Gravitational Wave Signals from Dark Higgs Inflation and Grand Unification

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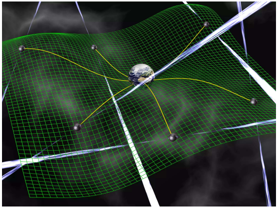
Based on work in collaboration with Injun Jeong, Seong Chan Park, Yeji Park, Stefano Scopel, Juhoon Son, Liliana Velasco Sevilla

Outline

A Window to the Very Early Universe?

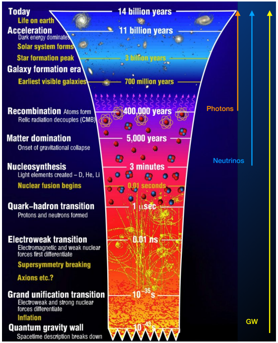
- Pulsar Timing Arrays
  $\rightsquigarrow$ Evidence for Stochastic GW Background
  NANOGrav, ApJL **951** (2023)
  EPTA, InPTA, A&A **678** (2023)
  Parkes PTA, ApJL **951** (2023)
  CPTA, RAA **23** (2023)



D. Champion/MPI for Radio Astronomy

- Lower frequency than LIGO/Virgo events
  $\rightsquigarrow$ Mergers of supermassive black holes?
- More interesting: particle physics origin
  - First-Order Phase Transition (FOPT)
  - Cosmic strings and domain walls
  - Primordial plasma
  - Inflation

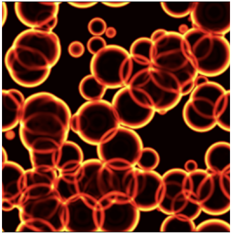
  $\rightsquigarrow$ GW backgrounds with different spectra



GW from First-Order Phase Transitions

Production processes

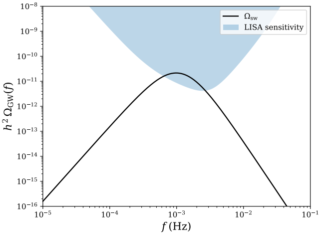
- Sound waves in plasma around bubbles
- Magnetohydrodynamic plasma turbulence
- Bubble collisions
- (Enhancement of scalar-induced GW)



I. Stomberg

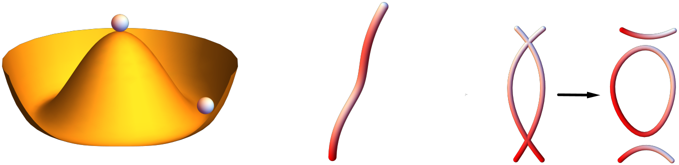
Peak frequency $\propto$ transition energy scale

f	v	Experiment
nHz	100 MeV	PTA
mHz	100 GeV	LISA
10^2 Hz	10^7 GeV	LVK



Caprini et al., JCAP **03** (2020)

Cosmic Strings and Domain Walls



Hinze, talk at *GW Probes of Physics BSM 4*, Warsaw, 2025

- Spontaneous symmetry breaking $G \to H$
- Non-trivial homotopy group $\pi_{2-D}(G/H)$

$\rightsquigarrow$ Topological defects: monopoles, cosmic strings, domain walls

- Cosmic strings form loops $\rightsquigarrow$ GW radiation
- Monopole nucleation $\rightsquigarrow$ string decay into GW

$\rightsquigarrow$ Explanation for PTA signal?

GW from the Primordial Plasma

- Collisions, hydrodynamic fluctuations in hot plasma $\rightsquigarrow$ GW
  Ghiglieri & Laine, JCAP **07** (2015)
- No new physics required
- Peak at frequency ~ GHz

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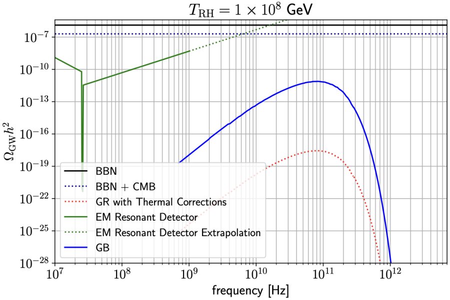
- Peak at frequency ~ GHz
- Amplitude can be enhanced by new physics

Example: Gauss-Bonnet Cosmology



Biswas et al., JCAP **09** (2024)

Higgs Inflation

Bezrukov & Shaposhnikov, PLB **659** (2008)

$$\mathcal{L}_J = \sqrt{-g_J}\left[\frac{M_P^2}{2} R_J + \xi \phi^\dagger \phi \, R_J + g_J^{\mu\nu} (D_\mu \phi)^\dagger (D_\nu \phi) - V_J(\phi) + \dots \right]$$

- J: Jordan frame
- R: Ricci scalar
- V_J: Standard Model scalar potential
- ξ: non-minimal coupling to gravity
    - Consistent with all symmetries
    - Required for renormalization in curved spacetime
    - Not necessarily small
    - ⚠ Unitarity violation?
      Burgess et al., JHEP **09** (2009), **07** (2010); Barbon & Espinosa, PRD **79** (2009); Hertzberg, JHEP **11** (2010); Giudice & Lee, PLB **694** (2011); ...
      Steingasser et al., `arXiv:2505.20386`
- Def.: conformal factor $\Omega^2(\phi) \equiv 1 + 2\xi \frac{\phi^\dagger \phi}{M_P^2}$

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Higgs Inflation

- Unitary gauge: $\phi = \begin{pmatrix} 0 \\ (\varphi + v)/\sqrt{2} \end{pmatrix}$
- Weyl transformation to Einstein frame: $g_J \to g_E \equiv \Omega^2(\varphi)\, g_J$
- Canonical normalization: $\chi = \int\limits_0^{\varphi} \mathrm{d}\varphi \sqrt{\frac{3}{2} \frac{(\Omega^2{}_{,\varphi})^2}{\Omega^4} + \frac{1}{\Omega^2}}$

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- Minimal coupling to gravity but modified potential
- Potential flat for large field values $\rightsquigarrow$ slow-roll

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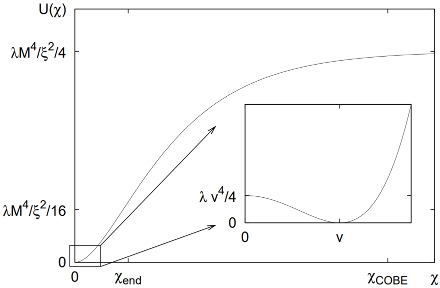

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Higgs Inflation in a Dark Sector

JK, Park[2], Son, Velasco Sevilla, JCAP **04** (2025)

$$\mathcal{L}_J = \sqrt{-g_J}\left[\frac{M_P^2}{2}\,\Omega^2(\phi)\,R_J + g_J^{\mu\nu}(D_\mu\phi)^\dagger(D_\nu\phi) - \frac{1}{4}g_J^{\mu\rho}g_J^{\nu\sigma}X_{\mu\nu}X_{\rho\sigma} - V_J(\phi)\right]$$

$$V_J(\phi) = -\mu^2\phi^\dagger\phi + \lambda\,(\phi^\dagger\phi)^2 = \lambda\left(\phi^\dagger\phi - \frac{v^2}{2}\right)^2 + \text{const.}$$

- $U(1)_X$ gauge symmetry with gauge coupling g
- SM singlet scalar ϕ with $U(1)_X$ charge
  - Spontaneously breaks $U(1)_X \rightsquigarrow$ FOPT $\rightsquigarrow$ GW
  - Non-minimal coupling to gravity $\rightsquigarrow$ Higgs-inflation-like inflation

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  - Spontaneously breaks $U(1)_X \rightsquigarrow$ FOPT $\rightsquigarrow$ GW
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- Optional: fermions with mass from Yukawa coupling
- Weak coupling to SM $\rightsquigarrow$ reheating

$$\mathcal{L}_\psi = \sqrt{-g_J}\sum_{i=1}^{n_\psi}\left[\overline{\psi}_{Li}i\not{D}\psi_{Li} + \overline{\psi}_{Ri}i\not{D}\psi_{Ri} - \frac{y}{2}\left(\overline{\psi}_{Li}\,\phi\,\psi_{Li}^c + \overline{\psi}_{Ri}\,\phi\,\psi_{Ri}^c + \text{h.c.}\right)\right]$$

Gravitational Waves from Phase Transition

Scalar potential with 1-loop and finite-temperature corrections

- $\rightsquigarrow$ Potential barrier around $T \sim v$
- $\rightsquigarrow$ FOPT possible

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GW spectrum determined by

- Nucleation temperature T_n
- $\alpha \leftrightsquigarrow$ strength of PT
- $\beta \leftrightsquigarrow$ duration
- Bubble wall velocity v_w

Calculated with help from `CosmoTransitions`

Wainwright, Comput. Phys. Commun. **183** (2012)

Effective Scalar Potential at Finite Temperature

"Traditional" 4D approach

$$V(h_c, T) = V_{\text{tree}}(h_c) + V_{\text{1-loop}}(h_c) + V_{\text{th}}(h_c, T)$$

$$V_{\text{tree}}(h_c) = -\frac{\mu^2}{2} h_c^2 + \frac{\lambda}{4} h_c^4$$

$$V_{\text{1-loop}}(h_c) = \sum_{i=h,\chi,g,f} \frac{n_i}{64\pi^2} m_i^4(h_c) \left[\ln \frac{|m_i^2(h_c)|}{v^2} - C_i \right]$$

$$V_{\text{th}}(h_c, T) = \sum_{i=h,\chi,g} \frac{n_i}{2\pi^2} T^4 \, \mathrm{Re}\, J_b\!\left(\frac{m_i^2(h_c)}{T^2} \right) + \frac{n_f}{2\pi^2} T^4 J_f\!\left(\frac{m_f^2(h_c)}{T^2} \right)$$

$$m_h^2(h_c) = -\mu^2 + 3\lambda h_c^2$$

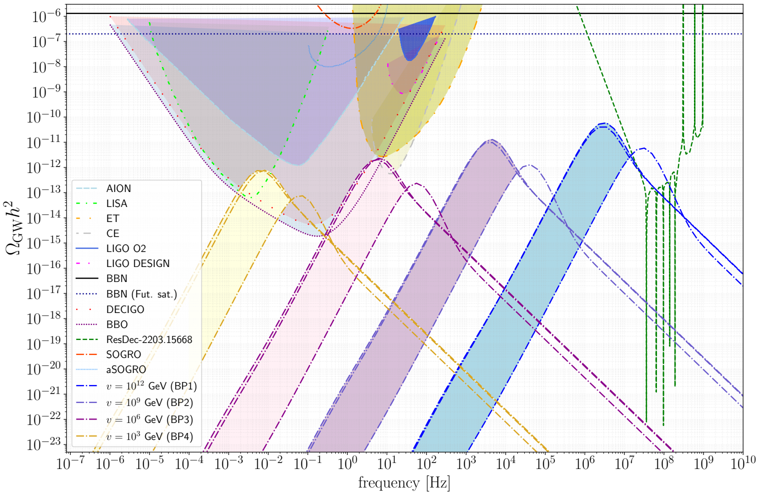
$$m_\chi^2(h_c) = -\mu^2 + \lambda h_c^2$$

$$m_g^2(h_c) = g^2 h_c^2$$

$$m_f^2(h_c) = \frac{y^2}{2} h_c^2$$

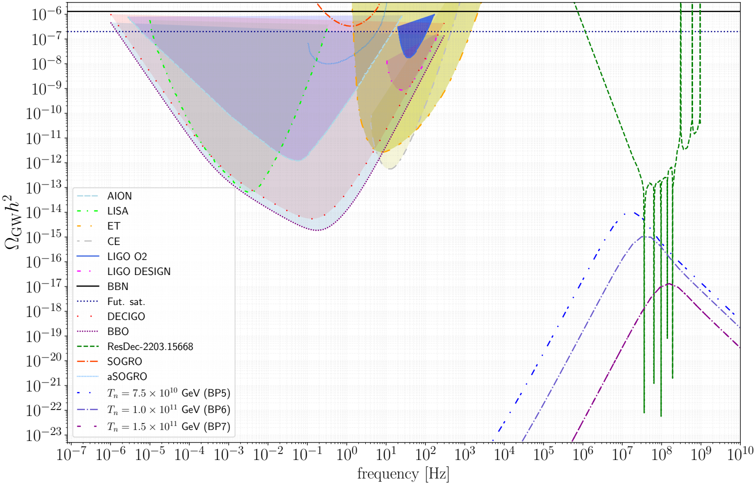
Gravitational Wave Spectrum I

No fermions, $g = 0.95$, $\lambda = 10^{-3}$
Bubble wall velocity $v_w = 1, v_{\text{det}}, 0.1$

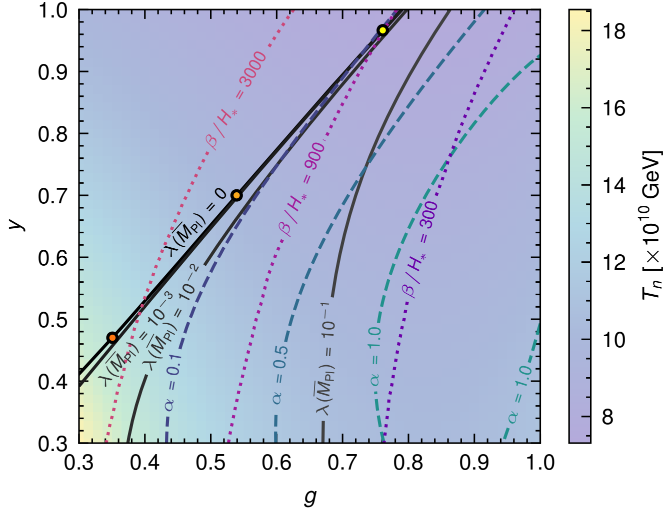


Gravitational Wave Spectrum II

1 fermion pair, $v_w = v_{\text{det}}$, $v = 10^{12}$ GeV, $\lambda = 10^{-3}$
$(g, y) \simeq (0.76, 0.97), (0.54, 0.70), (0.35, 0.47)$



Parameter Space with 1 Fermion Pair



Inflationary Observables

CMB measurements (pre-ACT)

Scalar power spectrum amplitude	$A_s = (2.098 \pm 0.023) \times 10^{-9}$
Scalar spectral index	$n_s = 0.9649 \pm 0.0042$
Tensor-to-scalar power ratio	$r < 0.036$ (95% CL)

Planck, A&A **641** (2020); BICEP/Keck, PRL **127** (2021)

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Model results (60 *e*-foldings)

- $n_s \simeq 0.965$
- $r \simeq 0.003$
- $A_s \simeq 5.1 \, \frac{\lambda(M_P)}{\xi^2} \quad \rightsquigarrow \quad \xi \simeq 5 \times 10^4 \sqrt{\lambda(M_P)}$

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	$g(M_P)$	$\lambda(M_P)$	$y(M_P)$	n_ψ	ξ
BP1	0.98	0.37	-	0	3×10^4
BP2	0.99	0.57	-	0	4×10^4
BP3	1.00	0.92	-	0	5×10^4
BP4	1.02	2.22	-	0	7×10^4
BP5	0.79	$\simeq 0$	1.10	1	$\mathcal{O}(1)$
BP6	0.55	$\simeq 0$	0.74	1	$\mathcal{O}(1)$
BP7	0.35	$\simeq 0$	0.48	1	$\mathcal{O}(1)$

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- With fermions: renormalization group running allows $\xi \sim 1$
- Analogous to Critical Higgs Inflation
  Hamada et al., PRL **112** (2014), PRD **91** (2015)

Grand Unification

- Standard Model gauge group

$$G_{\mathrm{SM}} = SU(3)_{\mathrm{c}} \otimes SU(2)_{\mathrm{L}} \otimes U(1)_Y \subset G = SU(5), SO(10), \ldots$$

- Spontaneous symmetry breaking $G \overset{\langle\phi\rangle}{\to} G_{\mathrm{SM}}$
  - ⇝ Many possibilities
  - ⇝ Rich phenomenology of topological defects, phase transitions
  - ⇝ Probe by GW

$$
\mathrm{SO}(10)\begin{cases}
\xrightarrow{1} & 4_{\mathrm{C}}\,2_{\mathrm{L}}\,2_{\mathrm{R}} & \longrightarrow & \\
\xrightarrow{1,2} & 4_{\mathrm{C}}\,2_{\mathrm{L}}\,2_{\mathrm{R}}\,Z_2^{\mathrm{C}} & \longrightarrow & \\
\xrightarrow{1,2} & 4_{\mathrm{C}}\,2_{\mathrm{L}}\,1_{\mathrm{R}}\,Z_2^{\mathrm{C}} & \longrightarrow & \cdots \\
\xrightarrow{1} & 4_{\mathrm{C}}\,2_{\mathrm{L}}\,1_{\mathrm{R}} & \longrightarrow & \cdots \\
\xrightarrow{1,2} & 3_{\mathrm{C}}\,2_{\mathrm{L}}\,2_{\mathrm{R}}\,1_{\mathrm{B-L}}\,Z_2^{\mathrm{C}} & \longrightarrow & \cdots \\
\xrightarrow{1} & 3_{\mathrm{C}}\,2_{\mathrm{L}}\,2_{\mathrm{R}}\,1_{\mathrm{B-L}} & \longrightarrow & \cdots \\
\xrightarrow{1} & 3_{\mathrm{C}}\,2_{\mathrm{L}}\,1_{\mathrm{R}}\,1_{\mathrm{B-L}} & \xrightarrow{2\ (2)} & \mathrm{G}_{\mathrm{SM}}\ (Z_2) \\
\xrightarrow{1\ (1,2)} & \mathrm{G}_{\mathrm{SM}}\ (Z_2) & &
\end{cases}
$$

$$
4_{\mathrm{C}}\,2_{\mathrm{L}}\,2_{\mathrm{R}}\begin{cases}
\xrightarrow{1}\ 3_{\mathrm{C}}\,2_{\mathrm{L}}\,2_{\mathrm{R}}\,1_{\mathrm{B-L}} & \begin{cases}\xrightarrow{1}\ 3_{\mathrm{C}}\,2_{\mathrm{L}}\,1_{\mathrm{R}}\,1_{\mathrm{B-L}} \xrightarrow{2\ (2)} \mathrm{G}_{\mathrm{SM}}\ (Z_2)\\ \xrightarrow{2'\ (2)}\ \mathrm{G}_{\mathrm{SM}}\ (Z_2)\end{cases} \\
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\end{cases}
$$

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4_{\mathrm{C}}\,2_{\mathrm{L}}\,2_{\mathrm{R}}\,Z_2^{\mathrm{C}}\begin{cases}
\xrightarrow{1}\ 3_{\mathrm{C}}\,2_{\mathrm{L}}\,2_{\mathrm{R}}\,1_{\mathrm{B-L}}\,Z_2^{\mathrm{C}} & \begin{cases}\xrightarrow{3}\ 3_{\mathrm{C}}\,2_{\mathrm{L}}\,2_{\mathrm{R}}\,1_{\mathrm{B-L}} \longrightarrow \cdots\\ \xrightarrow{1,3}\ 3_{\mathrm{C}}\,2_{\mathrm{L}}\,1_{\mathrm{R}}\,1_{\mathrm{B-L}} \xrightarrow{2\ (2)} \mathrm{G}_{\mathrm{SM}}\ (Z_2)\\ \xrightarrow{2',3\ (2,3)}\ \mathrm{G}_{\mathrm{SM}}\ (Z_2)\end{cases} \\
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\xrightarrow{3}\ 4_{\mathrm{C}}\,2_{\mathrm{L}}\,2_{\mathrm{R}} & \longrightarrow\ \text{Eq. (4.10)} \\
\xrightarrow{1}\ 4_{\mathrm{C}}\,2_{\mathrm{L}}\,1_{\mathrm{R}} & \longrightarrow\ \cdots \\
\xrightarrow{1,3}\ 3_{\mathrm{C}}\,2_{\mathrm{L}}\,2_{\mathrm{R}}\,1_{\mathrm{B-L}} & \longrightarrow\ \cdots \\
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\xrightarrow{1,3\ (1,2,3)}\ \mathrm{G}_{\mathrm{SM}}\ (Z_2). &
\end{cases}
$$

Jeannerot et al., PRD **68** (2003); figure by Liliana Velasco Sevilla

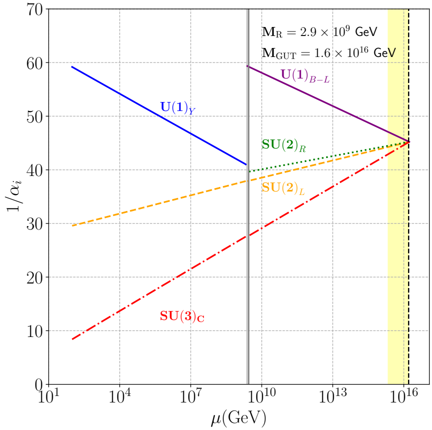
A Promising SO(10) Scenario

Bertolini et al., PRD **81** (2010), **85** (2012); Malinský et al., PRD **95** (2017), **105** (2022), **108** (2023)
Jeong, JK, Scopel, Velasco-Sevilla, `arXiv:2506.07182`

- Gauge bosons: adjoint representation **45** of $SO(10)$
- Fermions: 3 generations in **16**
- Scalars:
  - **45** $\rightsquigarrow$ break $SO(10)$ at $M_{\mathrm{GUT}} \sim 10^{16}$ GeV
  - **126** $\rightsquigarrow$ break $SU(2)_{\mathrm{R}} \times U(1)_{B-L}$ at $M_{\mathrm{R}} \sim 10^{9}$ GeV
  - **10** $\rightsquigarrow$ electroweak symmetry breaking
- 2-step symmetry breaking

$$SO(10) \xrightarrow{\langle \mathbf{45} \rangle \sim M_{\mathrm{GUT}}} SU(3)_{\mathrm{c}} \times SU(2)_{\mathrm{L}} \times SU(2)_{\mathrm{R}} \times U(1)_{B-L}$$
$$\xrightarrow{\langle \mathbf{126} \rangle \sim M_{\mathrm{R}}} SU(3)_{\mathrm{c}} \times SU(2)_{\mathrm{L}} \times U(1)_{Y}$$

Gauge Coupling Unification



- Yellow band: uncertainty from threshold corrections

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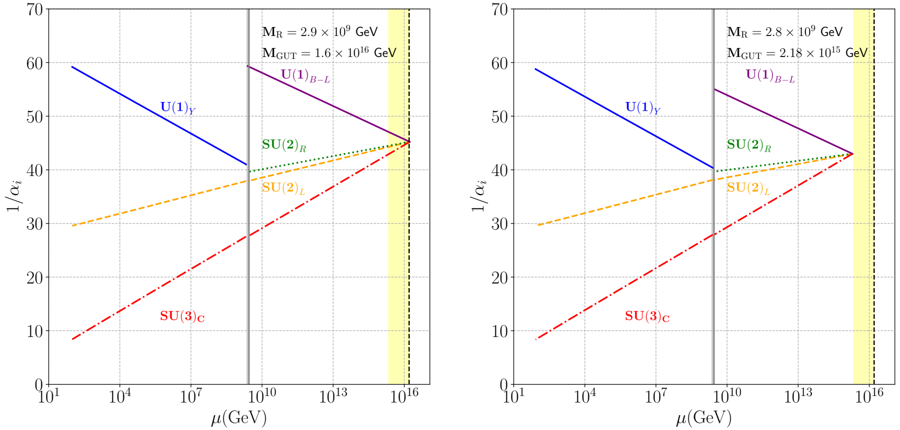

- Yellow band: uncertainty from threshold corrections
- Right plot: Dark Matter candidate added
  Biswas, Kar, Kim, Scopel, Velasco-Sevilla, PRD **106** (2022)

Constraints

- Gauge couplings unify
- Proton lifetime $\sim 10^{36}$ y
  Jeong, JK, Scopel, Velasco-Sevilla, `arXiv:2506.07182`
- Viable scalar mass spectrum for suitable parameters
  Malinský et al., PRD **105** (2022)

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  Malinský et al., PRD **105** (2022)

⚠ Monopoles from 1st symmetry breaking step
$\rightsquigarrow$ Reduction of density needed

- Different mechanisms, e.g., annihilation in interplay with strings
  Vilenkin, NPB **196** (1982), Preskill & Vilenkin, PRD **47** (1993)
- Promising GW signal?

⚠ Cosmic strings from 2nd symmetry breaking step
$\rightsquigarrow$ Promising GW signal or conflict with bounds?

⚠ Electroweak symmetry breaking and fermion masses challenging in minimal setup Malinský et al., PRD **108** (2023)

Effective Scalar Potential at Finite Temperature

- Potential for symmetry breaking at M_{GUT}

$$V_{\mathrm{tree}}(\phi) = -\frac{\mu^2}{2}\,\mathrm{Tr}\,\phi^2 + \frac{a_0}{4}\left(\mathrm{Tr}\,\phi^2\right)^2 + \frac{a_2}{4}\,\mathrm{Tr}\,\phi^4$$

- Classical field φ_c in vev direction

$$\phi_c = \varphi_c\,\sqrt{2}i\,U_{15}$$

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- Classical field φ_c in vev direction

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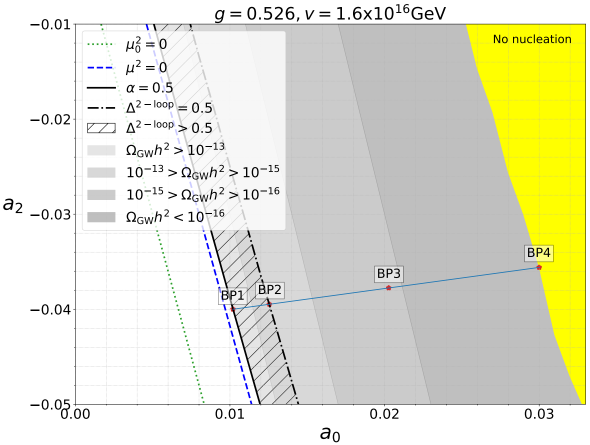
- Effective potential

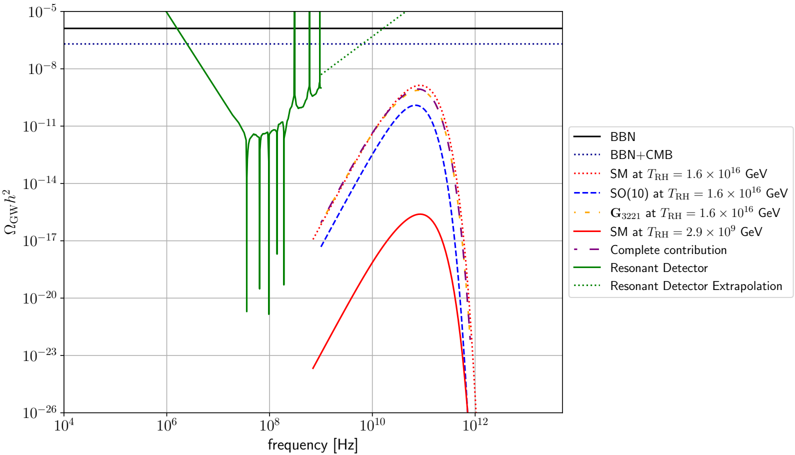
$$\begin{aligned}
V(\varphi_c, T) &= V_{\mathrm{tree}}(\varphi_c) + V_{\text{1-loop}}(\varphi_c) + V_{\mathrm{th}}(\varphi_c, T) \\
V_{\mathrm{tree}}(\varphi_c) &= -\frac{1}{2}\mu^2\varphi_c^2 + a_0\varphi_c^4 + \frac{1}{6}a_2\varphi_c^4 \\
V_{\text{1-loop}}(\varphi_c) &= \sum_{i=g,s,\chi} \frac{n_i}{64\pi^2} m_i^4(\varphi_c)\left[\ln\frac{|m_i^2(\varphi_c)|}{v^2} - C_i\right] \\
V_{\mathrm{th}}(\varphi_c, T) &= \sum_{i=g,s,\chi} \frac{n_i}{2\pi^2} T^4\,\mathrm{Re}\,J_b\!\left(\frac{m_i^2(\varphi_c)}{T^2}\right)
\end{aligned}$$

Gravitational Wave Density from Phase Transition



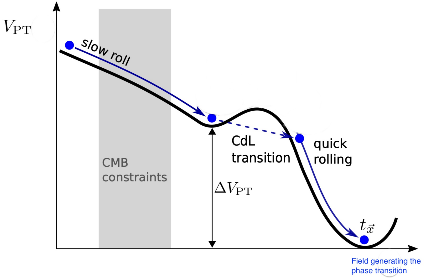
- Gauge coupling g and $M_{\text{GUT}} = v$ determined from unification
- Viable scalar mass spectrum $\rightsquigarrow$ a_0 and a_2 restricted as shown
  Malinský et al., PRD **105** (2022)

GW from the Primordial Plasma



- $M_{\rm GUT} = 1.6 \times 10^{16}$ GeV
- For comparison only: SM signal for same reheating temperature
- *Resonant Detector* proposed by Herman et al., PRD **108** (2023)

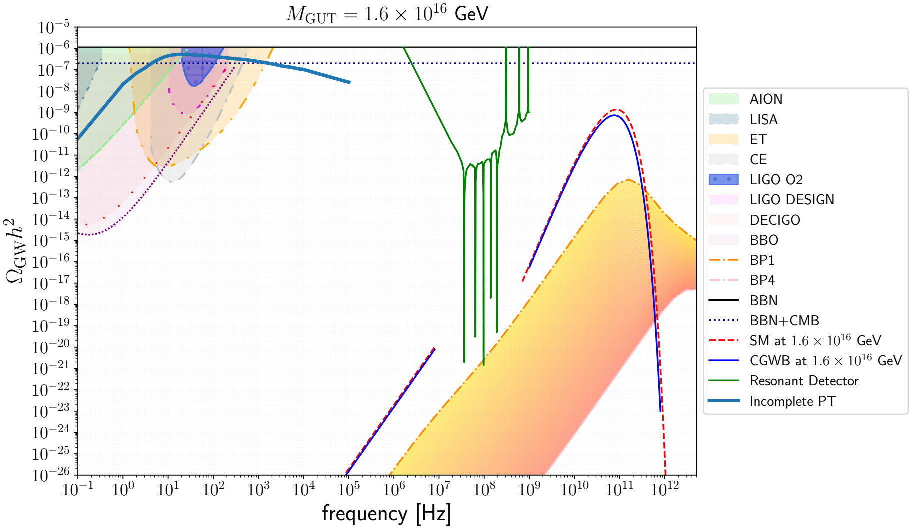
GW from Incomplete Phase Transition



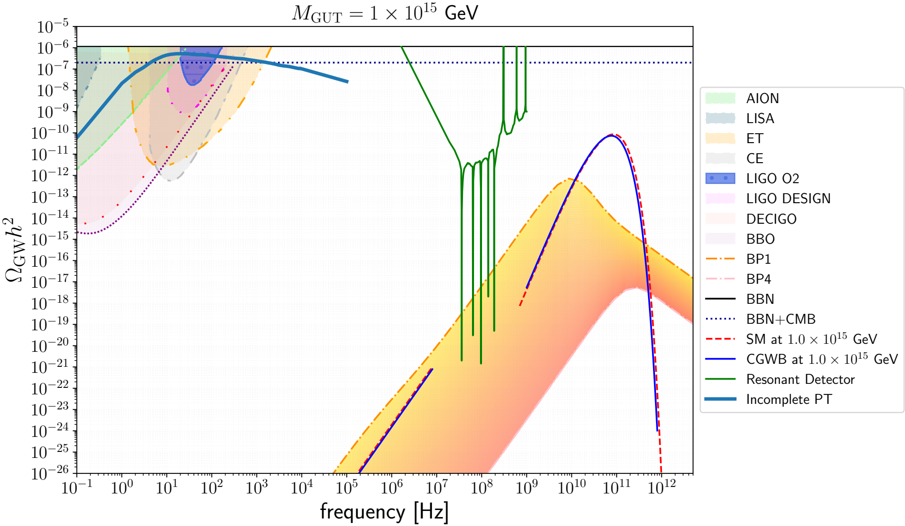
Adapted from Barir et al., PRD **108** (2023)

- During inflation: Bubbles form but do not fill whole space
- Phase transition completes after inflation
- Density inhomogeneities $\rightsquigarrow$ GW (non-linear effects at horizon re-entry)
- Low-frequency signal, depends on details of inflation

Overview of GW Signals



Overview of GW Signals



Conclusions and Outlook

- Unified framework for inflation and first-order phase transition
  - Dark $U(1)_X$ and non-minimal coupling to gravity
    $\rightsquigarrow$ gravitational waves and inflation from same scalar
  - Peak frequencies between 10^{-2} Hz and 10^{8} Hz
- Gravitational wave signals from $SO(10)$ Grand Unification
  - First-order phase transition, primordial plasma $\rightsquigarrow$ high frequencies
  - Incomplete phase transition during inflation $\rightsquigarrow$ low frequencies
- Future improvements
  - Calculation of effective potential via `DRalgo` (dimensional reduction)
    Ekstedt et al., Comput. Phys. Commun. **288** (2023)
  - Calculation of bubble wall velocity using `WallGo`
    Ekstedt et al., JHEP **04** (2025)
  - Gravitational waves from topological defects
  - Inflaton coupling to SM $\rightsquigarrow$ reheating
  - Dark Matter
  - Gravitational waves from later symmetry breaking steps

Parameters Determining the GW Signal

- T_n: temperature at which probability of nucleating 1 bubble per horizon volume is ~ 1
- $\alpha = \frac{1}{\rho_{\text{rad}}} \left[\Delta V(h_c, T) - T \frac{dV(h_c, T)}{dT} \right] \Big|_{T=T_n}$
- $\frac{\beta}{H(T_n)} = T_n \, \frac{d}{dT} \frac{S_3}{T} \Big|_{T_n} \simeq \left(\frac{T_n}{T - T_n} \right) \left(\frac{S_3(T)}{T} - \frac{S_3(T_n)}{T_n} \right)$
  (T: arbitrary reference temperature not too far from T_n)
- Expressions modified for large supercooling

Templates for GW Spectrum: Sound Waves

$$\Omega_{sw} h^2(f) = 2.65 \times 10^{-6}\, H_* \tau_{sw} \left(\frac{\beta}{H_*}\right)^{-1} v_w \times$$
$$\times \left(\frac{\kappa_\nu \alpha}{1+\alpha}\right)^2 \left(\frac{g_*}{100}\right)^{-\frac{1}{3}} \left(\frac{f}{f_{sw}}\right)^3 \left(\frac{7}{4+3\,(f/f_{sw})^2}\right)^{7/2}$$

$$f_{sw} = 1.9 \times 10^{-5}\, \frac{1}{v_w} \left(\frac{\beta}{H_*}\right) \left(\frac{T_n}{100\, GeV}\right) \left(\frac{g_*}{100}\right)^{1/6} \text{Hz}$$
$$\tau_{sw} = \min\left[\frac{1}{H_*}, \frac{R_*}{\bar{U}_f}\right]$$
$$H_* R_* = \max(v_w, c_s)\, (8\pi)^{1/3} (\beta/H_*)^{-1}$$
$$\bar{U}_f^2 \simeq \frac{3}{4}\left(\frac{\kappa_\nu \alpha}{1+\alpha}\right)$$
$$\kappa_\nu \simeq \begin{cases} \alpha\left(0.73 + 0.83\sqrt{\alpha} + \alpha\right)^{-1} & v_w \sim 1 \\ v_w^{6/5}\, 6.9\alpha\left(1.36 - 0.037\sqrt{\alpha} + \alpha\right)^{-1} & v_w \ll 1 \end{cases}$$

Templates for GW Spectrum: MHD turbulence

$$\Omega_{\text{turb}} h^2(f) = 3.35 \times 10^{-4} \left(\frac{\beta}{H_*}\right)^{-1} v_w \times$$
$$\times \left(\frac{\epsilon\,\kappa_\nu \alpha}{1+\alpha}\right)^{\frac{3}{2}} \left(\frac{g_*}{100}\right)^{-\frac{1}{3}} \frac{(f/f_{\text{turb}})^3 \, (1+f/f_{\text{turb}})^{-\frac{11}{3}}}{1+8\pi f/h_*}$$

$$h_* = 16.5 \frac{T_n}{10^8\,\text{GeV}} \left(\frac{g_*}{100}\right)^{1/6} \text{Hz}$$
$$f_{\text{turb}} = 2.7 \times 10^{-5} \frac{1}{v_w} \left(\frac{\beta}{H_*}\right) \left(\frac{T_n}{100\,\text{GeV}}\right) \left(\frac{g_*}{100}\right)^{1/6} \text{Hz}$$
$$\epsilon = 0.05$$

Gravitational Waves from Plasma

Ghiglieri, Laine, JCAP 07 (2015)

$$\Omega_{GW}(f, T_0)\, h^2 =$$

$$\Omega_{\gamma_0} h^2 \frac{\lambda}{M_P} \int_{T_{EW}}^{T_{Max}} dT \left(\frac{g_{*0}}{g_*(T)} \right)^{4/3} T^2 \, \hat{k}(f, T)^3 \frac{\eta(\hat{k}, T)}{\sqrt{\rho(T)}} \beta(T)$$

$\Omega_{GW}(f, T_0)$: fraction of energy released into GW per frequency octave

$$\lambda = 30\sqrt{3}/\pi^4$$

$$\hat{k}(f, T) \equiv \frac{k}{T} = \left[\frac{g_{*s}(T)}{g_{*s}(T_0)} \right]^{\frac{1}{3}} \frac{2\pi f}{T_0}$$

$$f = \frac{1}{2\pi} \left[\frac{g_{*s}(T_0)}{g_{*s}(T_{EW})} \right]^{\frac{1}{3}} \left(\frac{T_0}{T_{EW}} \right) k_{EW}$$

$$T_{EW} = 160\,\text{GeV}$$

$$k_{EW} = k(T)\,(g_{*s}(T_{EW})/g_{*s}(T))^{1/3}\, T_{EW}/T$$

Shear Viscosity

Simple functional form in terms of Debye masses

$$\eta(\hat{k}, T) = \begin{cases} \frac{1}{8\pi} \frac{16}{g_1^4 \ln(5T/m_{D_1})} & \hat{k} \lesssim \alpha_1^2 \\ \eta_{\mathrm{HTL}}(\hat{k}, T) + \eta^T(\hat{k}, T) & \hat{k} \gtrsim 3 \end{cases}$$

η_{HTL}: Hard Thermal Logarithmic (HTL) expression

Ghiglieri, Jackson, Yu, Laine, Zhu, JHEP 07 (2020)

$$\eta_{\mathrm{HTL}}(\hat{k}, T) = \frac{1}{16\pi} \hat{k} n_B(\hat{k}) \sum_{n=1}^{m} d_n \hat{m}^2_{D_{\mathcal{G}_n}} \ln\left(4 \frac{\hat{k}^2}{\hat{m}^2_{D_{\mathcal{G}_n}}} + 1\right)$$

Debye Masses of Group Factor $\mathcal{G}_n$

Weldon, PRD **26** (1982)

$$m^2_{D_{\mathcal{G}_n}}(T) = g_n^2(T)\, T^2 \left[\frac{1}{3} S(\mathrm{Adj}) + \frac{1}{6} \sum_i S(R_{F_i}) + \frac{1}{6} \sum_j S(R_{S_j}) \right]$$

- S: Dynkin index of representation (adjoint representation Adj for gauge bosons, i-th fermion representation R_{F_i}, j-th real scalar representation R_{S_j})
- $g_n(T)$: temperature-dependent gauge coupling

GW from Incomplete Phase Transition

- PT driven by spectator field χ
- $V_{\mathrm{PT}}(\chi, T) \ll V_{\mathrm{Inf}}(\phi_{\mathrm{Inf}})$,
- Signal governed by dimensionless parameter

$$\gamma_{\mathrm{PT}} = \frac{1}{H^4} \frac{\Gamma}{V} \left(\frac{\Delta V_{\mathrm{PT}}}{\dot{\phi}_{\mathrm{Inf}}^2} \right)^2$$

$$\Gamma \simeq T^4 \left(\frac{S_3}{2\pi T} \right)^{\frac{3}{2}} \exp\left(-S_3/T\right)$$

- Used $\gamma_{\mathrm{PT}} = 10^{-4}$ ($\sim$ upper limit) in overview plot