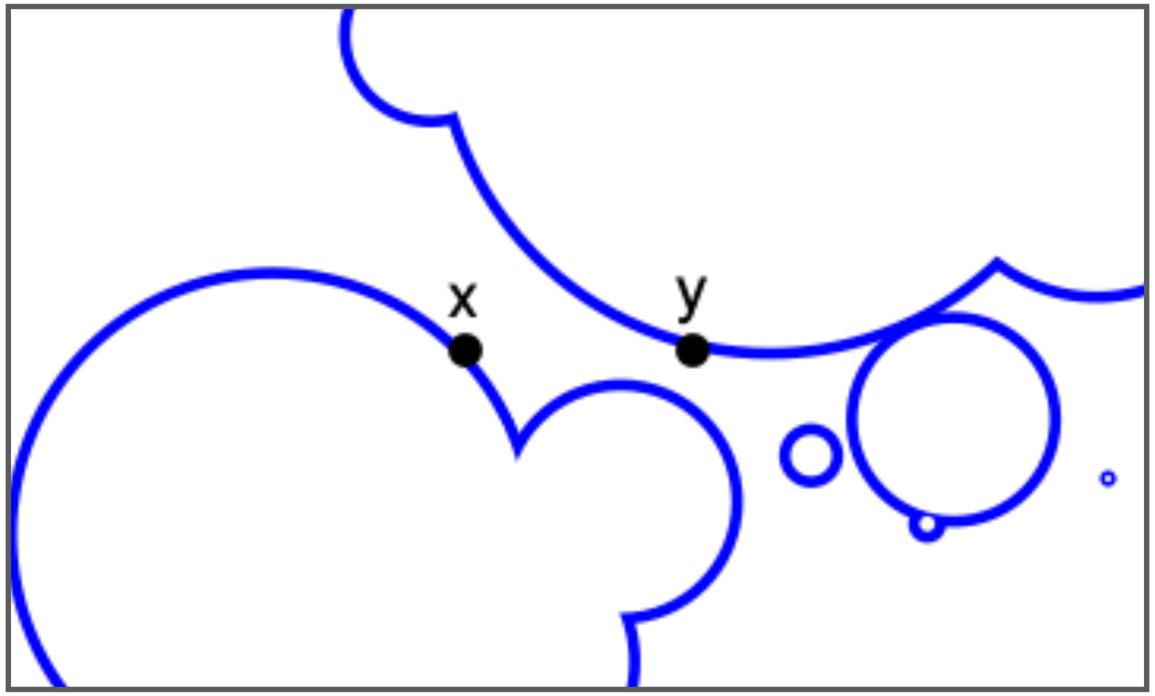

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- To estimate curvature perturbation, we need $\left\langle \delta t_c(\vec{x}) \delta t_c(\vec{y}) \right\rangle$

$t_c(\vec{x})$: transition time for each spatial point $\vec{x}$

$\delta t_c(\vec{x}) = t_c(\vec{x}) - \left\langle t_c \right\rangle$: difference of $t_c(\vec{x})$ from average

$\langle \cdots \rangle$: average over infinitely many realizations of bubble configurations

- $\left\langle \delta t_c(\vec{x}) \delta t_c(\vec{y}) \right\rangle$ can be decomposed as

$$(\text{expectation value}) = \sum (\text{probability}) \times (\text{value})$$

$$\left\langle \delta t_c(\vec{x}) \delta t_c(\vec{y}) \right\rangle = \int dt_x \int dt_y \begin{pmatrix} \text{prob. for} \\ t_x < t_c(\vec{x}) < t_x + dt_x \\ t_y < t_c(\vec{y}) < t_y + dt_y \end{pmatrix} \times \left(t_x - \left\langle t_c \right\rangle \right) \left(t_y - \left\langle t_c \right\rangle \right)$$

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LIGHT-CONE FORMALISM FOR VACUUM BUBBLES

e.g. [RJ, Takimoto '16]

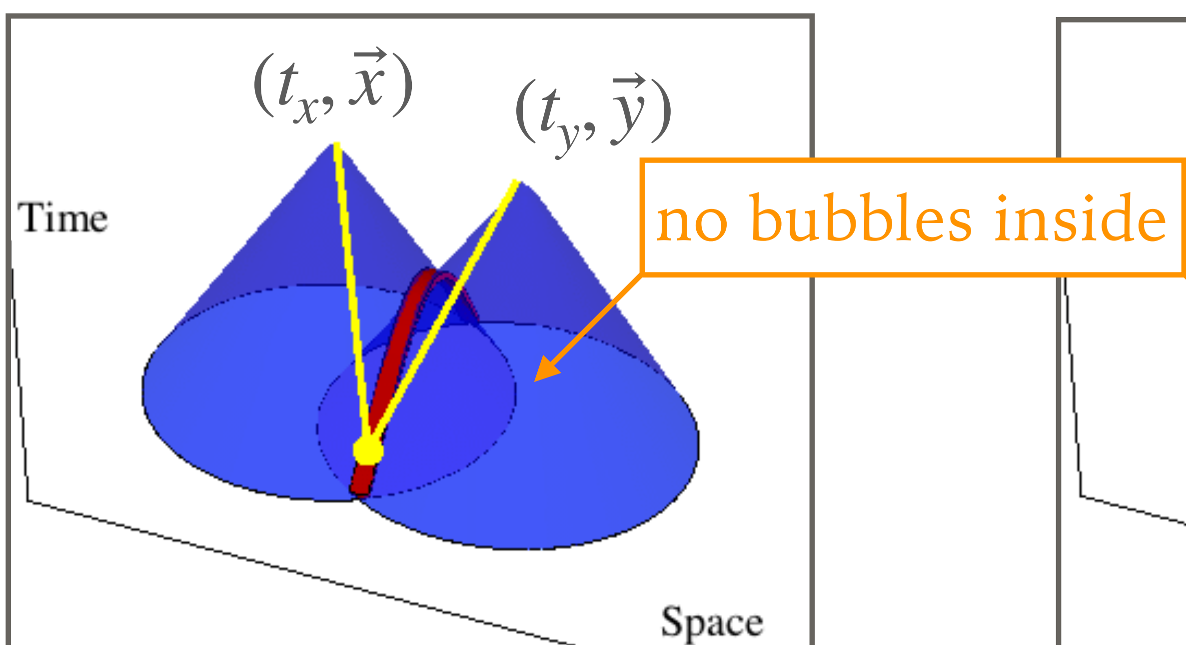
- Probability part can be decomposed into two factors

$$P = P_{\rm surv} \times P_{\rm nuc}$$

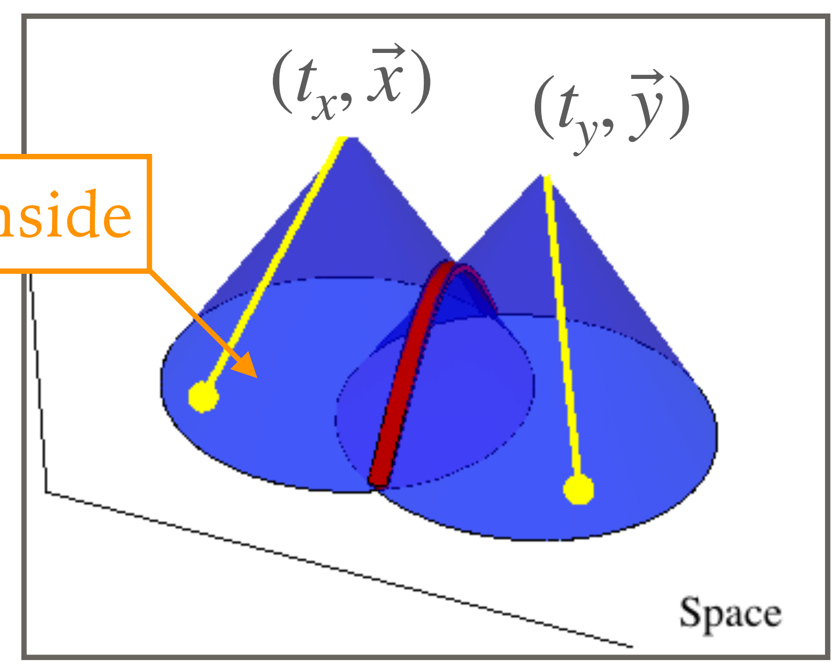
1) Survival probability $P_{\rm surv}$: no bubble must nucleate inside the blue past cones

(otherwise such bubbles hit $\vec{x}$ or $\vec{y}$ before the evaluation time t_x or t_y)

case 1 (single-bubble)



case 2 (double-bubble)



LIGHT-CONE FORMALISM FOR VACUUM BUBBLES

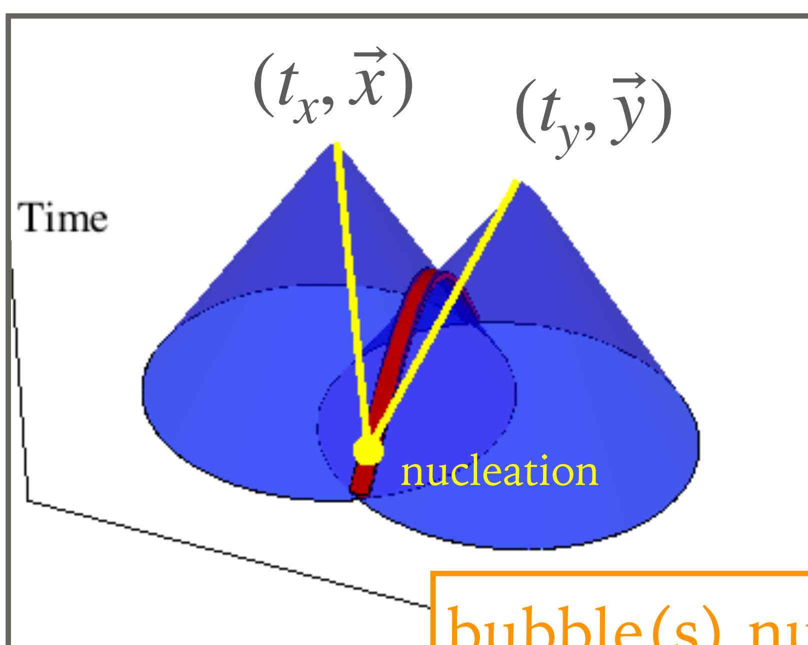
e.g. [RJ, Takimoto '16]

- Probability part can be decomposed into two factors

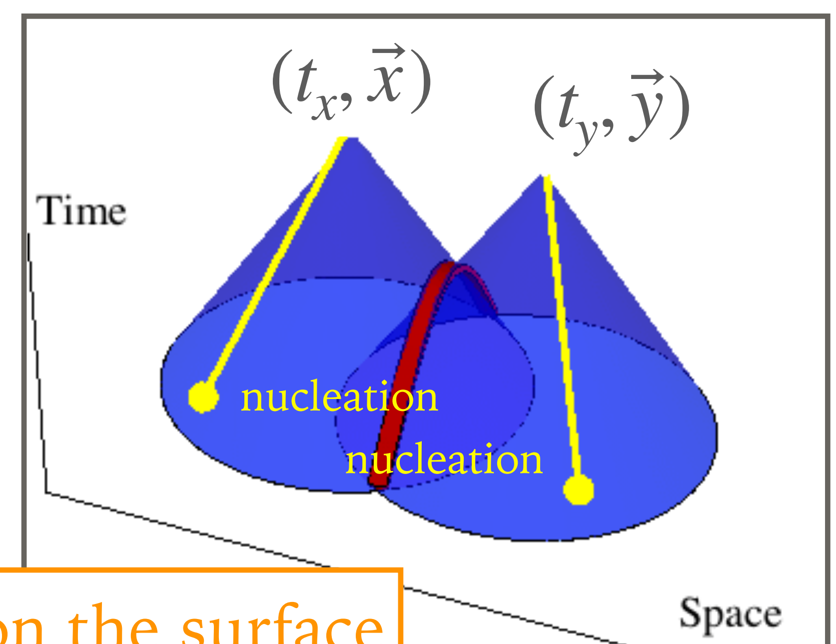
$$P = P_{\rm surv} \times P_{\rm nuc}$$

2) Nucleation probability $P_{\rm nuc}$: bubble(s) must nucleate at the right time and position on the surface of the past cones

case 1 (single-bubble)



case 2 (double-bubble)



bubble(s) nucleate on the surface

LIGHT-CONE FORMALISM FOR VACUUM BUBBLES

e.g. [RJ, Takimoto '16]

- Now calculation of $\left\langle \delta t_c(\vec{x}) \delta t_c(\vec{y}) \right\rangle$ is straightforward

$$\left\langle \delta t_c(\vec{x}) \delta t_c(\vec{y}) \right\rangle = \left\langle \delta t_c(\vec{x}) \delta t_c(\vec{y}) \right\rangle^{(s)} + \left\langle \delta t_c(\vec{x}) \delta t_c(\vec{y}) \right\rangle^{(d)}$$

$$\langle \delta t_c(\vec{x}) \delta t_c(\vec{y}) \rangle^{(s)} = \int_{-r}^{r} dt_{x,y} \, \frac{2\pi e^{-r/2}}{r \mathcal{I}(x,y)} \left(\frac{r^2}{4} + r + 2 - \frac{t_{x,y}^2}{4} \right) \left[\left(\ln \left(\frac{\mathcal{I}(x,y)}{8\pi} \right) \right)^2 - \frac{t_{x,y}^2}{4} + \frac{\pi^2}{6} \right]$$

$$\begin{aligned} \langle \delta t_c(\vec{x}) \delta t_c(\vec{y}) \rangle^{(d)} = & \int_{-r}^{r} dt_{x,y} \, \frac{16\pi^2}{\mathcal{I}^2(x,y)} \\ & \times \left[4 - \frac{e^{-t_{x,y}/2 - r/2}}{2r} (r + t_{x,y} + 4)(r - t_{x,y}) - \frac{e^{t_{x,y}/2 - r/2}}{2r} (r - t_{x,y} + 4)(r + t_{x,y}) \right. \\ & \left. + \frac{e^{-r}}{16r^2} ((r+4)^2 - t_{x,y}^2)(r^2 - t_{x,y}^2) \right] \left[\left(\ln \left(\frac{\mathcal{I}(x,y)}{8\pi} \right) - 1 \right)^2 - \frac{t_{x,y}^2}{4} + \frac{\pi^2}{6} - 1 \right]. \end{aligned}$$

$$\left(\beta = 1 \text{ unit}, \ r \equiv |\vec{x} - \vec{y}|, \ t_{x,y} = t_x - t_y, \ \mathcal{I}(x,y) = 8\pi \left[e^{t_{x,y}/2} + e^{-t_{x,y}/2} + \frac{t_{x,y}^2 - (r^2 + 4r)}{4r} e^{-r/2} \right] \right)$$

LIGHT-CONE FORMALISM FOR VACUUM BUBBLES

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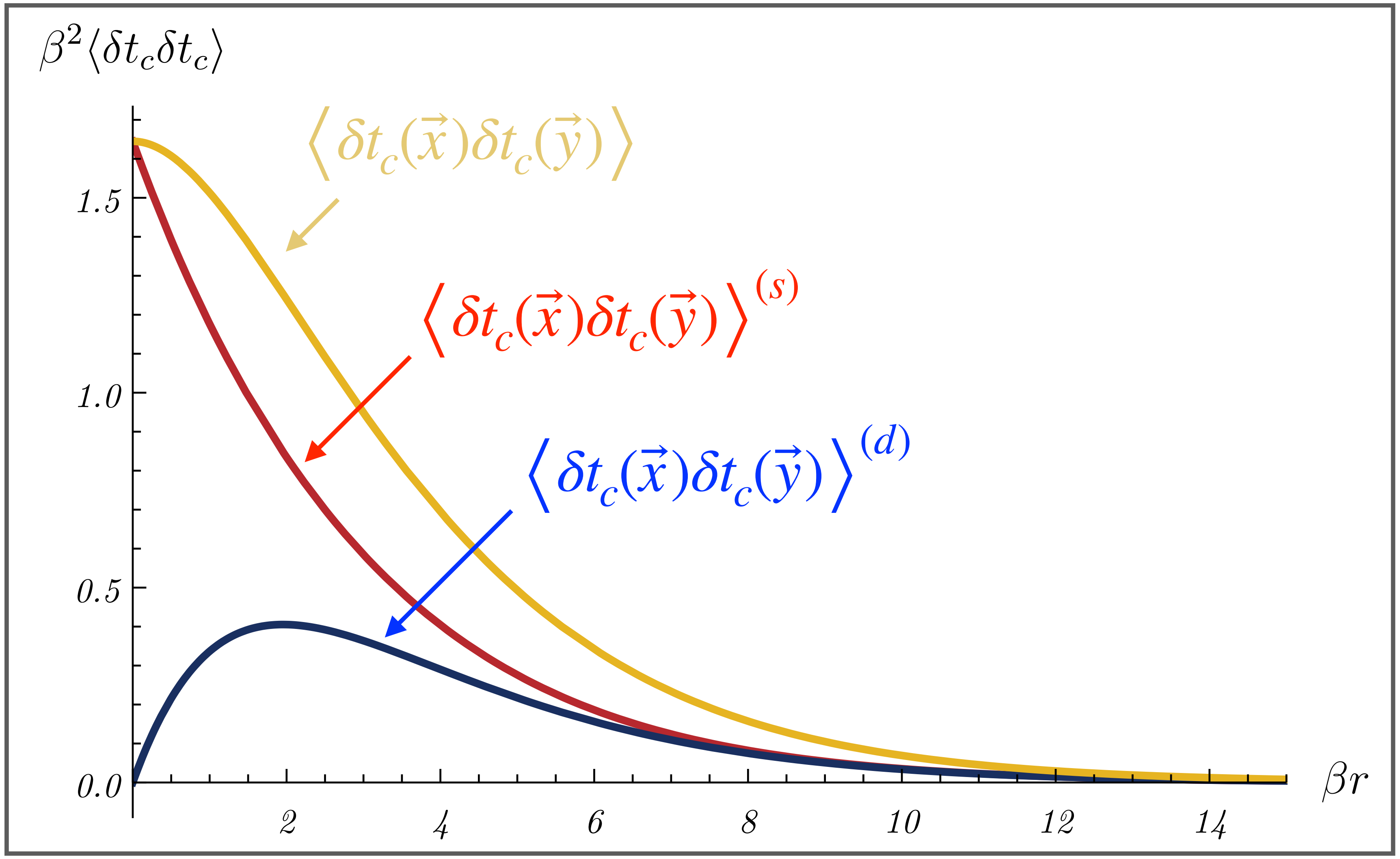

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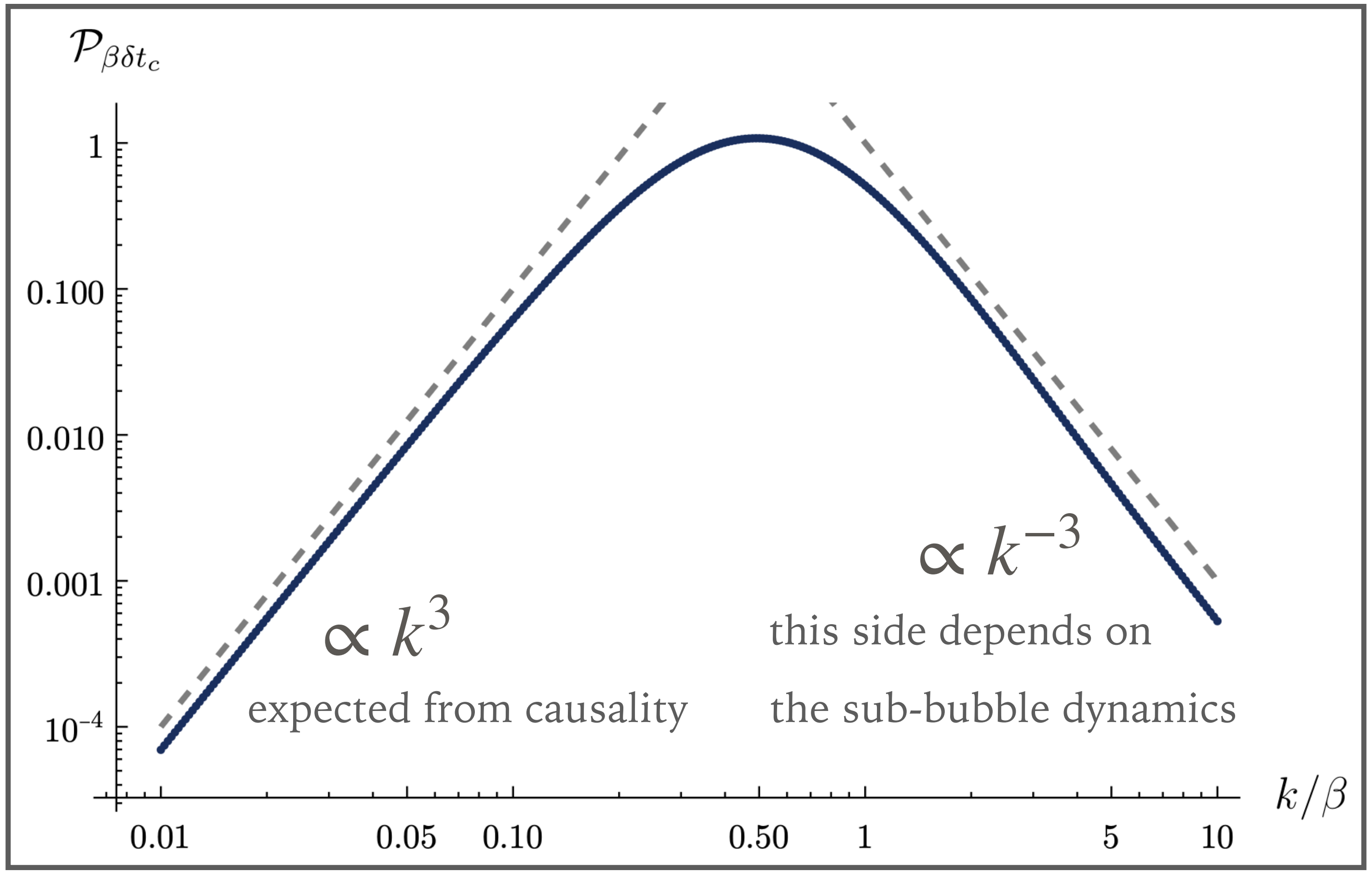
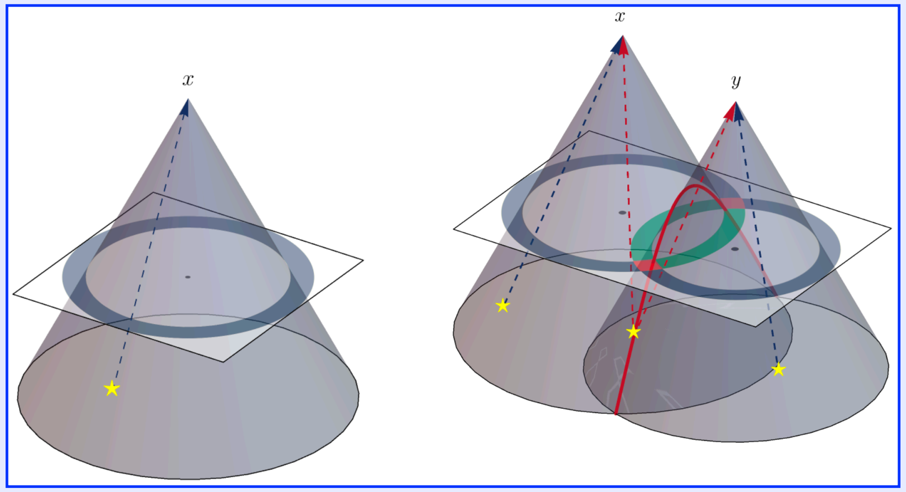
which can be translated into $\mathscr{P}_{\beta\delta t_c}(k) = \int d^3r\, e^{i\vec{k}\cdot\vec{r}} \beta^2 \left\langle \delta t_c(\vec{x})\delta t_c(\vec{y}) \right\rangle$

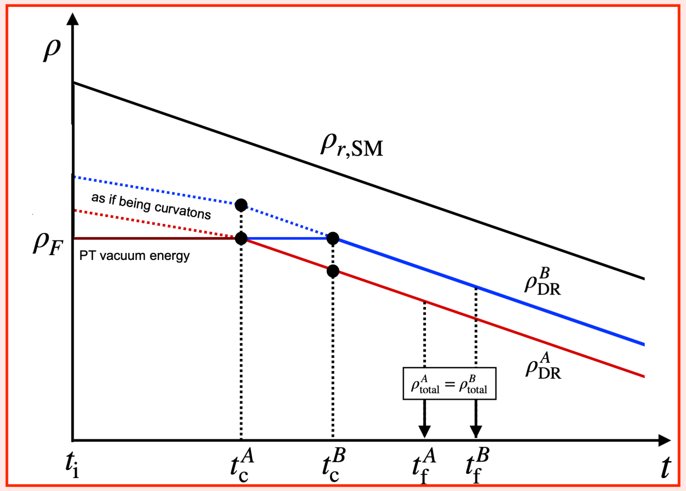


VACUUM BUBBLE δN-FORMALISM

Light-cone formalism for vac. bubbles



δN-formalism



e.g. [RJ, Takimoto '16]

[Starobinsky '85] [Salopek & Bond '90] [Sasaki & Stewart '96]
[Sasaki & Tanaka '98] [Wands, Malik, Lyth, Liddle '00] ...

vacuum bubble δN-formalism

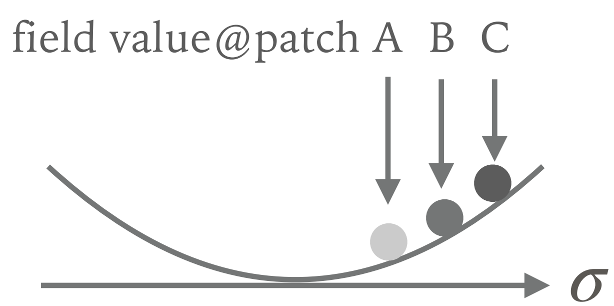
δN-FORMALISM

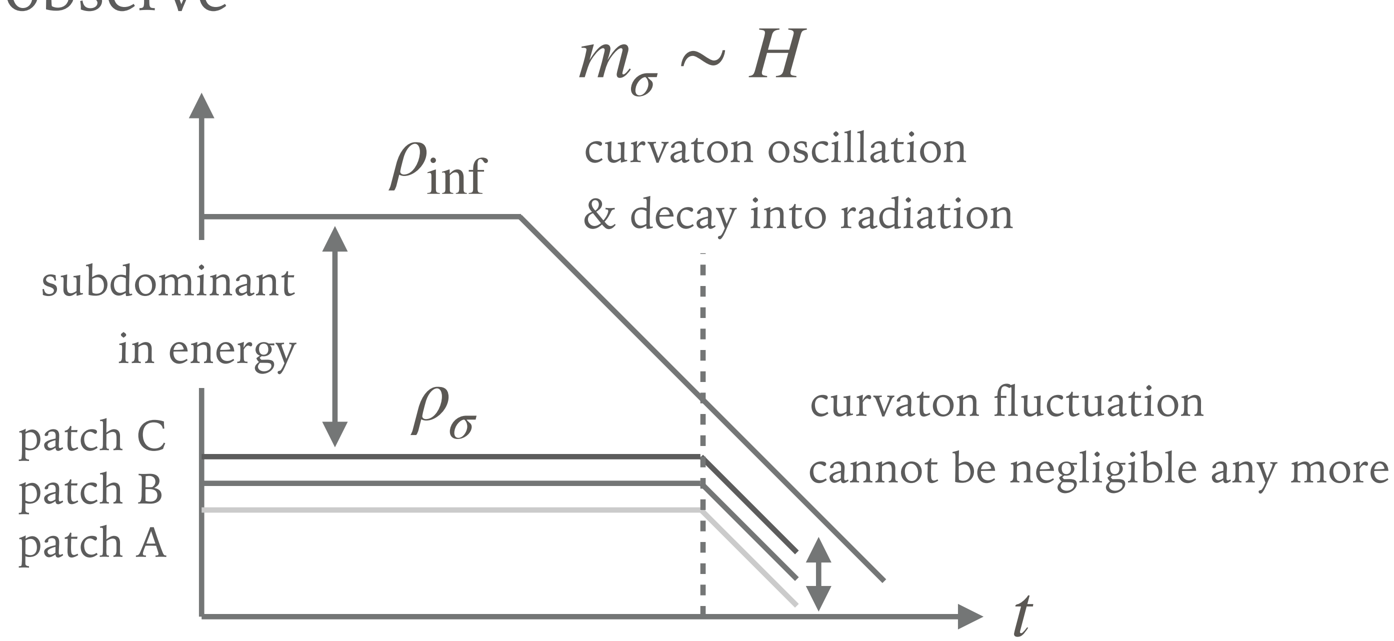
- Formalism to calculate curvature perturbation on superhorizon scales
- Often used to estimate curvature perturbation from curvatons

<u>Curvaton?</u>

① A hypothetical scalar field subdominant during inflation

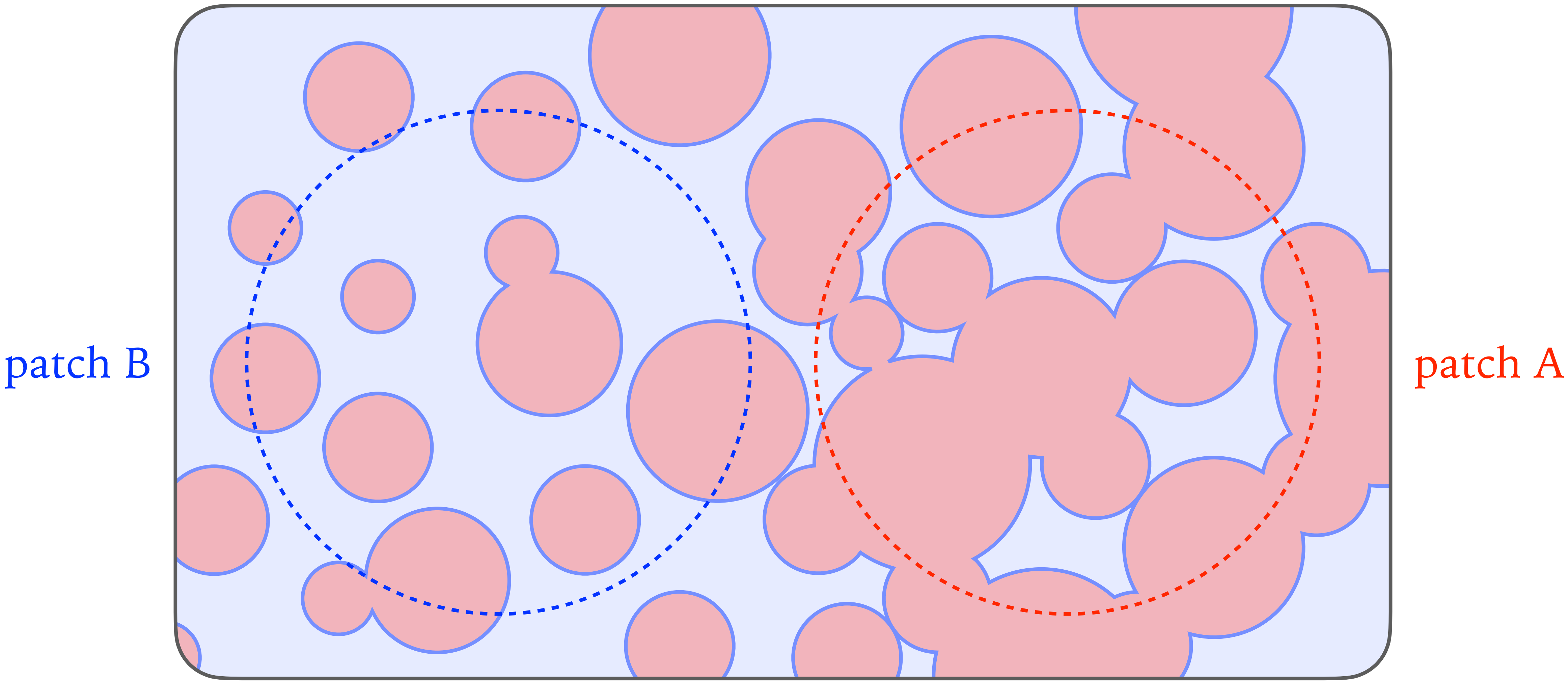
② Though subdominant in energy, it generates a dominant fraction of the curvature perturbation we observe





δN-FORMALISM

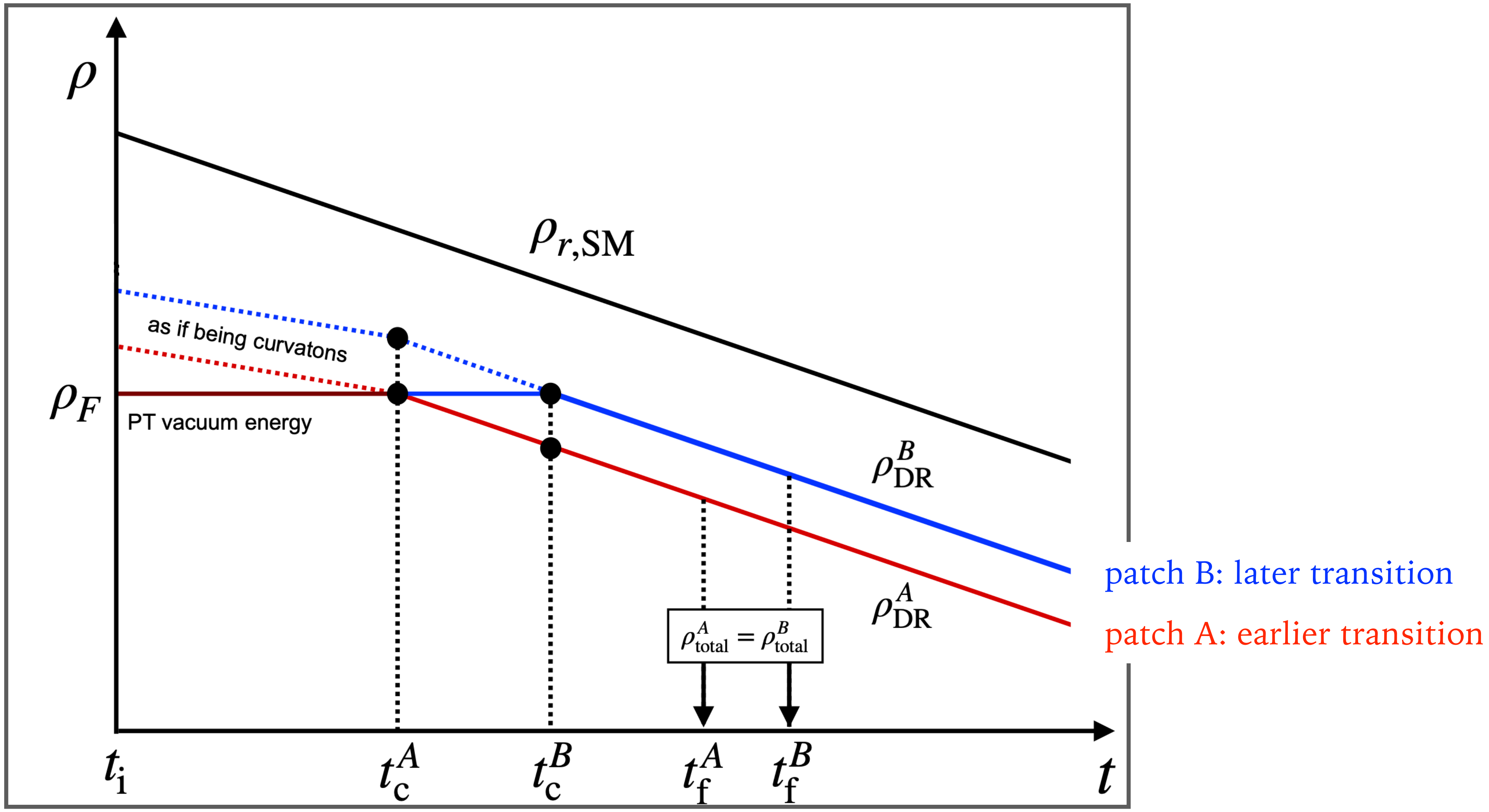
- The probabilistic completion of the transition by vacuum bubbles looks like a curvaton



- We assume a transition in the dark sector (producing dark radiation)

δN-FORMALISM

➤ The probabilistic completion of the transition by vacuum bubbles looks like a curvaton



δN-FORMALISM

- Prescription of the δN-formalism

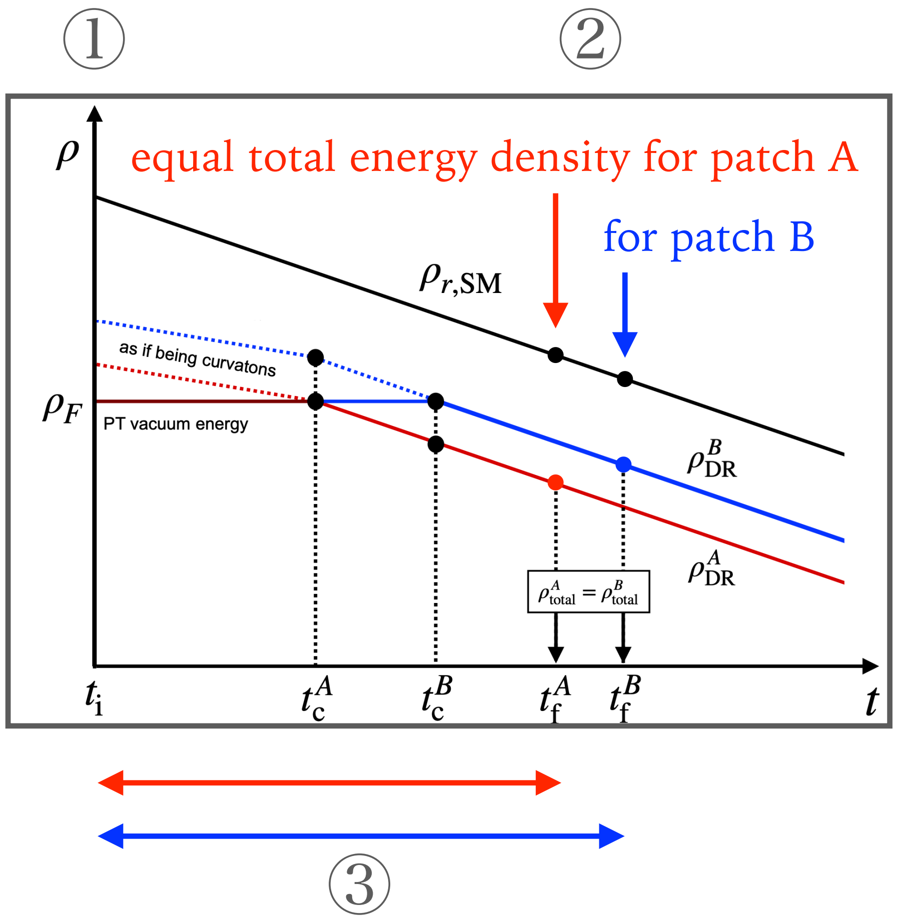
  ① Start from the spatially flat hypersurface

  ② Evaluate the e-folding N

  at the equal energy density hypersurface

  ③ Fluctuation δN in this e-folding N is

  the curvature perturbation ζ



- After all, we get the relation between ζ and δt_c

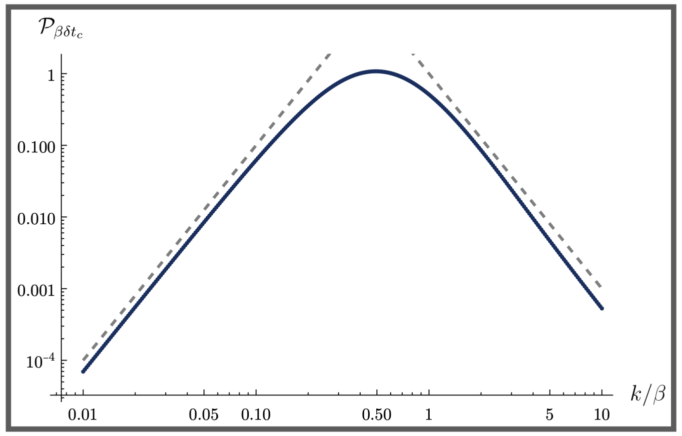
$$\zeta \simeq \frac{f_{\mathrm{DR}}}{2} \frac{\delta t_c}{\langle t_c \rangle} \; (+\zeta_{\mathrm{inf}}) = f_{\mathrm{DR}} H_* \delta t_c \; (+\zeta_{\mathrm{inf}})$$

inflationary ζ is an independent source

$f_{\mathrm{DR}} \ll 1$: average fraction of dark radiation after the completion of the transition

COMBINING THE TWO

- From the light-cone formalism, we get $\mathscr{P}_{\beta\delta t_c} =$

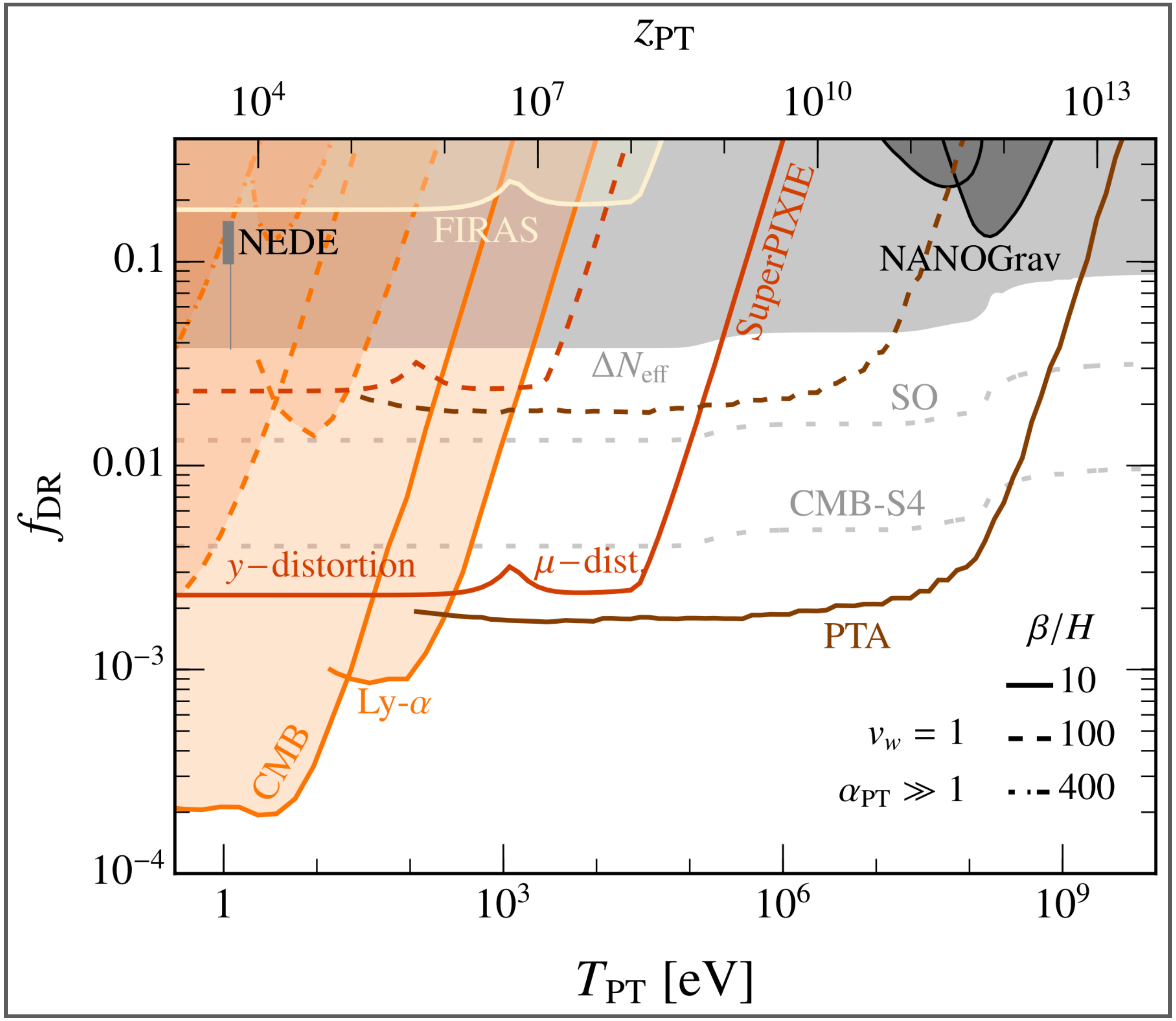


- From the δN-formalism, we get $\mathscr{P}_\zeta \simeq f_{\rm DR}^2 \mathscr{P}_{H_*\delta t_c} = \left(\dfrac{H_*}{\beta}\right)^2 f_{\rm DR}^2 \mathscr{P}_{\beta\delta t_c}$

power spectrum of $\langle H_*\delta t_c(x)\, H_*\delta t_c(y)\rangle$ power spectrum of $\langle \beta\delta t_c(x)\, \beta\delta t_c(y)\rangle$

- Using the $\mathscr{P}_\zeta$ obtained as an input for Boltzmann solvers (like CLASS), we can derive constraints on late-time transitions in the dark sector

CONSTRAINTS ON LATE-TIME TRANSITIONS IN A DARK SECTOR



[Elor, RJ, Kumar, McGehee, Tsai '24]

SUMMARY FOR PART 2-1

- To estimate superhorizon curvature perturbation in FOPTs, we develop "vacuum-bubble δN-formalism"

- Based on this, we report constraints on late-time FOPTs in a dark sector

OUTLINE

1. A positive definite formulation of tunneling

[Espinosa, RJ, Konstandin JCAP 02 (2023) 021, 2209.03293] [+Matake, Miyachi in progress]

2. Curvature perturbation from first-order phase transitions

2-1) Superhorizon scales [Ellor, RJ, Kumar, McGhee, Tsai PRL 133 (2024) 21, 211003, 2311.16222]

2-2) Horizon scales [Franciolini, RJ, Gouttenoire 2503.01962]

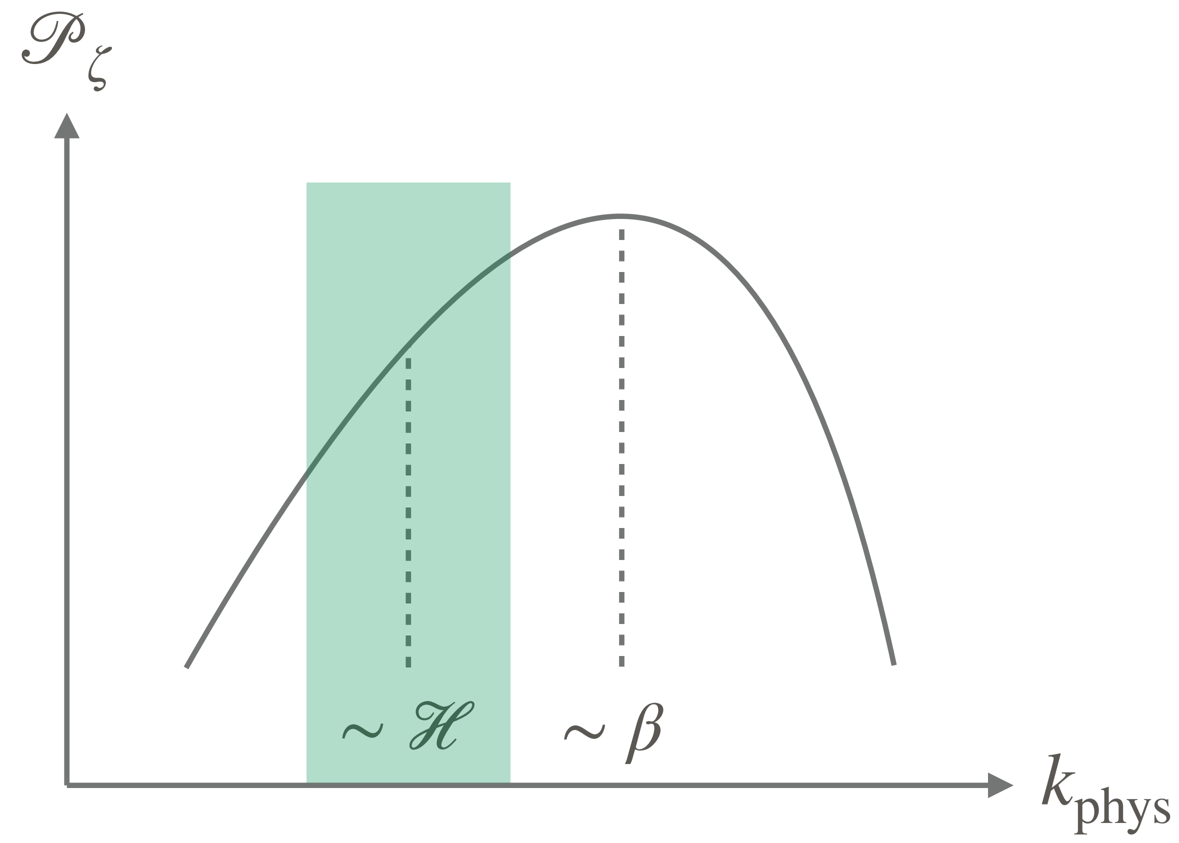
CURVATURE PERTURBATION FROM FIRST-ORDER PHASE TRANSITIONS

$k_{\rm phys} \ll H$

(plus, we assume $H \ll \beta$ for technical reasons)

[Ellor, Kumar, McGhee, Tsai PRL 133 (2024) 21, 211003, 2311.16222]

$k_{\rm phys} \sim H$



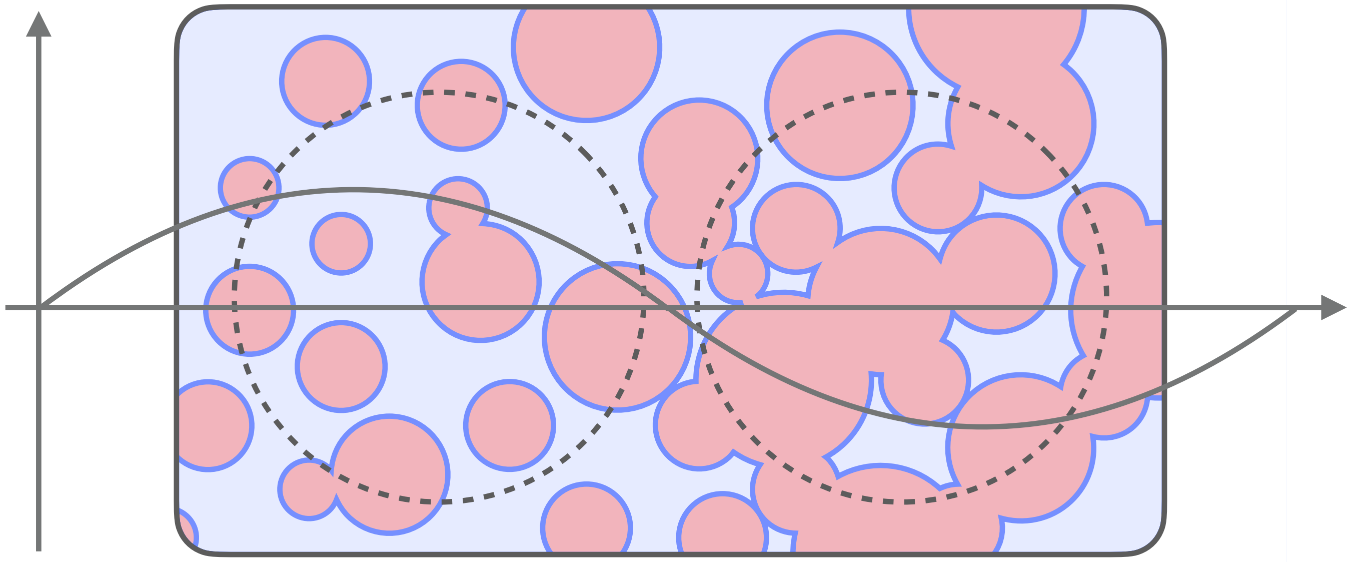
[Franciolini, RJ, Gouttenoire 2503.01962]

SLOW TRANSITION AND PBH FORMATION

- Can PBHs form from curvature perturbation generated by small β/H (but still $\gtrsim$ a few) FOPTs?

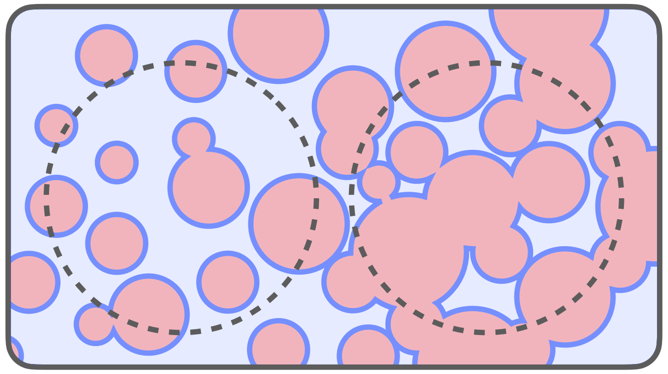
Intuitively

$$\delta \left(= \frac{\delta\rho}{\rho}\right)$$



- With a careful treatment of gauges (in cosmological perturbations), we answered to this question in the negative

SLOW TRANSITION AND PBH FORMATION



➤ Setup & findings of [Lewicki, Troczek, Vaskonen '24]

① Background

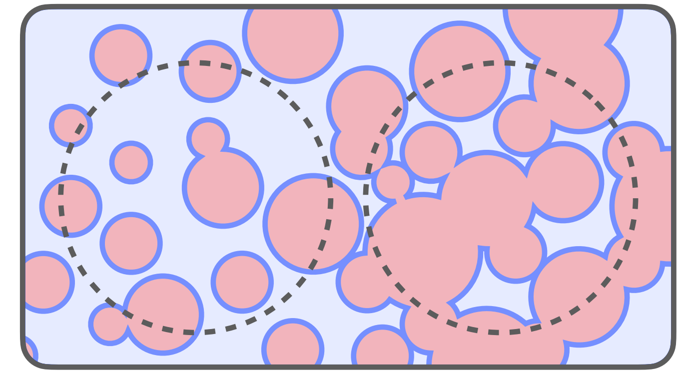
- Radiation & vacuum energy $\bar{\rho}_r' + 4\mathscr{H}\bar{\rho}_r = -\bar{\rho}_V'$
- Initially the universe is vacuum energy dominated $\bar{\rho}_V(t = -\infty) = \Delta V$, and then radiation takes over
- Vacuum energy decays with the exponential nucleation of bubbles

$$\Gamma(t) = H_*^4 \, e^{\beta(t - t_*)}$$

meaning that $\bar{\rho}_V$ decreases with the average false vacuum fraction $\bar{F}(t)$ as

$$\bar{\rho}_V = \bar{F}(t) \times \Delta V \qquad \bar{F}(t) = \exp\left[-\frac{4\pi}{3} \int_{-\infty}^{t} dt_n \, \Gamma(t_n) \, a(t_n)^3 \left(\int_{t_n}^{t} \frac{d\tilde{t}}{a(\tilde{t})} \right)^3 \right]$$

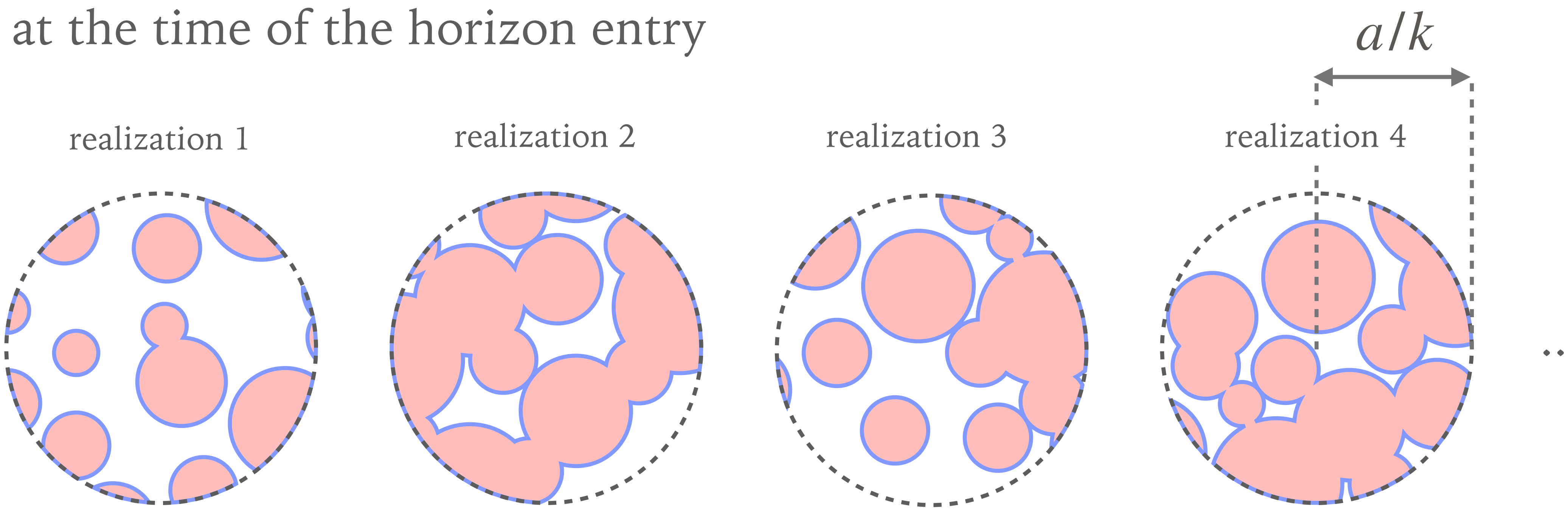
SLOW TRANSITION AND PBH FORMATION



➤ Setup & findings of [Lewicki, Troczek, Vaskonen '24]

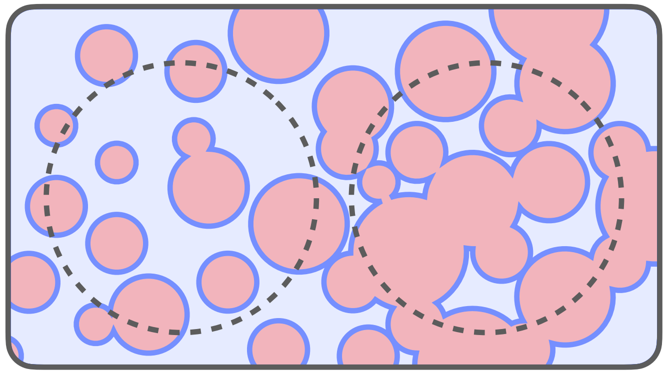
② Perturbation

- Stochastic process of bubble nucleation induces density fluctuations
- For a fixed comoving wavenumber k, consider a sphere of comoving radius $1/k$, and numerically calculate the PDF of the density contrast of this region at the time of the horizon entry



These pictures are just for illustration: they develop a much more efficient algorithm than naively generating bubbles

SLOW TRANSITION AND PBH FORMATION



- Setup & findings of [Lewicki, Troczek, Vaskonen '24]

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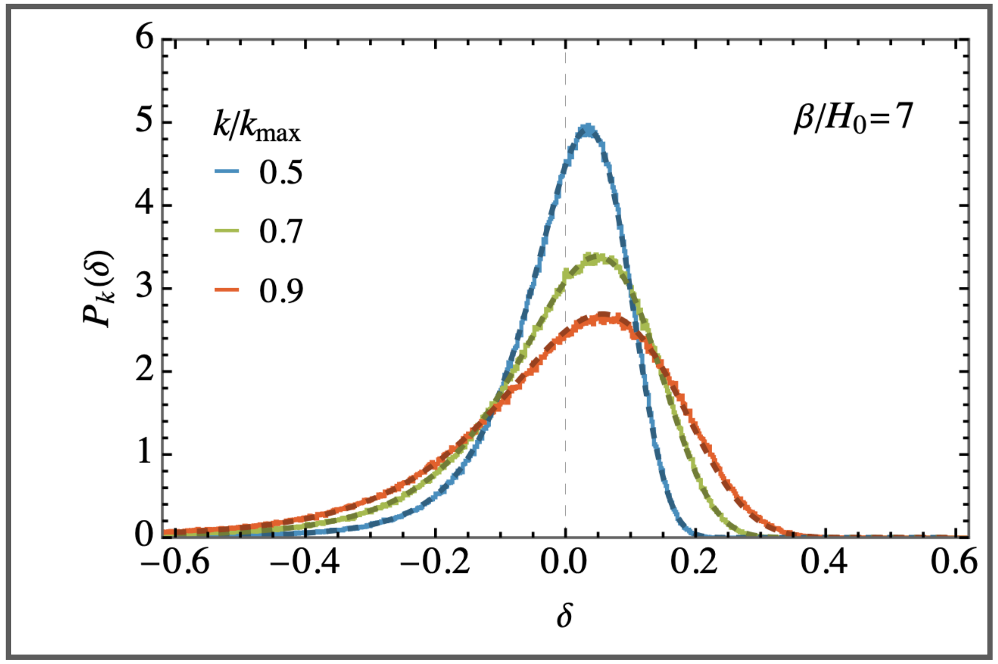
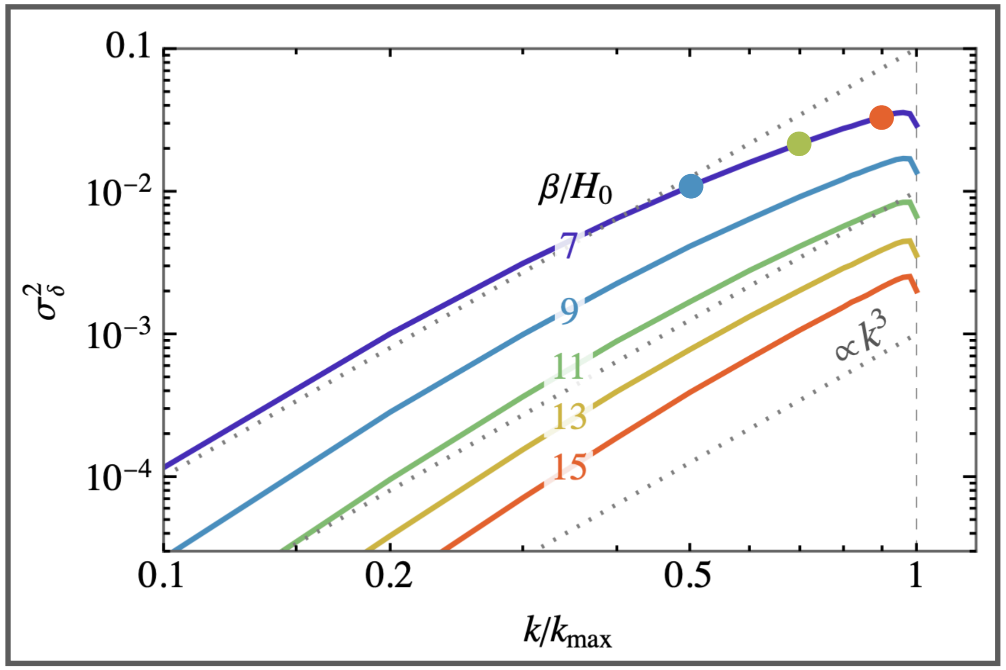
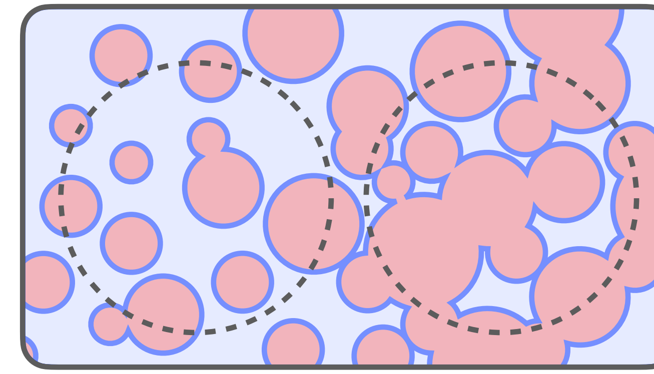




SLOW TRANSITION AND PBH FORMATION

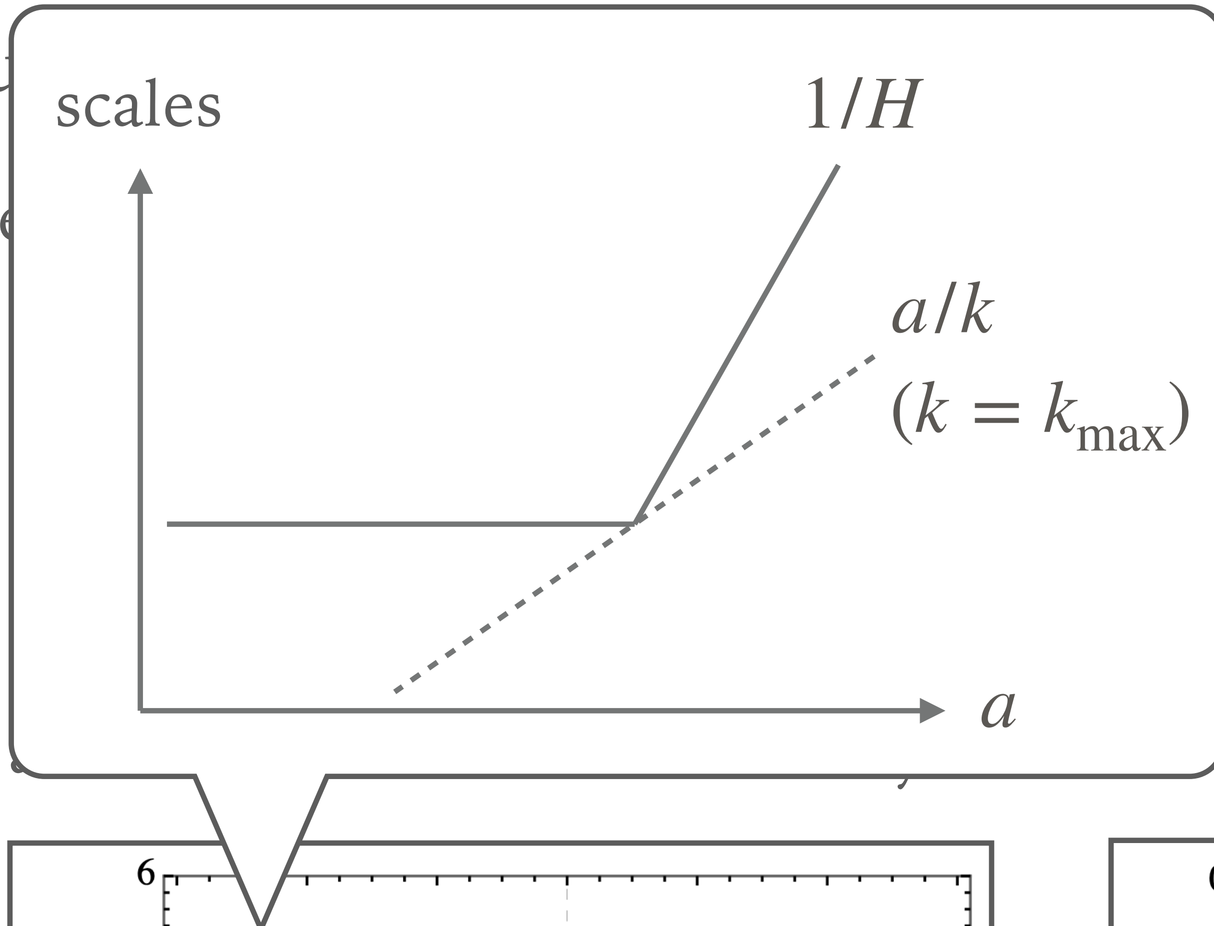


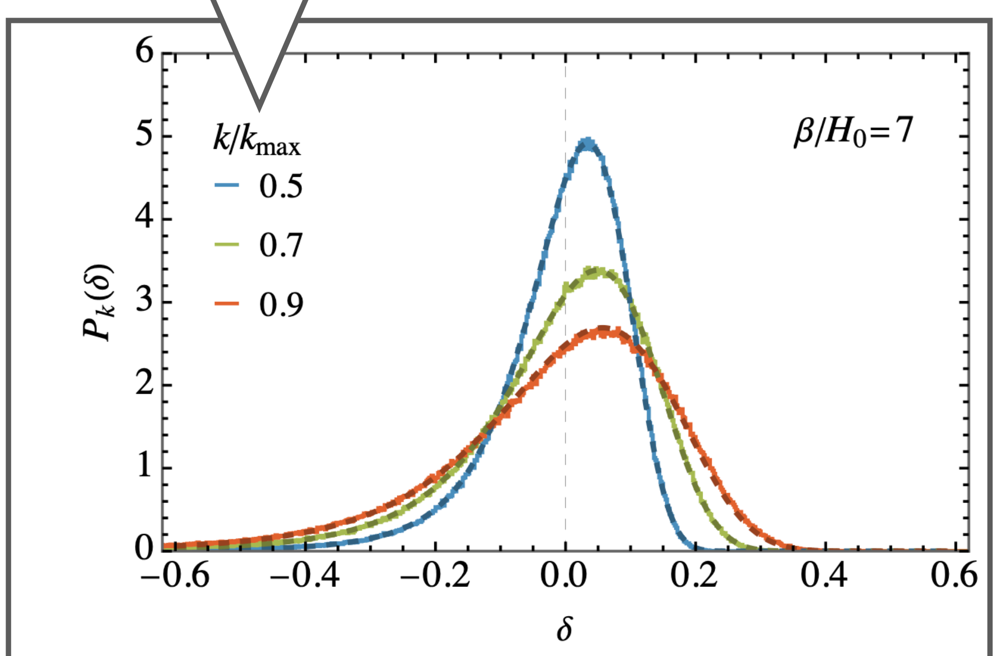
➤ Setu

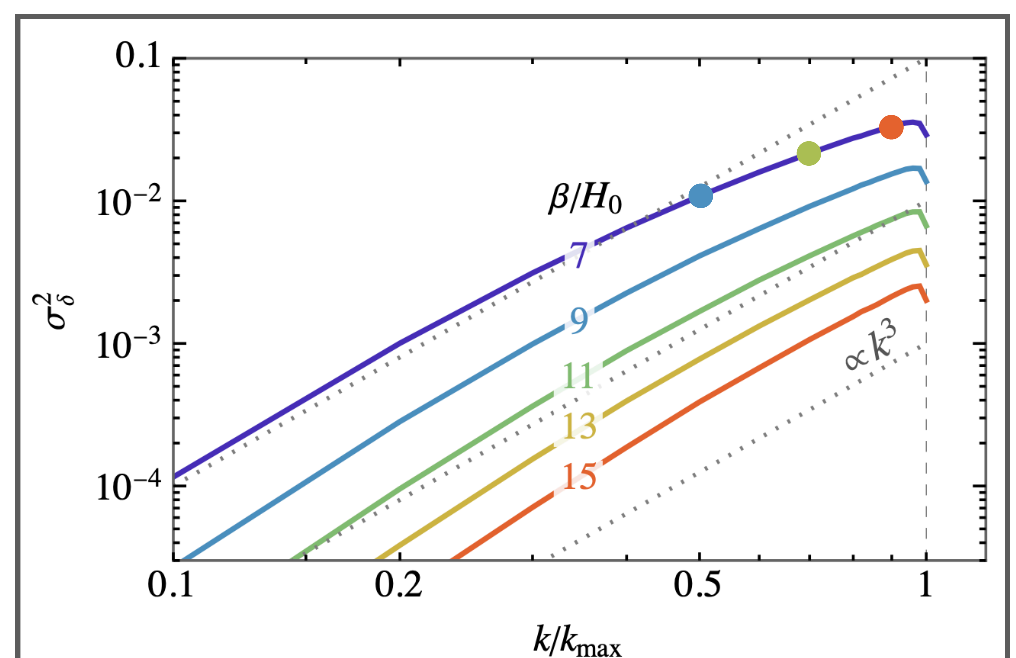
② Pe

- nduces density fluctuations
- sider a sphere of comoving radius $1/k$,

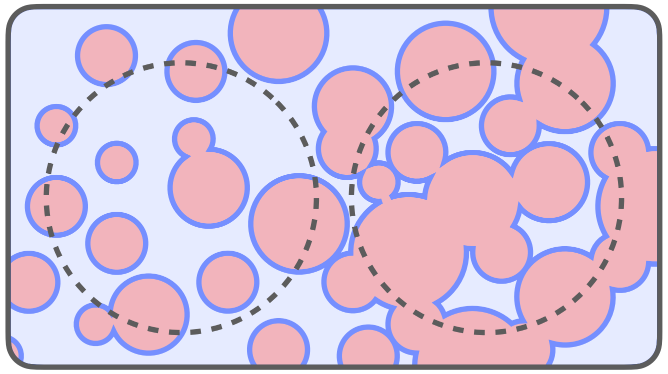
density contrast of this region







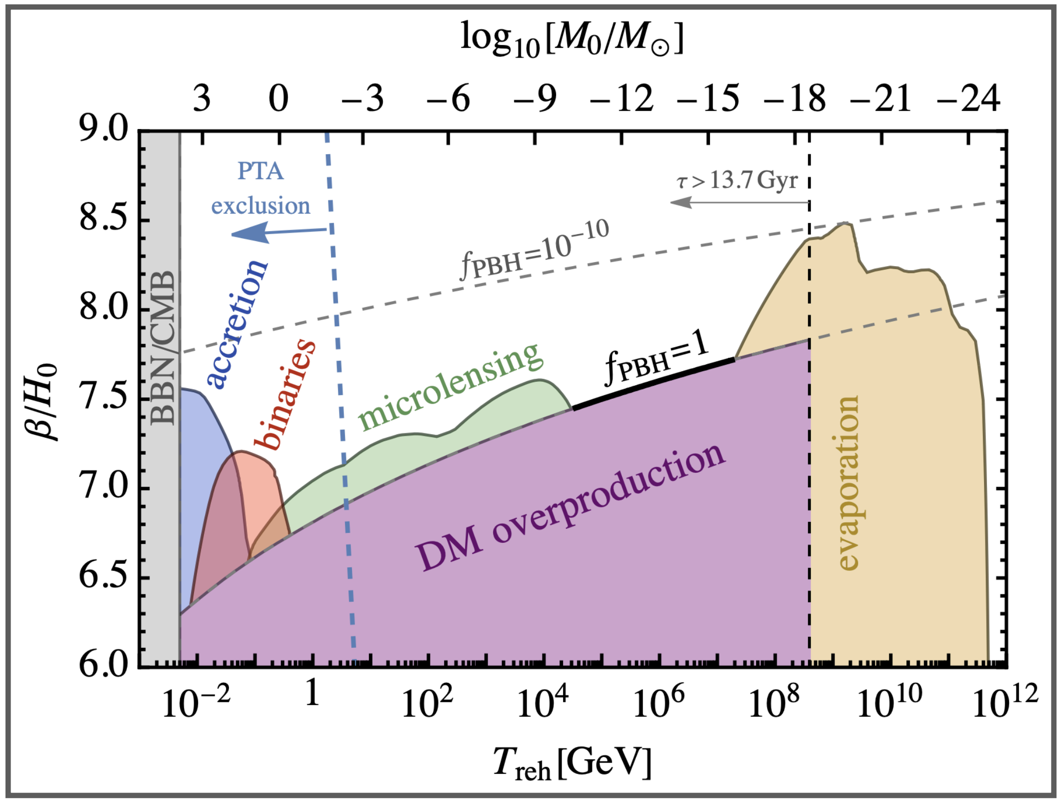
SLOW TRANSITION AND PBH FORMATION



- ➤ Setup & findings of [Lewicki, Troczek, Vaskonen '24]

  ② Perturbation

  - For $\beta/H_* \lesssim 7$ the variance of the density contrast is so large that the density contrast δ exceeds the threshold for PBH formation $\delta_c = 0.55$ frequently enough to explain the whole DM by PBHs



GAUGE ISSUES?

[Franciolini, RJ, Gouttenoire 2503.01962]

- δ is the density contrast, but in which gauge?

- Our point: δ should be interpreted as the density contrast in the flat gauge $\delta^{(F)}$, since in the algorithm of [Lewicki, Troczek, Vaskonen '24] the density contrast is computed in a ***flat*** FLRW universe

- On the other hand, the threshold $\delta_c \sim 0.5$ is estimated in the comoving gauge

- How would the conclusion change if we use the gauge consistently?

CONSISTENT TREATMENT OF THE GAUGE

[Franciolini, RJ, Gouttenoire 2503.01962]

- Perturbation equations we solve

encodes the false-vacuum fraction

$$\delta_{\boldsymbol{k}}^{(F)\prime} + 3\mathscr{H}(c_s^2 - w)\delta_{\boldsymbol{k}}^{(F)} = (1+w)\mathscr{V}_{\boldsymbol{k}} - 3\mathscr{H}\underline{\delta_{p,\mathrm{nad},\boldsymbol{k}}}$$

$$\Phi_{\boldsymbol{k}}'' + 3(1+c_s^2)\mathscr{H}\Phi_{\boldsymbol{k}}' + \left[3(c_s^2 - w)\mathscr{H}^2 + c_s^2 k^2\right]\Phi_{\boldsymbol{k}} = \frac{3}{2}\mathscr{H}\underline{\delta_{p,\mathrm{nad},\boldsymbol{k}}}$$

$$\mathscr{V}_{\boldsymbol{k}} = -\frac{2}{3(1+w)}\frac{\Phi_{\boldsymbol{k}}' + \mathscr{H}\Phi_{\boldsymbol{k}}}{\mathscr{H}}$$

- Equation of state $w = \bar{p}/\bar{\rho}$ & sound speed $c_s^2 = \bar{p}'/\bar{\rho}'$

- Gauge-invariant Newtonian potential Φ & scalar velocity $\mathscr{V}$

- Gauge-invariant non-adiabatic pressure $\delta_{p,\mathrm{nad}} = \dfrac{\delta p_{\mathrm{nad}}}{\bar{\rho}}$, $\delta p_{\mathrm{nad}} = \delta p^{(F)} - c_s^2 \delta\rho^{(F)}$

- In the present case $\delta p_{\mathrm{nad}} = \dfrac{1-3c_s^2}{3}\bar{\rho}\delta^{(F)} + \dfrac{4}{3}\Delta V \underline{\delta F^{(F)}}$

fluctuation in the false-vacuum fraction

CONSISTENT TREATMENT OF THE GAUGE

[Franciolini, RJ, Gouttenoire 2503.01962]

- We use the (very efficient) code developed in [Lewicki, Troczek, Vaskonen '24] to calculate the distribution of the fluctuation $\delta F^{(F)}$

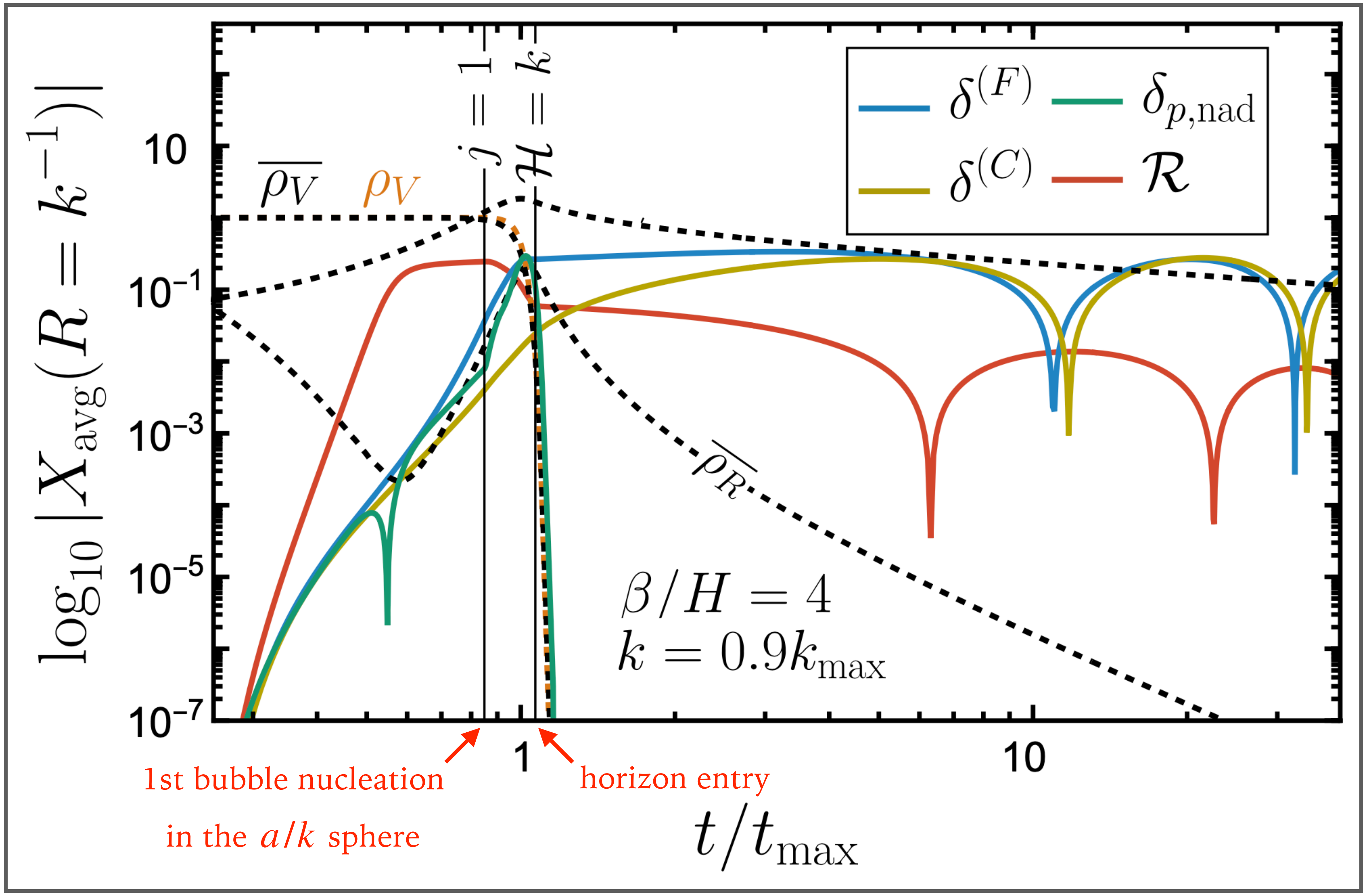
- The only difference is we identify it as the quantity in the flat gauge

- Once the perturbation equations are solved, we also estimate $\delta_{\boldsymbol{k}}^{(C)}$ with

$$\delta_{\boldsymbol{k}}^{(C)} = \delta_{\boldsymbol{k}}^{(F)} + (5+3w)\Phi_{\boldsymbol{k}} + \frac{2\Phi'_{\boldsymbol{k}}}{\mathscr{H}}$$

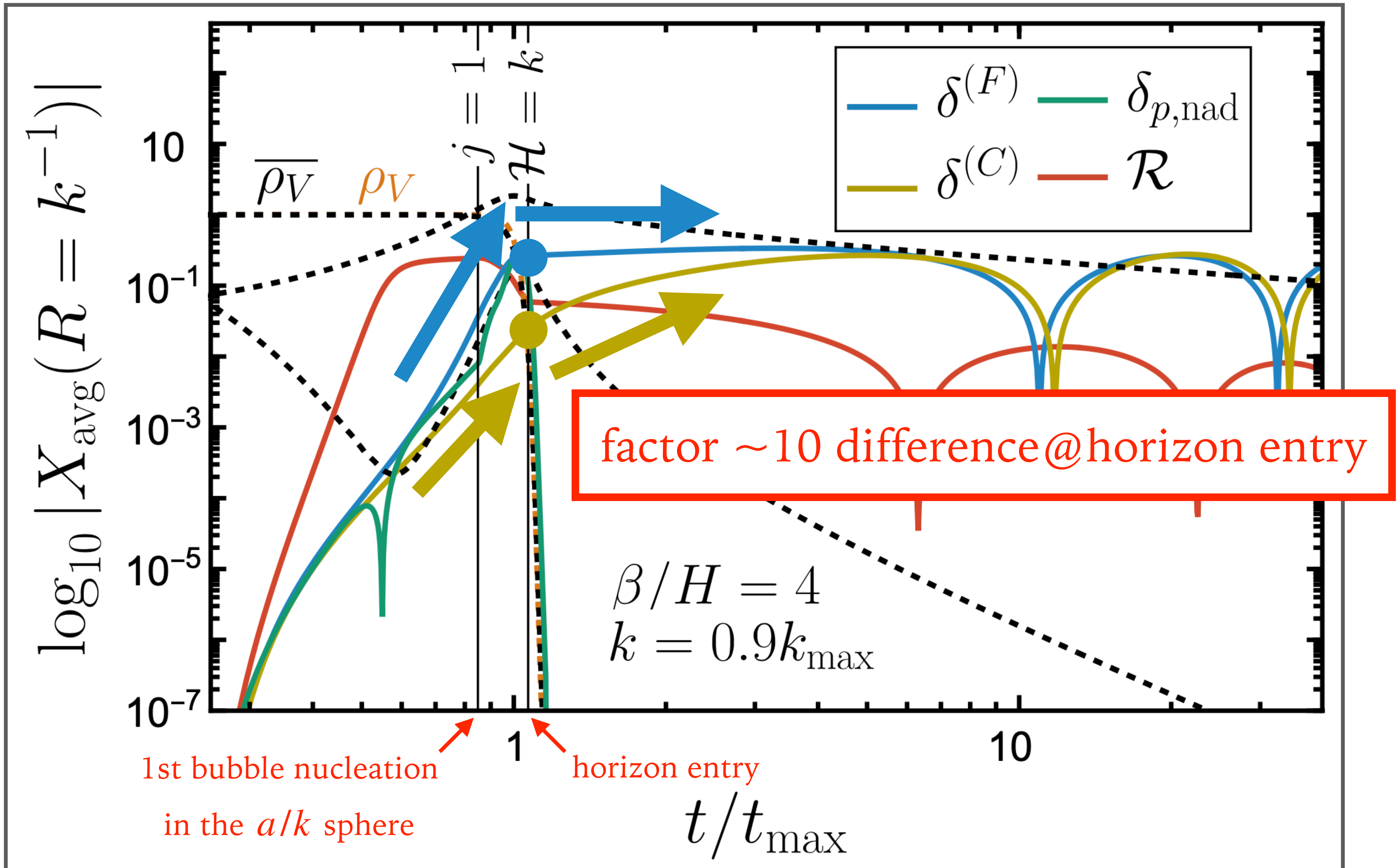
TYPICAL TIME EVOLUTION

- Point: difference between $\delta_{\boldsymbol{k}}^{(F)}$ and $\delta_{\boldsymbol{k}}^{(C)}$ around the horizon entry

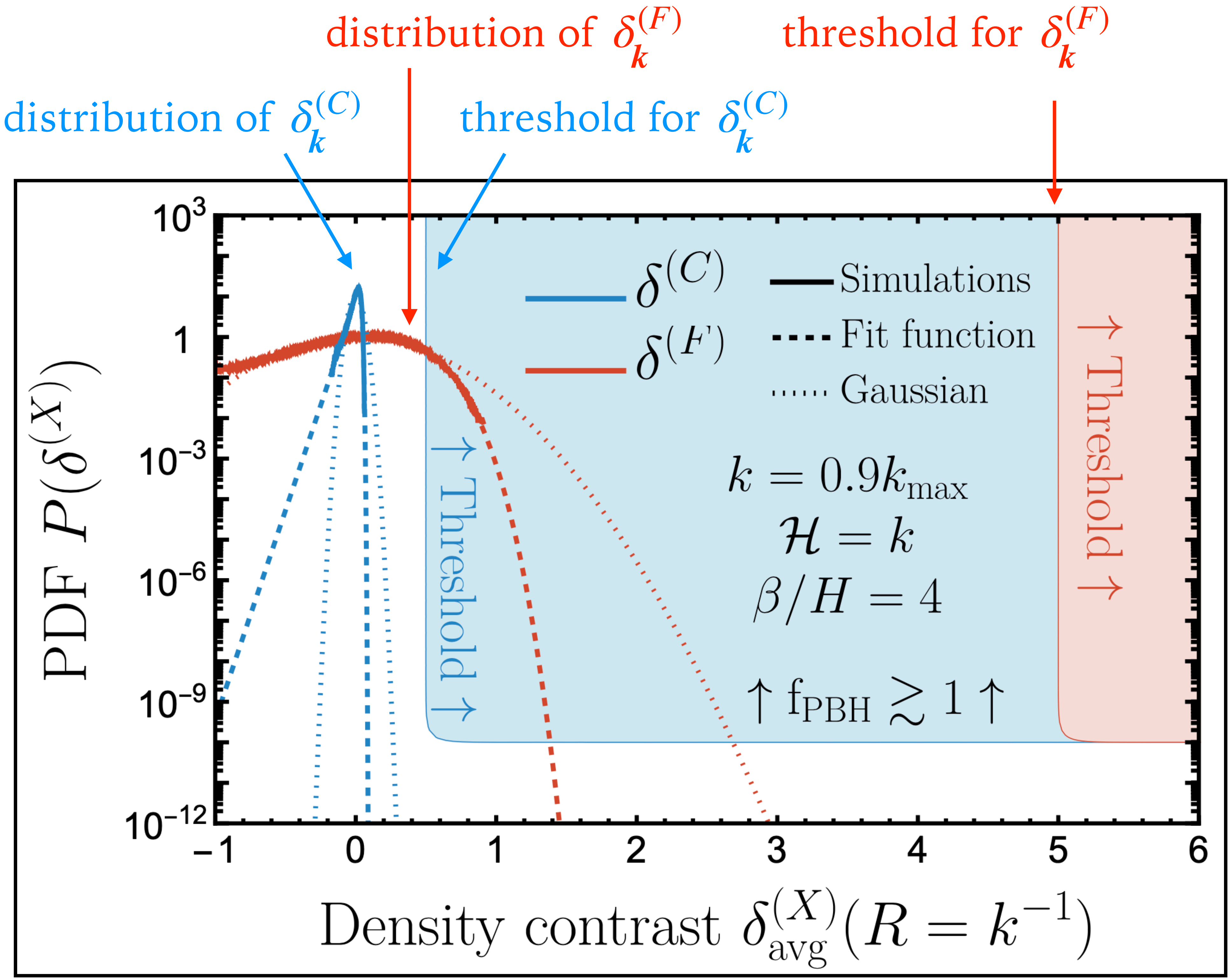


TYPICAL TIME EVOLUTION

➤ Point: difference between $\delta_k^{(F)}$ and $\delta_k^{(C)}$ around the horizon entry



IMPLICATION TO PBH FORMATION



SUMMARY FOR PART 2-2

- After carefully treating the gauge, PBH formation in supercooled FOPTs with $\beta/H \sim 7$ seems difficult

- Still missing some aspects?